Our Beloved Leon Henkin

María Manzano*
Universidad de Salamanca

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Abstract

Leon Henkin was born in 1921 in New York, in particular in Brooklyn, in the heart of a Jewish family that originally came from Russia. He died at the beginning of November in 2006. He was an extraordinary logician, and excellent teacher, a dedicated professor and an exceptional person overall. Henkin was an extraordinary insightful professor in the clarity of his expositions and was well loved by his students, who on his last day of class in the academic years would applause his efforts with great emotion.

He was also very aware that we are beings immersed in the crucible of history from which we find it hard to escape.

The reason I am presenting this paper here is because Henkin acts as an emotional bond between Istvan and me. Henkin was the first person to introduce Istvan's and Hajnal's work to me. It was during his trip to Europe in 1982, and in particular after his visit to Yugoslavia, were he met Hajnal Andreka and Istvan Nemeti, from Budapest.

1 His Life

Leon Albert Henkin was born in 1921 in New York, in particular in Brooklyn, in the heart of a Jewish family that originally came from Russia. He died at the beginning of November in 2006, as we are told by mutual friends from the same cause as the mathematician Eratosthenes of Cyrene. He in fact died in Oakland, since some years previously he have had to move there with his wife Ginette, leaving behind his beautiful, simple, vanguard, and minimalist home in the hills of Berkeley, decorated with exquisite *objets d' art*, among which those from American Indian cultures were outstanding. From his dining room you could look out over the gorgeous sight of the Golden Gate of San Francisco. Ginette Henkin told me:

> Leon came from a close-unit family, since two brothers married three sisters. Eldest uncle of Leon came to the United States from Russia,

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> were he went to medical school and after earning some money he brought Leon's father to the States (father was twenty years old just after Russian Revolution). They later brought grandmother, sisters and father. Eldest uncle married one sister and Leon's father another sister and after Leon's mother passed away, he married the third. Leon lost his mother while he was a child, shortly after birth of his sister Estelle.

Leon was very shy when he was young, he was always on chess team in high school but he was a strong student. Leon's father had always being very proud of him, since the very beginning. In a footnote of [15], Henkin said:

> In fact, he had shown his high expectations for me at the time of my birth by choosing my middle name to be "Albert." He once told me that at that time (April 1921) the New York Times had run a series of articles publicizing Einstein's revolutionary theory of relativity, so my father decided to borrow Einstein's first name for his newborn son.

The family was not religious, even grandfather (he was a Hebrew scholar), the only Jewish rite observed was Seder. They were all socialist who voted for Norman Thomas, who was an American Presbyterian minister who achieved fame as a socialist, pacifist, and was six-time presidential candidate for the Socialist Party of America.

Ginette was Canadian and she told me how they met:

> Two couples met on the same weekend: Ginette met Leon and Harold met Estelle. Leon was driving a Montreal friend from grad-school who wanted a lift, Maurice A'Abbé. Estelle went with Leon and L'Abbé to Montreal. Estelle, Leon, Harold Kuhn, L'Abbé, Ginette and another person had dinner together and went dancing after. Less than two years later Ginette and Leon got married, she was 25. Leon and Ginette had two sons, Julian and Paul.

He was an extraordinary logician, and excellent teacher, a dedicated professor and an exceptional person overall. His heart was huge and he was passionately devoted to his ideas of pacifism and socialism (in the sense of belonging to the left). He did not only believe in equality but also worked actively to see that it was brought about. He became involved in minority defence programs, in the fight against social exclusion, and in programs of Excellence and Diversity in Mathematics. He devoted a large part of his life to developing teaching programs at elementary level for both school children and their teachers. In a personal letter Steve Givant told me:

> ...his contributions to education. These include: (a) helping to start at Berkeley an educational program for underrepresented minorities that developed first into the S.O.S. program for high school students,

> and then the P.D.P. program for undergraduate students; (b) collaborating with Uri Treisman to start the Summer Math Institute at Berkeley to prepare mathematically talented minority undergraduate students (nationally) for graduate school in mathematics; (c) collaborating with me to start the Summer Math Institute at Mills to prepare mathematically talented undergraduate woman student (nationally) for graduate school in mathematics.

In the obituary in the *San Francisco Chronicle* [Monday, November 20, 2006], the also logician J.W. Addison and a very good friend of Henkin, an emeritus professor at the same department of the University of California at Berkeley, we read:

> You could say he was an academic triple threat –very strong in teaching, very strong in research, very strong in administration.

Leon Henkin left us an important collection of documents, some of them so exciting as his proof of the theorem of completeness —both for the theory of types [6] and for first-order logic [5]— by means of an innovative and highly versatile method which was later to be used in many other logics, even in those known as non-classical. He himself used the method to prove completeness from a modified version of the interpolation theorem [11]. For some of his results we know the process of discovery, which observed facts he was trying to explain, and why he ended up discovering things that were not originally the target of his enquiries. Thus, in these cases we do not have to engage in risky hypotheses or explain his ideas on the mere basis of the later, cold elaboration of scientific articles. It is well known that the *logic of discovery* is not what is adopted on organizing the final exposition of our research through their different propositions, lemmas, theorems and corollaries.

Henkin was an extraordinary insightful professor in the clarity of his expositions and was well loved by his students, who on his last day of class in the academic years would applause his efforts with great emotion. Indeed, Henkin always wondered whether his classes should be easy to follow or whether they should force his students to make important efforts, since he believed that what can be learned with great ease also tends to be forgotten with great ease. Many of us who knew him believe that he reached a perfect balance and that he would never be obscure on purpose.

He was the Director of the Department of Mathematics at the University of California at Berkeley on at least two occasions: during 66-68 and during 83-85. Although it seems strange, in 1983 he wrote[1]:

> It is much harder now than when I served during 66-68. One big difference is that the University budget has suffered greatly through a combination of political and economic conditions.
>
> ...

[1] Personal letter to María Manzano.

> I send a clipping from our student newspaper —on the first day of our academic year— describing some of the problems. (The Dean was unhappy, but the Chancellor gave us $20,000 more to open 2 new courses!).

In that clipping from the *The Daily Californian*, the situation is explained and the writer tells us that Henkin and the Vice-Dean David Goldschmidt had sent a letter to the Republican Governor George Deukmeijian in which they set forth their demands. The situation spiralled to the extent that a year and a half later Henkin said he wanted to resign, although he was eventually convinced to carry one for a further year.

He was also very aware that we are beings immersed in the crucible of history from which we find it hard to escape. This is in fact reflected at the beginning of an interesting article about the teaching of mathematics [14]:

> Waves of history wash over our nation, stirring up our society and our institutions. Soon we see changes in the way that all of us do things, including our mathematics and our teaching. These changes form themselves into rivulets and streams that merge at various angles with those arising in parts of our society quite different from education, mathematics, or science. Rivers are formed, contributing powerful currents that will produce future waves of history.
>
> The Great Depression and World War II formed the background of my years of study; the Cold War and the Civil Rights Movement were the backdrop against which I began my career as a research mathematicians, and later began to involve myself with mathematics education.

1.1 The Second World War

In an interview from the series entitled *"The Princeton Mathematics Community in the 1930s"* ([24] and [25]) Henkin recounts how the atmosphere of relative calm with respect to WWII changed radically towards the end of 1942, when America entered the conflict. He tells us that he came upon Mrs Eisenhart in the street, and we learn that in their conversation she said:

> We must all do our duty and get on with it.

Henkin says that very soon everybody was expressing similar opinions. He also stresses that his professor, Herman Weyl, decided not to change his work schedule and that he was positively impressed by this:

> I also remember that I had a lecture by Hermann Weyl that same morning, Monday the 8th. It was 9:00. He said, 'I know that all of you are very excited and upset and cannot let go of these great world events that have engulfed us'. But, he said, 'I've learned from my experience that in the most tempestuous of times, there is a

> great value in giving some of your attention and your energy to your continuing work'. Therefore, he said, ‚I am just going to give the regular lecture now that I planned with you last week.' So he did, and I think there is something of real value in those opening remarks.

Like many scientists, he felt he had to be committed and he worked for four years on the Manhattan project. As he tells us (in [15], page 133):

> During the period May, 1942-March, 1946 I worked as a mathematician, first on radar problems and then, beginning January 1943, on the design of a plan to separate uranium isotopes. Most of my work involved numerical analysis to obtain solutions of certain partial-diferential equations.

The project gathered together a large number of eminent scientists, including exiled Jews, pacifists and people on the Left.

1.2 Other facts about Henkin

In an interview [24], Henkin recounts an amusing anecdote about his years at Princeton. Professor Alfred Tucker asked him to create a sort of disturbance on the last day of the academic year:

> Like every great teacher he wanted some dramatic incident to imprint the course on the minds of the young students.

So Henkin started dancing around and contorting himself before the class fellows, whose eyes bulged because they were out of the know of the theatrical nature of the event and were unable to understand his lack of respect towards his teacher. He ended up by removing his waistcoat without taking off his jacket!. Henkin was very fond of ballet and dancing, both ballroom and modern.

There are other amusing anecdotes in these interviews ([24] y [25]), and here we shall see one that had to do with his personal dilemma about teaching: presenting the topics clearly to his students or obliging them to make an effort:

> That effortless way in which the ideas came made them too easy to slip away. I probably learned more densely packed material from what we called the 'baby seminar', in point set topology conducted by Arthur Stone. I learned more because he made us do all the work.

In relation to that same seminar, he describes how he had to face up to professor Lefschetz, although he was fairly unworried because he defines himself as very bold:

> I was giving my solution to one of the problems that Arthur Stone has set to me before, and being a logician I wanted to make all the details very clear and Lefschetz became impatient. As I got into some of those details he said, 'Well, that's all obvious. Just go on toward

> the end.' I was a very brash young man. I said, 'Professor Lefschetz, it may be obvious to you, but I have come from an environment where a proof requires us to give all the details.' And I just went ahead.

He was also interested in literature. In 1983, in a personal letter to María Manzano, in reply to her praise of the novel *Rabbit is Rich*, by the newly awarded Pullitzer Prize winner John Updike, Henkin writes:

> I am glad you enjoyed the language of Updike's writing. I took a volume of his stories on my camping trip last summer, and I admired his writing very much. I still have my school-boy dream of becoming a story writer, but I'm aware I could never achieve such a master style as Updike.

In July 1984 he told María Manzano that for the first time in his country a woman had been nominated as Vice-President of the Democratic Party. However, he was saddened by the fact that Reagan might win the election again, as in fact happened. Today he would possibly have been thrilled that not only a woman but also a black person from the Democratic Party both hold the highest positions. Henkin undoubtedly would have liked to see Obama as President of the United States and Hillay Clinton as Secretary of State.

1.3 Henkin in Spain

This part corresponds to the contribution by María Manzano, who during the academic year of 1977-1978 enjoyed a Fulbright fellowship in *The Group in Logic and Methodology of Science* at the University of California at Berkeley, Leon Henkin being her director and mentor, as he thought of himself.

Henkin did not wish to visit Spain during the Franco era, but during the academic year of 1977-78 this was fortunately no longer the situation. His first trip was in 1982 and he visited several Spanish Universities, such as Barcelona Madrid and Seville. I have kept some of the abundant correspondence related to that visit and at the present juncture the complexity of such communication —as compared with what has happened after the advent of e-mail, etc.— seems very strange. The *Hotel Colón* in Barcelona, was chosen by me as a venue and the couple agreed to this since Ginette and Leon preferred a local touch rather than one that was

> Both of us share your preference for a place with distinctive Spanish character, rather than the ugly, commonplace international style that infects big cities.

In their later letters they always said:

> Not only the warm hospitality... but also the friendliness of people in the other 4 Spanish cities we visited, and the depth and variety of their cultures, made the whole experience a memorable one.

He wished to know all about Spain, especially its social and political developments, and in 1982 he commented:

> Yes, we too were very pleased with the clear and strong victory of the Socialist Party in Spain. Along with all the mathematicians and logicians we met, we were worried about the role of the military and relieved that they did not interfere.

He bewailed the absence of news in the American press and in 1984 he wrote:

> I recall your Spanish election in the week following our visit in 1982. I hope that your new government is working out well... we get little news of Spain in our journals.

In contrast, I personally was always informed about the developments in their own country:

> Our own election in California was a disaster, with a new Governor and Senator, each more conservative than the other. The State is in a fiscal crisis and I'm afraid the University will be in for a rough time...

In 1982 Leon Henkin came to Barcelona and gave two talks [20]. In his letter to the Department, he explained:

> The former would deal with algebraic structures that I have investigated for many years, with Alfred Tarski and Donald Monk, which stand in the same relation to first-order logic as Boolean algebras have to sentential logic. The latter would describe structures that I defined some years ago to elucidate the relation between mathematical induction and recursive definition in the theory of Peano arithmetic. There are both new and old results in each domain.

The talk about inductive models [10] was especially interesting because he told us something that would never appear in any formal article: motivation. Henkin tried to convince a mathematical colleague about the reasons that made a given argument completely wrong, even though at first sight it might seen convincing.

Regarding algebraic structures, Henkin spent many years investigating this, together with Alfred Tarski and Donald Monk ([17] and [16]). Just after his trip to Europe in 1982, and in particular after his visit to Yugoslavia, his relations with Hajnal Andreka and Istvan Nemeti, from Budapest, allowed a second, equally fecund period devoted to cylindrical algebras.

2 As a Student and Professor

Between 1937 and 1947, Henkin began his preparation, studying at Columbia in NY and obtaining a diploma in mathematics and philosophy. Over those

years he studied logic with Ernest Nagel at the Dept of Philosophy and this led him to become interested in the field to a point where he even read Russell's *Principles of Mathematics*. It was there that he first heard about the axiom of choice and he tells us that he was impressed by the amusing and intimate way Russell used to explain it, contrasting how easy is to choose a shoe from an infinite collection of pairs of shoes, but how difficult it is to do so when instead we have pairs of socks. This reading led him to an incursion into the *Principia Mathematica* of Russell and Whitehead [27], and he became infatuated by the theory of types and by the axiom of reducibility. At Columbia, he also followed another course on logic given by Nagel, and he read an article by Quine, with a proof of completeness for propositional logic. He also had occasion to listen to Tarski, whom the invasion of Poland had surprised while at Harvard and had prevented him from returning to Poland. Tarski spoke of Gödel's work on undecidable propositions in type theory. However, his first contact with the work of Gödel arose from a sort of reading seminar promoted by Von Neumann to comment on Gödel's results on the consistency of the continuum hypothesis. At that time, Henkin had still not graduated, but he was the only student who seemed to be interested in these issues and prepared to invest time and energy in studying them.

Upon finishing his studies at Columbia, he applied for admission to universities where logic was well established: Harvard (where Quine was), Princeton (Church) and Columbia. He was accepted at all three, but he rejected the first one because it did not offer grants and he finally opted for Princeton.

2.1 Doctorate studies

It was at Princeton that Henkin followed his master's and doctorate studies, although between both, as commented above, he worked on the famous Manhattan project, an initiative of the United States although with the collaboration of Canada and the United Kingdom.

During his initial term at Princeton, he followed a logic course by Church in which both propositional and first-order calculuses were studied and normal form and completeness theorems were proved for them, and the Löwenheim-Skolem results were analyzed. The completeness proof was that of Gödel (Henkin's, evidently, did still not exist) and the *reductive nature* of the proof was remarked on. During the second semester, a second-order language was studied and, in particular, Peano Arithmetic was introduced in great detail and the results of incompleteness were proved, both for arithmetic and second-order logic. To prove incompleteness, recursive functions were introduced, although only the primitive ones. General recursive functions were not studied, but the role they play in the proof of certain results of undecidability was mentioned. Henkin tells us that although the content of the course was not, as you see, in the least surprising, what was striking was the style of his maestro; the way he had in transmitting his conception of logic. It appears that he would make frequent halts in his discourse to clarify the idea that he was following the *logistic method*: clearly delimiting what was language and what was metalanguage; how formal

language should be established in a completely effective way, and why metalanguage (English, in his case) should be limited. One can gain an idea of how by reading Church's book *Introduction to Mathematical Logic, vol I.* The second volume, whose index appears in the first, was never published, although some of its chapters were circulated among his students.

The Frege theory about the notions of *sense and denotation*[2] were the topic of another subject that Henkin followed with Church when he returned from his fur years leave. Henkin affirms that Church convincingly defended the notion that as well as formal language and that of the universe of mathematical objects where we interpret their formulae, there is a third dimension of abstract objects denominated concepts or senses. A sentence expresses a proposition but also names a truth-value. In [15] Henkin writes:

> Under this theory a symbolic expression functioning as a name denotes an object of the universe of discourse, and expresses some sense of that object; a sentence is construed as a name of its truth value, and the sense it expresses is called a proposition.

The Completeness of Formal Systems is the title of the thesis that he presented at Princeton in 1947, and his director was Alonzo Church[3].

Between 1947 and 1957, Henkin spent two years following post-doctorate courses at Princeton, four teaching and investigating in Los Angeles, at the University of California; one year in Amsterdam with a Fulbright fellowship, and three years at Berkeley, also in the University of California. He refused a job offer at the university in 1952 because of loyalty oath. He said [15]:

> After two post-doctoral years at Princeton, I took my first position as assistant professor at the University of Southern California in 1949. In 1952 I was invited to join Tarski at Berkeley, but I declined because of a "loyalty oath" required of all faculty members by the University of California at that time. Subsequently the oath was abolished, and I moved to Berkeley in 1953.

He valued the role of Tarski very highly as regards his own decision to set up at Berkeley. On 30 October 1983, in a personal communication to María Manzano he wrote:

> I write to tell you that Alfred Tarski, who came to Berkeley in 1942 and founded our great center for the study of logic and foundations, died Wednesday night (Oct. 26), at age 82. All through this year he has been getting weaker; his wife Maria worked heroically to confort and protect him, but finally he gave up his life...
> ...
> It was he who brought me to Berkeley in 1953, so I owe much to him personally as well as scientifically.

[2] Provably Church explained what was to become his paper on that subject [4].

[3] We recommend the reading of: *"Alonzo Church: His Life, His Work and Some of His Miracles"* [22] and *"Diagonalization and Church's Thesis: Kleene's homework"* [1].

From 1957, until his death Henkin devoted himself to mathematics, mainly logic and algebra. From 1953 he was at Berkeley; he was a professor as from 1957 and as of 1991 he was an Emeritus Professor. From 1991 to 2006 he was at Mills College, Oackland, collaborating with Steve Givant. He spent research stays at Dartmouth College in Hanover (New Hampshire), at the Institute for Advanced Studies in Princeton, at the University of Oxford (UK), in Colorado, and at several European Universities; in the then Yugoslavia, Spain and Portugal and also in France at the University of Paris VII.

2.2 Henkin the teacher

The story behind this is that of María Manzano, who during the academic year 1977-78 attended his classes in *algebra* for students in the first years of the degree course, and of *metamathematics* for doctorate students.

The textbook used in the algebra course was that of Birkhoff MacLane, *A Survey of Modern Algebra*, but we did not follow the order of the book. The topics were the usual ones in an Algebra course: Rings, Fields, Polynomials, Homomorphisms, Vector Spaces, etc. Before each class Henkin would give us a text of some 4-5 pages that summarized what was to be addressed in the class. The texts were printed in purple ink, done with the old multicopiers that we called *"Vietnamese copiers"* and that were so often used to (illegally) print pamphlets in our revolutionary days here in Spain against Franco regime. Before starting the sessions, as well as the copies of the topics to be addressed he would give us a sheet explaining the tasks of the week: revision of class notes and of the corresponding sections of the book (indeed, *exactly* which ones) and some 8 problems to solve. As well as giving us back the exercises corrected, he would give us a copy with exercises containing problems solved by him. Detailed information of everything about the course that might be of interest to us was announced a long time before strictly necessary: the dates for handing in the various tasks, the dates of our exams to be done in class with our books and notes, and his tutorial schedule. In the courses given in the first years of the degree, he was always very enthusiastic, even jovial, in class and he transmitted a feeling of confidence. His tutorials were always well attended.

As mentioned above, he was extraordinarily clear in his explanations, although he himself sometimes doubted whether this was the right way of doing things. In the above-commented interview [25] he said:

> That effortless way in which the ideas came made them too easy to slip away. I probably learned more densely packed material from what we called the 'baby seminar', in point set topology conducted by Arthur Stone.

Proof of this internal debate are the following words that formed part of the summary of his course on *metamathematics*.

> Many bright students find my lectures a little slow, and they consider my concern with the machinery of logic (as distinct from the

> results) as pedantic. Concerning the first of these judgments, it is valid, but since many of the students are as slow-thinking as I, and the quick-thinking ones can always skip lectures and study the references, the pace as a whole is not bad –and indeed, the poorly-prepared students may find themselves struggling a bit to keep up. Concerning the second charge, however, I think it can be at least partially turned aside by adverting to pedagogical principles –which I am quite willing to explicate and discuss in office hours, or even in class if demand warrants.

In the summary of the course Henkin defines metamathematics as the mathematical theory of mathematical theories, and he introduces the latter as the study of structures and their interrelations. There are three classes of structures and their interrelations that are studied in metamathematics: formal grammars, deductive calculus, and semantics (called interpreted languages). The language of sentential logic, of first-order and equational logic, many-sorted logic and higher-order logic were dealt with in a unified way in the course. In each case, we studied the relationship between the semantic notion of consequence and the syntactic notion of derivability, with proofs of the theorems of soundness and completeness. It is curious that the first-order proof of completeness that Henkin developed in class was not his own but was developed by using the reduction of Herbrand to propositional logic. I cite from my purple notes.

> Since we use the completeness of sentential logic in our proof, we effectively reduce the completeness problem for first order logic to that of sentential logic.

The issue of implicit and explicit definability was addressed in detail and the Beth/Padoa theorems relating them and the interpolation theorems of Craig-Lyndon were proved [11]. The theorems of Löwenheim-Skolem and of compactness were proved and commented. Naturally, the notions of universal algebra were introduced to relate structures: basically, substructures, homomorphisms, direct products and also ultraproducts. In particular, ultraproduct construction was used to prove compactness. Henkin did not forget classic themes such as those of the elimination of quantifiers, and of categoricity. The theory of types and Gödel's theorem were important parts of the course; indeed, they accounted for 2/5th of the whole. The language of the theory of types introduced by Henkin is that based on identity, very similar to that of his works ([12] and [13]), which contained a selector operator that would allow the axiom of choice to be expressed. The recursive functions, the arithmetization of formal language, the Gödelization and self-reference inevitably led to Gödel's theorems of incompleteness. The last topic was that of general recursive functions and relations.

Between 1957 and 1972, Henkin shared his work in research into mathematics with enquiries into its teaching. As from 1972, he devoted himself mainly to investigating the teaching of the subject. In fact in 1979, with a Fulbright

fellowship, he spent time in Israel devoted to looking into the teaching of mathematics. He was then at the *Department of Education in Science* at *Tehnion University* in Haifa. On this occasion he also visited two universities in Egypt.

All these aspects were addressed by Henkin in his article about mathematical induction [10], which from our point of view is the best paper on logic to offer students as a first reading of a *real-life* article.

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