Maximum measures of spherical sets avoiding orthogonal pairs of points

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(joint work with Oleg Pikhurko.)

Let a_n be the supremum of the Lebesgue (surface) measure of I, where I ranges over all measurable sets of unit vectors in R^n such that no two vectors in I are orthogonal, and where the surface measure is normalized so that the whole sphere gets measure 1. The problem of determining a_n was first stated in a 1974 note by H. S. Witsenhausen, where he gave the upper bound of $1/n$ using a simple averaging argument. In a 1981 paper by Frankl and Wilson, they prove their well-known theorem and use it to attack this problem; there it was shown that a_n decreases exponentially. In this talk, we focus on the case $n = 3$, where we improve Witsenhausen's $1/3$ upper bound to 0.313. The proof involves some basic harmonic analysis and infinite-dimensional linear programming.