On the number of K_4-saturating edges

Hong Liu, UIUC

(Joint work with József Balogh.)

Let G be a K_4-free graph, an edge in its complement is a K_4-*saturating* edge if the addition of this edge to G creates a copy of K_4. Erdős and Tuza conjectured that for any n-vertex K_4-free graph G with $\lfloor n^2/4 \rfloor + 1$ edges, one can find at least $(1+o(1))\frac{n^2}{16}$ K_4-saturating edges. We construct a graph with only $\frac{2n^2}{33}$ K_4-saturating edges. Furthermore, we prove that it is best possible, i.e., one can always find at least $(1+o(1))\frac{2n^2}{33}$ K_4-saturating edges in an n-vertex K_4-free graph with $\lfloor n^2/4 \rfloor + 1$ edges.