An efficient Extension for Sperner families: based upon from m-cardination Sperners sets to m+1-cardination

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Abstract

Sperner family (or Sperner system) is a set system {S(E)}where S(E), the Sperner set with the generating set E, is a set in which no element is contained in another. Formally, S(E) is composed of the elements, i.e. Sperner elements, from power set of E and for any different *X*, *Y* in S(E), *X* is not contained in *Y* and *Y* is not contained in *X*. Great efforts have been made to compute or estimate the number N, the cardination of the{S(E)} with n, the cardination of E. N increases rapidly with n. For example, N=7579 for n=5 and N=7828352 for n=6. So far the largest N is known for n=7. Therefore, the construction of Sperner family for large n is still a changeling problem in present days.

It is noted that the Sperner family is the set of the all Sperner sets for a specified E. We therefore classify the Sperner family into different categories (sub-family) by the cardination of the Sperner set. Furthermore, we recursively construct the Sperner set with cardination increasing from m to m+1. A direct recursive computation from m to m+1 is to check the 2^n different 1-cardination Sperner elements for compatibilities with the interested m-cardination Sperner sets, which costs $2^n * m$ inclusion comparisons. Instead, we find the compatibility for a specific child Sperner element from its possibly r younger brothers with a same parent of m-1-cardination Sperner set. It is noted that the above specific child is corresponding to a specific Sperner set of m-cardination and thus reduces the inclusion comparisons from 2 n * m to r instead of C(r, 2), selecting 2 from r. The number r is small in general and m is from 1 to C(n, (n-1)/2) according to the Sperner theorem.

A proof of correctness for no missing Sperners set and no repeating one in extension by the proposed strategy is also discussed in this paper. An example diagram is also provided for illustrating the realization idea.

Keywords: Sperner family, Sperner set, Sperner element, recursive computation

An example diagram for illustrating the realization idea:

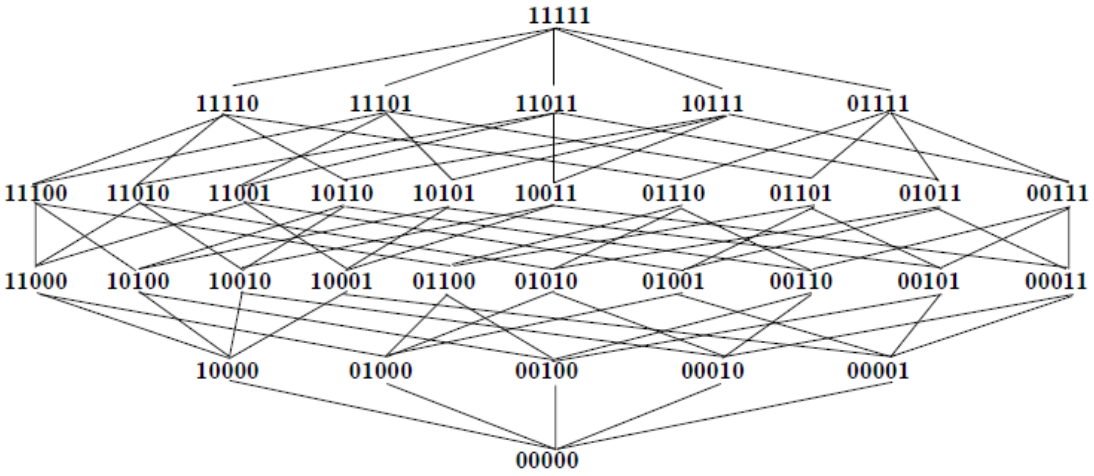

Fig. 1 Sperner elements (n=5) with their containing relationship represented in graph

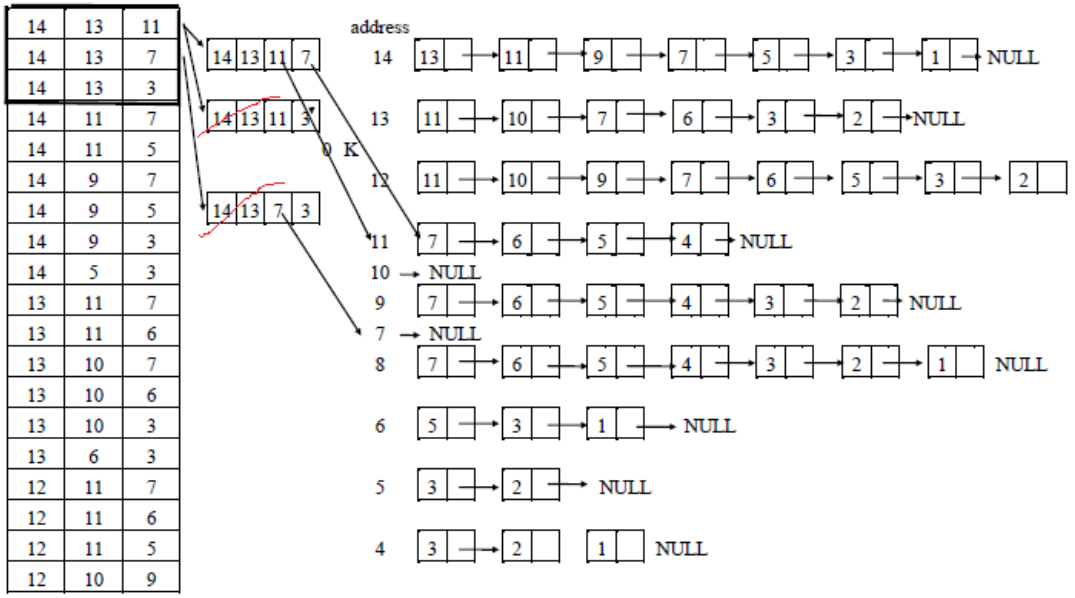

Fig. 2 One instance of Sperner set extension from 3-cardination to 4-cardination: The first extension is successful but the second and the third are not. (14 = 01110, …..)