Copies of fixed graphs in random distance graphs

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The problem of appearance of an arbitrary fixed graph in Erdős–Rényi random graph $G_{n,p}$ was studied thoroughly by a number of authors (see, e.g., [1]). For a certain model of random distance graphs this issue was investigated by M. Zhukovskii in [2]. We consider a class of distance graphs $G(n,r,s) = (V(n,r), E(n,r,s))$ defined as follows:

$$V(n,r) = \{\mathbf{x} = (x_1, \ldots, x_n) \colon x_i \in \{0,1\},\ x_1 + \ldots + x_n = r\}, \quad E(n,r,s) = \{\{\mathbf{x},\mathbf{y}\} \colon (\mathbf{x},\mathbf{y}) = s\},$$

where $(\mathbf{x},\mathbf{y})$ is the Euclidean scalar product.

We study the random distance graphs $\mathcal{G}(G(n,r,s),p)$ where each edge from the set $E(n,r,s)$ is included in the graph with probability p independent of other edges. We find the threshold probabilities for the property of containing a fixed graph and investigate the distribution of the number of subgraphs isomorphic to a given graph when the probability p is critical. We also find the theshold probabilities for planarity in these graphs.

References

[1] B. Bollobás, *Random graphs*, 2nd Edition, Cambridge Stud. Adv. Math., 73, Cambridge Univ. Press, Cambridge, 2001, section 4: Small Subgraphs.

[2] M. E. Zhukovskii, *On the probability of the occurrence of a copy of a fixed graph in a random distance graph*, Math. Notes, 2012, 92:6, 756–766 (Russian original: Mat. Zametki, 2012, 92:6, 844–855).

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