Polynomial Time Algorithms for the 3-Dimensional VLSI Routing in the Cube

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Abstract. In previous works some polynomial time algorithms were presented for special cases of the 3-Dimensional VLSI Routing problem. Solutions were given to problems when all the terminals are either on a single face (SALP - Single Active Layer Problem) or on two opposite faces (3DCRP - 3-Dimensional Channel Routing Problem) or on two adjacent faces (3DΓRP - 3-Dimensional Gamma Routing Problem) of a rectangular cuboid. We prove that combining these algorithms one can solve any given problem on cubes and we give some polynomial time algorithms to find these solutions.

Keywords: VLSI design, 3-dimensional routing, Steiner-tree

Routing in the design of VLSI (Very Large Scale Integrated) circuits is an important area of modern applied mathematics, in particular combinatorial optimization. There were a lot of interesting results in this area in the last four decades. Although more and more problems are proved to be NP-complete, there are a lot of heuristic solutions, approximating the optimum of these problems with a good rate of efficiency (for a further view read [1]). There are many well known technologies to construct electric circuits for this model. We give a new theoretical model that can be a future direction in the development of new circuits. We would like to construct "routing boxes" that means one can place terminals in all the faces of a rectangular cuboid formed by the circuit boards layered together.
From a graph-theoretical point of view the 2-dimensional detailed routing problems (in particular, the most often studied channel routing and switchbox routing problems) search for vertex-disjoint Steiner-trees (trees with given sets containing specific terminals) on a (2-dimensional) square grid while the 3-dimensional ones search on a (3-dimensional) cu-

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bic grid.
Since even the Channel Routing Problem (when all the terminals are on two opposite boundaries of the square grid) cannot always be solved, one has to introduce several parallel layers. In the last four decades hundreds or perhaps thousands of papers studied the possibilities of routing a channel or switchbox using a possibly small number of layers. However, no universal constant exists for the number of layers to make every switchbox routing (with any size and shape) possible.
Similarly, even the simplest 3-dimensional problem (the Single Active Layer Routing Problem, when all the terminals are on a single external face of the cubic grid) cannot always be solved, one has to make the grid "finer". A *spacing* of size s in a given direction means that we introduce s extra lines between any two consecutive lines in that direction (plus s extra lines after the last original one). In a complete analogy with the 2-dimensional case, no universal refinement (that is, maximal spacing size) will do for 3-dimensional switchboxes of any size and shape.
In this paper we prove the existence of a universal spacing size if the switchbox is a cube (of arbitrary size). Our upper bound will be of theoretical interest only but we expect that it can drastically be reduced (we successfully decreased the size of the spacing in some special cases).
To our best knowledge three special cases of the 3-dimensional switchbox routing problem have already been studied, namely: Single Active Layer Routing Problem [2],[3], 3-Dimensional Channel Routing Problem [4], 3-Dimensional Γ Routing Problem [5],[6]. In addition to these three subproblems we have six further cases. We proved the following:

Theorem 1. *Combining the methods used by the three previous subproblems all the six new cases on a cube can be solved with a fixed maximum number of spacings needed. Our solution can be found in polynomial time.*

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