Turán problems and shadows

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(joint work with Dhruv Mubayi and Jacques Verstraëte)

The *Turán number*, $\text{ex}_r(n, F)$ of an r-uniform hypergraph F is the maximum number of edges in an r-uniform hypergraph with n vertices not containing copies of F. All the four Honorees of the meeting have strong results on Turán numbers. The expansion G^+ of a graph G is the 3-uniform hypergraph obtained from G by enlarging each edge of G with a vertex disjoint from $V(G)$ such that distinct edges are enlarged by distinct vertices.

We determine $\text{ex}_3(n, G^+)$ exactly when G is a path or cycle, thus settling conjectures of Füredi and Jiang (for cycles) and Füredi, Jiang and Seiver (for paths). We find the asymptotics for $\text{ex}_3(n, G^+)$ when G is any fixed forest. This settles a conjecture of Füredi. We also show that for each graph G, either $\text{ex}_3(n, G^+) \leq \left(\frac{1}{2} + o(1)\right) n^2$ or $\text{ex}_3(n, G^+) \geq (1 + o(1))n^2$, thereby exhibiting a jump for the Turán number of expansions. In addition, for the graph Q_3 of the 3-dimensional unit cube, we show $\text{ex}_3(n, Q_3) = \Theta(n^2)$.