Sharp Bounds on Davenport-Schinzel Sequences of Every Order

Seth Pettie

A Davenport-Schinzel sequence with order s is a sequence over an n-letter alphabet that avoids subsequences of the form $a..b..a..b..$ with length $s+2$. They were originally used to bound the complexity of the lower envelope of degree-s polynomials or any class of functions that cross at most s times. They have numerous applications in discrete geometry and the analysis of algorithms.
Let $DS_s(n)$ be the maximum length of such a sequence. In this talk I'll present a new method for obtaining sharp bounds on $DS_s(n)$ for every order s. This work reveals the unexpected fact that sequences with odd order s behave essentially like even order $s-1$. The results refute both common sense and a conjecture of Alon, Kaplan, Nivasch, Sharir, and Smorodinsky [2008]. Prior to this work, tight upper and lower bounds were only known for s up to 3 and even $s > 3$.