On Saturated $k$-Sperner Systems

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(joint work with Jonathan Noel and Alex Scott)

Given a set $X$, a collection $\mathcal{F} \subseteq \mathcal{P}(X)$ is said to be $k$-Sperner if it does not contain a chain of length $k + 1$ under set inclusion and it is saturated if it is maximal with respect to this property. Gerbner, Keszegh, Lemons, Palmer, Pálvölgyi and Patkós conjectured that, if $|X|$ is sufficiently large with respect to $k$, then the minimum size of a saturated $k$-Sperner system $\mathcal{F} \subseteq \mathcal{P}(X)$ is $2^{k-1}$. In this talk we disprove this conjecture by showing that there exists $\varepsilon > 0$ such that for every $k$ and $|X| \geq n_0(k)$ there exists a saturated $k$-Sperner system $\mathcal{F} \subseteq \mathcal{P}(X)$ with cardinality at most $2^{(1-\varepsilon)k}$. 