Dedicated to colleague and friend George J. Spix whose interest to foundation of Special relativity is inspirational for us

On the different forms of the electromagnetic equations in a uniform medium, an alternative to the Minkowski theory

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Two known, alternative to each other, forms of presenting the Maxwell electromagnetic equations in a moving uniform medium are investigated and discussed. Approach commonly used after Minkowski is based on the two tensors: $H^{ab} = (\mathbf{D}, \mathbf{H}/c)$ and $F^{ab} = (\mathbf{E}, c\mathbf{B})$; relationships between fields $\mathbf{D} = \epsilon_0\epsilon\mathbf{E}, \mathbf{B} = \mu_0\mu\mathbf{H}$ change their form at Lorentz transformations and have the form of the Minkowski equations. Minkowski equations may be written in covariant form

$$H_{ab} = \Delta_{abmn}\, F^{mn} \;, \qquad \Delta_{abmn} = \epsilon_0\epsilon k^2\, g_{am}\, g_{bn} + \epsilon_0\epsilon(k^2-1)\, u_n(g_{bm}u_a - g_{am}u_b) \;.$$

where u^a is a 4-velocity of the moving medium under an inertial reference frame. In this approach, the wave equation for electromagnetic potential has the form

$$\epsilon_0\epsilon k^2\, \partial_a(\partial^a A^b - \partial^b A^a) + \epsilon_0\epsilon\, (k^2-1)\, \partial_a(\, u^a\partial^b - u^b\partial^a\,)\, (u^n A_n) = j^b \;;$$

it involves explicitly the u^a-velocity of a moving medium. So, the electrodynamics by Minkowski implies the absolute nature of the mechanical motion. An alternative formalism (Rosen's and others) may be developed in the new variables ($c = 1/\sqrt{\epsilon_0\mu_0},\ k = 1/\sqrt{\epsilon\mu}$)

$$x^0 = kct, \qquad j^0 = J^0,\ \ \mathbf{j} = \mathbf{J}/kc, \qquad \mathbf{d} = \epsilon_0\mu_0\, \mathbf{E},\ \ \mathbf{h} = \mathbf{H}/kc) \;,$$

In these variables, the Maxwell equations can be written in terms of a single tensor $f^{BC} = (\mathbf{d}, \mathbf{h})$:

$$\partial_B\, f^{BC} = j^C \;,\ \partial_C\, f_{AB} + \partial_A\, f_{BC} + \partial_B\, f_{CA} = 0 \;.$$

This form of the the Maxwell's equations exhibits symmetry under modified Lorentz transformations in which everywhere instead of the vacuum speed of light c is used the speed of light in the medium $\tilde{c} = kc$. In virtue of this symmetry we might consider such a formulation of the Maxwell theory in the medium as invariant under the mechanical motion of the reference frame; at this the velocity transition must be done with the use of modified Lorentz formulas. Transition to 4-potential leads to the simple wave equation

$$f_{CB} = \partial_C A_B - \partial_B A_C \;, \qquad \partial^B\partial_B\, A^C - \partial_C\partial_B A^B = j^C \;.$$

No additional 4-velocity parameter enters this equation, so this form of the electrodynamics presumes a relative nature of the mechanical motion; also this equation describes waves propagating in space with the light velocity kc, which is invariant under modified Lorentz formulas.

In connection with these two theoretical schemes, a point of principle must be stressed: it might seem reasonable to perform Poincaré-Einstein clock synchronization in the uniform medias with the help of real light signals influenced by the medium, which leads us to the modified Lorentz symmetry.

Similar approach is developed for a spin 1/2 particle obeying the Dirac equation in a uniform medium.

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