Partition regularity and the columns property

Ben Barber, University of Birmingham

A system of linear equations with integer coefficients is partition regular if, whenever the natural numbers are finitely coloured, there is a monochromatic solution. In 1933 Rado showed that a finite system of equations is partition regular if and only if its matrix of coefficients has the "columns property".

It is easy to write down infinite systems which have the columns property but are not partition regular. However, all known examples of infinite partition regular systems do have the columns property. Must all infinite partition regular systems have the columns property?