On the typical structure of sum-free sets.

József Balogh, Szeged University and UIUC
(Based on joint work results with Alon, Morris, Samotij and Warnke)

First we study sum-free subsets of the set $\{1, \ldots, n\}$, that is, subsets of the first n positive integers which contain no solution to the equation $x + y = z$. Cameron and Erdős conjectured in 1990 that the number of such sets is $O(2^{n/2})$. This conjecture was confirmed by Green and, independently, by Sapozhenko. We prove a refined version of their theorem, by showing that the number of sum-free subsets of $[n]$ of size m is $2^{O(n/m)}\binom{\lceil n/2 \rceil}{m}$, for every $1 \leq m \leq \lceil n/2 \rceil$. For $m \geq \sqrt{n}$, this result is sharp up to the constant implicit in the $O(\cdot)$. Our proof uses a general bound on the number of independent sets of size m in 3-uniform hypergraphs, proved recently by the authors, and new bounds on the number of integer partitions with small sumset.
Then we study sum-free sets of order m in finite Abelian groups. We determine the typical structure and asymptotic number of sum-free sets of order m in Abelian groups G whose order n is divisible by a prime q with $q \equiv 2 \pmod 3$, for every $m \geq C(q)\sqrt{n \log n}$, thus extending and refining a theorem of Green and Ruzsa. In particular, we prove that almost all sum-free subsets of size m are contained in a maximum-size sum-free subset of G.
Finally, we explain connection with recent "independent sets in hypergraph" general theorems, and describing typical structure of graphs.
In the talk I try to have different approach from other talks on "independent sets in hypergraph" general theorems.