Tutorial on logical analysis of relativity theories

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In this talk we will overview the axiomatic framework for relativity theories developed by our research school lead by Hajnal Andréka and István Németi.

Our school's general aims are to axiomatize relativity theories using simple, comprehensible and transparent basic assumptions (axioms); and to prove the surprising predictions (theorems) of relativity theories using a minimal number of convincing axioms. We are building a whole net-like hierarchy of axiom systems and logical connections between them. We not only axiomatize relativity theories, but also analyze their logical and conceptual structures and, in general, investigate them in various ways (using our logical framework as a starting point).

In physics the role of the axioms (the role of basic statements that we assume without proofs) is even more fundamental than in mathematics. That is why we aim to formulate natural, simple and convincing axioms. All the surprising or unusual statements should be provable as theorems and not assumed as axioms. For example, the statement "no observer can move faster than light" is a theorem in our approach and not an axiom.

We work within first-order logic for several reasons, e.g., because it can be viewed as a fragment of natural language with unambiguous syntax and semantics. Being a *fragment of natural language* is useful in our project because one of our aims is to make relativity theory accessible to a broad audience. *Unambiguous syntax and semantics* are important for the same reason, because they make it possible for the reader to always know what is stated and what is not stated by the axioms. Therefore, they can use the axioms without being familiar with all the tacit assumptions and rules of thumb of physics (which one usually learns via many, many years of practice).

A novelty in our approach is that we try to keep the transition from special relativity to general relativity logically transparent and illuminating. We are going to “derive” the axioms of general relativity from those of special relativity in two natural steps. In the first step we extend special relativity of inertial observers to accelerated observers. In the second step we eliminate the difference between inertial and noninertial observers in the level of axioms. This second natural step provides a first-order logic axiomatization of general relativity suitable for further extensions and logical analysis.

Some of the questions we study to clarify the logical structure of relativity theories are:

- What is believed and why?
- Which axioms are responsible for certain predictions?
- What happens if we discard some axioms?
- Can we change the axioms and at what price?

Among others, logical analysis makes relativity theory modular: we can replace some axioms with other ones, and our logical machinery ensures that we can continue working in the modified theory. This modularity might come handy, e.g., when we want to extend general relativity and quantum theory to a unified theory of quantum gravity.