On the Notion of Possibility in Relativity Theory

Attila Molnár

Eötvös University, Budapest Department of Logic

The Logic and Theory of Relativity group lead by Andréka, H. and Németi, I. developed several axiom systems for relativity theory to investigate it within mathematical

logic.

One of the simplest and most commonly used axiom system is an axiom system of kinematics, the so-called SpecRel. Although this axiom system is very simple, it implies all the main predictions (theorems) of relativity theory. However, as it is proposed by the group in many articles, sometimes the classical first-order logic framework of SpecRel does not seem to be sufficient to give back the appropriate physical meaning. For example, the main axiom of SpecRel, the axiom which is about the *possibility* of sending out light signals, states that there *could be* a photon which crosses certain points. This "could be" indicates some kind of notion of possibility, which is barely accessible from a classical first-order logic. According to the classical logic, an entity either exists somewhere, or does *not* exist somewhere. There are no options such as *"it does not exist there but it is possible to exists there"* or *"does not exist there and it could never ever happen that it exists there."*

This problem becomes more serious when we try to expand the system SpecRel by certain dynamical axioms (to get SpecRelDyn). For example, we would like to postulate that for every observer, everywhere any kind of *possible collision is realizable*. It is worth to investigate this type of axioms, because this way leads to an experimental understanding of the notion of possibility.

We will investigate axiom systems of special relativity based on *modal* logic, which is the standard tool for formally handle dynamical notions – such as performing an experiment, for instance "send out a light signal" or "realize a collision". Modal logic is a logic containing classical logic and an operator to manage formally the notion of *possibility*. For us it means the *possibility* of the existence of a light signal in certain points and the *possibility* of the existence of bodies making collisions.

Our axiom systems will be built with the following goals:

- Give a plausible but formal notion of *possibility* based on the informal explanations of the classical SpecRel and SpecRelDyn.
- Save the theorems and the ideas of their proofs from SpecRel and SpecRelDyn.
- Make space for notions unavailable in SpecRelDyn, e.g., an operational notion of mass.