Abstract, outline & bibliography for:

A century of axiomatic systems for ordinal approaches to Special Relativity Theory

by

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(This article presents a part of my master's thesis in philosophy. I am currently still translating/rewriting my text from Dutch into English. Also, since I wrote my master's thesis in 2004, some new material has become available and I am extending my text to include those.

Since my article has not been finished yet, below is -for now- an abstract, an outline an a bibliography. I will upload the complete article as soon as possible.)

Abstract:

This article presents an introductory overview of the history of axiomatic systems for the ordinal approach to special relativity.

Those axiomatic systems are based on the observation by Robb in the early twentieth century that axiomatic systems for Minkowski time-space can be constructed from a single causal or temporal order relation.

Depending on what the designers of the axiomatic systems are interested in, we get different results. Axiomatic systems can be of first or second order. If they are of first order, they may use an axiom scheme to generate an infinite set of axioms. Some systems are decidable, others are categorical. Some systems contain a metric, others don't. A comparison between the different systems is being made.

Outline:

0. Introduction: special relativity can be defined in terms of a partial order relation, which expresses causality, which means that two events are each others light-cone. Between events where this causal relation exists, there is a "time-like" relation; between events where this causal relation does not exist, there is a "space-like" relation. Several axiomatic systems which express this concept have been built during the past one-hundred years.
1. Alfred Arthur Robb started the tradition of ordinal axiomatic systems for special relativity between 1911 and 1936.
2. The Theorem of Aleksandrov-Zeeman-Hua: Aleksandrov, Zeeman (1964) and Hua independently proved that the Lorentz Group follows from causality.
3. "Foundations of Special Relativity - Kinematic Axioms for Minkowski Space-Time" (1973) by John W. Schutz
4. In 1977, John A. Winnie "The Causal Theory of Space-time".
5. "The elementary foundations of spacetime" by James Ax (1978)
6. Brent Mundy builds in "Optical Axiomatization of Minkowski Space-Time Geometry" (1986) an axiomatic system which is intended as a simplification of Robb's work. It uses ten first order axioms and a second order continuity axiom.
7. Robert Goldblatt builds in "Orthogonality and Spacetime Geometry" (1987) a theory on quaternary "orthogonality" and ternary "betweenness" relations; continues by proving that both these relations can be constructed from Robb' "after" relation and concludes by using the Aleksandrov-Zeeman-Hua theorem to obtain the Lorentz transformations. Goldblatt' system is first order, decidable and consistent but not categorical nor independent.
8. John W. Schutz builds in "Independent axioms for Minkowski space-time " (1997) a categorical and independent but second order and non decidable axiom system on a single ternary "betweenness" relation. Based on this single betweenness relation, he defines the binary partial temporal order relation.
9. "Branching Space-Time by Belnap" (1992), is not an axiom system for Minkowski time-space but intends to be more general and includes phenomena like the Einstein-Podolsky-Rosen paradox. The point of Belnap' s BST is to prove there is no formal inconsistency between relativity theory and indeterminism. BST uses a single causal order relation, but has no notion of dimensionality nor metric.
10. We end our discussion of ordinal axiomatic systems for special relativity with the systems by Hajnal Andréka, Judith Madarász & István Németi, which is rather a construction set for logical systems for relativity. This "specrel" system consists of five first order axioms which results in an undecidable system. Six or seven extra axioms can be added to make the system categorical.
11. Conclusions: similarities and differences between the different axiomatic systems: first vs. second order logic, decidability vs. categorical, representation theorem, metric...

Bibliography:

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