Abstract potential theory and applications to rendezvous numbers

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We analyze relations between various forms of energies (reciprocal capacities), the transfinite diameter, various Chebyshev constants and the so called rendezvous or average number. The latter is originally defined for compact connected metric spaces (X, d) as the (in this case unique) nonnegative real number r with the property that for arbitrary finite point systems $\{x_1, \ldots, x_n\} \subset X$, there exists some point $x \in X$ with the average of the distances $d(x, x_j)$ being exactly r. Existence of such a miraculous number has fascinated many people; its normalized version was even named "the magic number" of the metric space. Exploring related notions of general potential theory, as set up, e.g., in the fundamental works of Fuglede and Ohtsuka, we present an alternative, potential theoretic approach to rendezvous numbers and thereby arrive at understanding how more general principles explain the existence and uniqueness of these miraculous numbers. In particular, we generalize and explain results on invariant measures, hypermetric spaces and maximal energy measures, when showing how more general, potential analytic, proofs can be found to them. The talk will focus therefore on the following topics:

- The development of the Choquet-Fuglede-Ohtsuka theory (some elementary parts of it)
- Rendezvous numbers and their generalisations (like weak rendezvous numbers and co.); Rendezvous intervals
- Capacitary measures, the maximum principle and their relations to rendezvous numbers