Generalized multiplicative Sidon-sequences

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As a generalization of multiplicative Sidon-sequences we investigate the following question: What is the maximal number of elements which can be chosen from the set $\{1, 2, \ldots, n\}$ in such a way that the equation $a_1a_2 \ldots a_k = b_1b_2 \ldots b_k$ does not have a solution of distinct elements? Let us denote this maximal number by $G_k(n)$. Erdős studied the case $k = 2$: In 1938 he proved that $\pi(n) + c_1n^{3/4}/(\log n)^{3/2} \leq G_2(n) \leq \pi(n) + c_2n^{3/4}$ and 31 years later improved the upper bound to $\pi(n) + c_2n^{3/4}/(\log n)^{3/2}$. Hence, in the lower- and upper bounds for $G_2(n)$ not only the main terms are the same, but the error terms only differ by a constant factor. We study $G_k(n)$ for $k > 2$, give asymptotically precise bounds for every k, and prove some estimates on the error terms.

To estimate $G_k(n)$ extremal graph theoretic results are used, namely results about the maximal number of edges of C_{2k}-free graphs and of such C_{2k}-free bipartite graphs, where the number of vertices in the two classes are fixed.

Note that our question is strongly connected to the following problem: Erdős, Sárközy, T. Sós and Győri investigated how many numbers can be chosen from $\{1, 2, \ldots, n\}$ in such a way that the product of any $2k$ of them is not a perfect square. The maximal size of such a subset is denoted by $F_{2k}(n)$. The functions F and G clearly satisfy the inequality $F_{2k}(n) \leq G_k(n)$.