Saturation Games

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Let $\mathcal{P}$ be a monotone increasing graph property and let G be a graph on n vertices which does not satisfy $\mathcal{P}$. An edge $e \in K_n \setminus G$ is called *legal* (with respect to G and $\mathcal{P}$) if $G \cup \{e\}$ does not satisfy $\mathcal{P}$. In the saturation game $(n, \mathcal{P})$ two players, called Shorty and Prolonger, build together a graph $G \subseteq K_n$ which does not satisfy $\mathcal{P}$. Shorty and Prolonger take turns is claiming legal edges (starting from the empty graph on n vertices) until none exist. At this point the game is over, and the resulting graph G is said to be $\mathcal{P}$ *saturated*. The *score* of the game is the number of edges in G at the end of the game. Shorty's goal is to minimize the score of the game, while Prolonger's goal it to maximize the score of the game.

We analyze saturation games for several graph properties, including $\mathcal{P} =$ "having chromatic number at least k" and $\mathcal{P} =$ "containing a k-matching", showing some surprising results.

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