Some Covering Problems in Geometry

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We discuss variations of the following problem: given a set in Euclidean n-space (resp. on the sphere). Bound the minimum number of translates (resp. rotated copies) that cover another given set (resp. the sphere). We present a method to obtain upper bounds for these problems. As applications of this method, we generalize some results of Rogers, and sharpen an estimate by Artstein–Avidan and Slomka. The key idea which makes our proofs rather simple and uniform throughout distinct geometric settings is the application of an algorithmic result of Lovász as opposed to the probabilistic approach taken by others.
If time permits, we discuss a lower bound, too. We consider the illumination problem (the problem of covering a convex body by translates of its interior). By a probabilistic argument, we show that arbitrarily close to the Euclidean ball there is a centrally symmetric convex body of illumination number exponentially large in the dimension.