The characterization of NP within interval-valued computing

Benedek Nagy
Department of Computer Science
Faculty of Informatics, University of Debrecen
nbenedek@inf.unideb.hu

Sándor Vályi
Faculty of Natural Sciences and Informatics
College of Nyíregyháza
valyis@nyf.hu

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Abstract

In [CiE2005], B. Nagy introduced a new computing paradigm, namely, interval-valued computing. It uses finite unions of subintervals of [0,1) as basic data processing unit (known as generalized intervals). While Turing machines are abstractions where the memory size can be arbitrarily extended in its length, this system assumes the unlimited density of data units. In [TCS2008], the notion of interval-valued computations was formalized. A computation is a sequence of operator applications on generalized intervals (interval-values). Operators are the usual Boolean ones, two shifts (left and right) and a so-called product which works as zooming. A discrete language L is decidable by (linear, polynomial, etc.) interval-valued computation if and only if there is a classical logspace algorithm that for any input word w constructs an interval-valued computation (of a size that linearly, polynomially depends on the length of w) that ends with a nonempty interval-value if and only if w is in L. This tastes like Boolean network computing style but not on discrete bit sequences but on full interval-values. In the same paper we showed that QSAT (the language of true quantified Boolean formulae) can be decided by a linear interval-valued computation and that PSPACE coincides with tha languages decidable by a polynomial size *restricted* interval-valued computations where restricted means that product operator may occur only as product by [0,1/2).

In [VLL2007] we demonstrated the connections of this system to visual computing. In [PMD2011] and in an other paper (accepted for publication, presented on [DCM2012]), we defined the notion of computable function by interval-valued computing and showed two functions to be com-

putable by polynomial length interval-valued computations which were interesting for the cryptography audience. The methods of these papers are relatively simple and a summarizing conclusion can be drawn: exactly languages in NP can be decided by a specific type of interval-valued computations, which is characterized by the following conditions:

- the computation starts with generation of all the posible witnesses
- and continues only with Boolean operators

This observation is just the interval-valued counterpart of the characterization of NP by languages having polynomially checkable witnesses. The main purpose of this talk is to acquaint this paradigm with the logic community.

Acknowledgements

Dedicated to the 70th birthday of István Németi.

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