Matroid union,
Graphic? Binary? Neither?

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Graphic matroids form one of the most significant classes in matroid theory. When introducing matroids, Whitney concentrated on relations to graphs. The definition of some basic operations like deletion, contraction and direct sum were straightforward generalizations of the respective concepts in graph theory. Most matroid classes, for example those of binary, regular or graphic matroids, are closed with respect to these operations. This is not the case for the union (also referred to as sum). The union of two graphic matroids can be nongraphic.

The first paper in this area was that of Lovász and Recski: they examined the case if several copies of the same graphic matroid are given. Then Recski conjectured thirty years ago that if the union of graphic matroids is not graphic then it is nonbinary. He also studied the case if we fix one simple graphic matroid and take its union with every possible graphic matroid.

If there are two matroids and the first one can be drawn as a graph with two points, then a necessary and sufficient condition is given for the other matroid to ensure the graphicity of the union. A similar case has been proved where the first matroid is a circuit with loops and bridges.

Theorem. *If $M(G_0)$ consists of loops and a single circuit of length n or n parallel edges ($n \geq 2$) and $M(G_1)$ is an arbitrary graphic matroid in the same ground set then the graphicity of the union can be decided in polynomial time.*

Applying some steps of the proof of this theorem we also prove that the above conjecture holds for these cases.

One can ask further questions about the classes formed by those graphic (or arbitrary) matroids whose union with any graphic (or arbitrary) matroid is graphic (or either graphic or nonbinary). These 2^3 variations define 8 matroid classes. We present some results about their relations and properties.

Acknowledgement: Part of the research has been supported by the grant OTKA-108947.