On Saturated k-Sperner Systems

Natasha Morrison, Oxford University

(joint work with Jonathan Noel and Alex Scott)

Given a set X, a collection $\mathcal{F} \subseteq \mathcal{P}(X)$ is said to be *k-Sperner* if it does not contain a chain of length $k+1$ under set inclusion and it is *saturated* if it is maximal with respect to this property. Gerbner, Keszegh, Lemons, Palmer, Pálvölgyi and Patkós conjectured that, if $|X|$ is sufficiently large with respect to k, then the minimum size of a saturated k-Sperner system $\mathcal{F} \subseteq \mathcal{P}(X)$ is 2^{k-1}. In this talk we disprove this conjecture by showing that there exists $\varepsilon > 0$ such that for every k and $|X| \geq n_0(k)$ there exists a saturated k-Sperner system $\mathcal{F} \subseteq \mathcal{P}(X)$ with cardinality at most $2^{(1-\varepsilon)k}$.