Ramsey numbers of ordered graphs

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(joint work with Karel Kral and Jan Kyncl)

An ordered graph $G_<$ is a graph G with vertices ordered by the linear ordering $<$. The ordered Ramsey number $R(G_<, c)$ is the minimum number N such that every ordered complete graph with c-colored edges and at least N vertices contains a monochromatic copy of $G_<$.
For unordered graphs it is known that Ramsey numbers of graphs with degrees bounded by a constant are linear with respect to the number of vertices. In contrast with this result we show that there are arbitrarily large ordered matchings $M_<(n)$ on n vertices for which $R(M_<(n), 2)$ grows super-polynomially in n. This implies that ordered Ramsey numbers of the same graph can grow super-polynomially in the size of the graph in one ordering and remain polynomial in another ordering.
We also prove that for every ordered graph its ordered Ramsey number grows either polynomially or exponentially in the number of colors.
For a few special classes of ordered paths, stars or matchings, we give asymptotically tight bounds on their ordered Ramsey numbers. For so-called monotone cycles we compute their ordered Ramsey numbers exactly. This result implies exact formulas for geometric Ramsey numbers of cycles introduced by Károlyi et al.