The clique, independence and chromatic numbers of random subgraphs of distance graphs

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Our talk is concerned with the classical Nelson–Hadwiger problem on finding the chromatic numbers of distance graphs in $\mathbb{R}^n$. We introduce a class of graphs $G(n,r,s) = (V(n,r), E(n,r,s))$ defined as follows:

$$V(n,r) = \{x = (x_1, x_2, \ldots, x_n) : \; x_i \in \{0,1\}, \; x_1 + x_2 + \ldots + x_n = r\}, \quad E(n,r,s) = \{\{x,y\} : (x,y) = s\},$$

where (x,y) is the Euclidean scalar product.

We study the random graphs $\mathcal{G}(G(n,r,s),p)$ whose edges are chosen independently from the set $E(n,r,s)$ each with probability p. We obtain sharp asymptotic bounds for the clique, independence and chromatic numbers of such graphs depending on some relations between the parameters n, r, s.