Spanning quadrangulations of triangulated surfaces

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(joint work with Carsten Thomassen)

While on any fixed surface there are only finitely many minimal graphs that are not 5-vertex-colorable, there is no such characterization for 4-vertex-coloring on any surface other than the sphere. On the positive side, a triangulation of a surface is 4-vertex-colorable if and only if the edges can be labeled with 3 colors such that the union of any two color classes forms a bipartite spanning quadrangulation. We explore this idea by establishing connections between spanning quadrangulations and cycles in the dual graph which are noncontractible and alternating with respect to a perfect matching.
We show that the dual graph of an Eulerian triangulation of an orientable surface other than the sphere has a perfect matching M and an M-alternating noncontractible cycle. As a consequence, every Eulerian triangulation of the torus has a nonbipartite spanning quadrangulation. For an Eulerian triangulation G of the projective plane the situation is different: If the dual graph of G is nonbipartite, then it has no noncontractible alternating cycle, and all spanning quadrangulations of G are bipartite. If the dual graph of G is bipartite, then it has a noncontractible, M-alternating cycle for some (and hence any) perfect matching M, and thus G has a bipartite spanning quadrangulation and also a nonbipartite spanning quadrangulation.