$K_{(s,t)}$-saturated bipartite graphs

Dániel Korándi, ETH Zürich

(joint work with Wenying Gan and Benny Sudakov)

An n-by-n bipartite graph is H-saturated if the addition of any missing edge between its two parts creates a new copy of H. In 1964, Erdős, Hajnal and Moon made a conjecture on the minimum number of edges in a $K_{(s,s)}$-saturated bipartite graph. This conjecture was proved independently by Wessel and Bollobás in a more general, but ordered, setting: they showed that the minimum number of edges in a $K_{(s,t)}$-saturated bipartite graph is $n^2-(n-s+1)(n-t+1)$, where $K_{(s,t)}$ is the "ordered" complete bipartite graph with s vertices in the first color class and t vertices in the second. However, the very natural question of determining the minimum number of edges in the unordered $K_{(s,t)}$-saturated case remained unsolved. This problem was considered recently by Moshkovitz and Shapira who also conjectured what its answer should be. We give a bound on the minimum number of edges in a $K_{(s,t)}$-saturated bipartite graph that is only smaller by an additive constant than the conjectured value. In this talk we sketch the ideas behind the proof.