# The Time of Bootstrap Percolation – Extremal and Probabilistic Results

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Classical $r$-neighbour bootstrap percolation, introduced by Chalupa, Leath and Reich in 1979, can be viewed as an oversimplified model of the spread of an infection: given a graph $G$ and a set $A_0$ of 'infected' vertices at time 0, for $t \geq 0$ we define

$$A_{t+1} = A_t \bigcup \big\{x \in V(G) : \ x \text{ has at least } r \text{ neighbours in } A_t\big\}$$

to be the set of infected vertices at time $t+1$. The set $A_0$ is said to *percolate* if its *closure*, $[A_0] = \bigcup_t A_t$, is the entire vertex set $V(G)$. The *percolation time* of $A_0$ is

$$T(A_0) = \min\{t : \ A_t = V(G)\}.$$

Most of the work in the last thirty years has been about what happens when the initial set $A_0$ is chosen at random, with major contributions by Aizenman, Balogh, Bollobás, Cerf, Cirillo, Duminil-Copin, Holroyd, Lebowitz, Manzo, Morris, and others.

In the first half of this talk I shall sketch the most significant of these results, and then I shall turn to some recent work of Balister, Benevides, Bollobás, Holmgren, Przykucki and Smith, emphasizing the extremal problems that arise.

The second part of the talk concerns *graph bootstrap percolation*, a rather different kind of bootstrap percolation I introduced in 1968 under another name, about which the first beautiful results were obtained by Frankl, Kalai and Alon in the 1980s. My aim is to say a little about a number of recent extremal and probabilistic results of Balogh, Bollobás, Koch, Morris and Przykucki.