# A prolegomenon to a quantum-information-theoretic complement to a general-relativistic implementation of a beyond-Turing computer

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**First International Conference on Logic and Relativity: honoring István Németi's 70th birthday**

Budapest, 10 September 2012

# 2012: István Németi at 70, relativistic computers at 25

- István Németi's 1987 PhD seminar at Iowa State U: "On logic, relativity, and the limitations of human knowledge"

⇒ concept of relativistic computer formulated

- David Malament's private communication to John Earman in 1988
- Itamar Pitowsky, *Iyyun* **39** (1990): 81-99
- Mark Hogarth, *Found Phys Lett* **5** (1992): 173-181

⇒ Malament-Hogarth spacetimes

Gábor Etesi and István Németi, 'Turing computability and Malament-Hogarth spacetimes', *International Journal of Theoretical Physics* **41** (2002): 342-370.

István Németi and Gyula Dávid, 'Relativistic computers and the Turing barrier', *Applied Mathematics and Computation* **178** (2006): 118-142.

# The physical Church-Turing thesis

**Thesis (Physical Church-Turing Thesis (PhCT))**

*No computing device that is physically realizable (even in principle) can exceed the computational barriers of a Turing machine.*

- Not to be confused with the logically independent Church-Turing thesis, according to which a function is 'effectively' or 'algorithmically' computable just in case it is computable by a Turing machine.
- Two ways to flesh out 'physically realizable':
  1. realizable by idealized device in context of a specific theory; here: classical general relativity (GR)
  2. realizable by idealized device as judged by all our fundamental theories, or our currently best candidates

⇒ PhCT (unlike CT, or Goldbach's conjecture) not just a hypothesis in pure mathematics, but involves physics (what kind of physics depends on how we contextualize 'physically realizable')

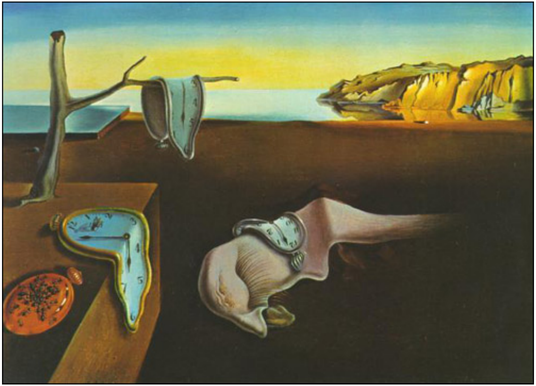
- Let's start by confining ourselves to GR.

⇒ Question: does GR permit, at least in principle, a scenario in which 'hyper-Turing' computations, i.e., computations not executable by Turing machines, are possible?

**Thesis (E.g. Etesi and Németi 2002)**

*"[I]t is consistent with Einstein's equations, i.e. with general relativity, that by certain kinds of relativistic experiments, future generations might find the answer to non-computable questions like the halting problem of Turing machines or the consistency of Zermelo Fraenkel set theory [and hence PhCT is false, at least in the context of GR]." (Németi and Dávid 2006)*

# Gravitational slowing of clocks



Salvador Dalí, *La persistència de la memòria*, Oil on canvas, 1931

- Clocks ‘deeper’ in gravitational field run more slowly than their initially synchronized counterparts ‘higher up’ in gravitational field.

$\Rightarrow$ use different in field strength to achieve computational speed-up—beyond any bounds

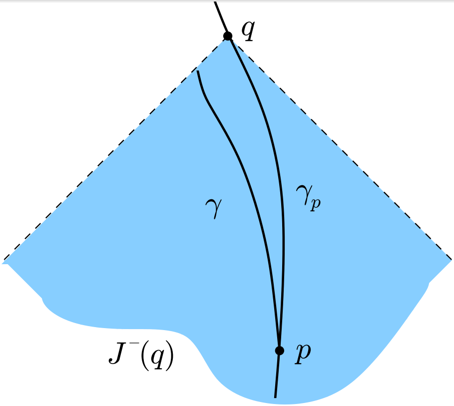
# What you need: a Malament-Hogarth spacetime

**In a nutshell**

*The gravitational field must be such that the computer can run an infinite number of computations in the causal past of a spacetime event the programmer can reach within finite time.*

- Important: no intrinsic speed-up of computer required, no supertask
- computer's worldline $\gamma$ has infinite proper length in the causal past of the event reached by programmer in finite time, i.e.

$$\int_\gamma ds = \infty$$




**Definition (Malament-Hogarth spacetimes)**

*A relativistic spacetime $\langle \mathcal{M}, g_{ab} \rangle$ is a Malament-Hogarth spacetime just in case there is a future-directed timelike half-curve $\gamma : \mathbb{R}^+ \rightarrow \mathcal{M}$ such that $\int_\gamma ds = \infty$ and there exists a point $q \in \mathcal{M}$ (the MH event) satisfying $\text{im}\gamma \subset J^-(q)$, where $J^-(q)$ is the causal past of $q$.*

# Relativistic computers

**Definition (Relativistic computer, Németi and Dávid, Def. 4.5)**

*A relativistic computer in a Malament-Hogarth spacetime $\langle \mathcal{M}, g_{ab} \rangle$ is a triple $\langle \gamma_p, \gamma, q \rangle$ such that $\gamma$ is an upward-infinite future-directed curve fully in $J^-(q)$, $\gamma_p$ is a timelike curve such that $q$ lies on it and an initial segment of $\gamma_p$ coincides with $\gamma$ (up to, say, point $p$).*

Let's look at concrete implementations of relativistic computers as physically realistic as possible...

# Examples of Malament-Hogarth spacetimes

- Minkowski spacetime if punctuated and conformally blown up in compact region around punctuation
- Anti de Sitter spacetime: Lorentzian analogue of hyperbolic space with constant negative scalar curvature
- Kerr spacetime: vacuum spacetime of uncharged, rotating black hole (if $|a| < M$)
- Reissner-Nordström spacetime: charged, non-rotating black hole
- Kerr-Newman spacetime: charged, rotating black hole (if $|a| + |Q| \neq 0$)

# A programmer falls into a Kerr black hole...

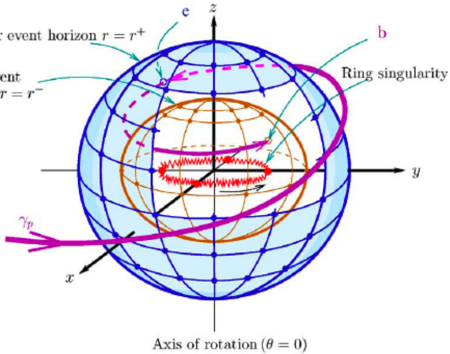



Fig. 1. A slowly rotating (Kerr) black hole has two event horizons and a ring-shape singularity. The ring singularity is inside the inner horizon $r = r^-$ in the "equatorial" plane of axes $x$, $y$. Time coordinate is suppressed. See Fig. 2 for a spacetime diagram with $x$, $y$ suppressed. (Fig. 2 denotes $z$ as $r$.) Rotation of ring is indicated by an arrow. Orbit of infalling programmer $\gamma_p$ is indicated, it enters outer horizon at point $e$, and meets inner horizon at point $b$.

# The causal structure of a Kerr black hole

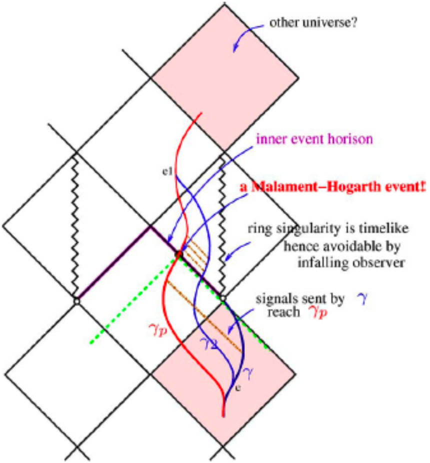



Fig. 5. Penrose diagram of slowly rotating black hole along the symmetry axis. This is a Malament–Hogart spacetime. The length of $\gamma_p$ is finite, while the length of $\gamma$ is infinite. ($\gamma_2$ will be used later, in Section 5.3.2.)

# How to check the consistency of ZFC

- Question (known to be non-Turing computable): check for consistency of Zermelo-Fraenkel set theory (ZFC): can we derive formula FALSE from axioms of ZFC?

$\Rightarrow$ computer checks theorems of set theory one by one (e.g. one a day)

- If the computer finds a proof of formula FALSE, then it send a signal to the programmer.
- If it doesn't, then no signal is sent.
- How does the programmer learn whether ZFC is consistent?
    - If he receives the signal, he knows that ZFC is inconsistent.
    - If he reaches the MH event without having received a signal, he knows that ZFC is consistent.
    - But how does she know she passed the MH event?

# The photonic communication protocol

- Thus, communication is necessary for the programmer to learn the outcome.
- communication is going to be one-way from computer to programmer

$\Rightarrow$ protocol needed

- This protocol is usually based on coded photon signals.
- Unfortunately, this poses a big obstacle to implementing a relativistic computer as there are serious problems...

# The trouble with light

1. The blueshift problem I: if the computer sends weekly digest messages, their accumulated energy will fry the programmer who observers the computer speeding up without bounds.
   - ⇒ That's why it only sends a message if it derives the formula FALSE.
2. The blueshift problem II: but the signal will be the shorter and higher-frequency the later it is sent, which will make it hard to recognize (and impossible to see).
   - computer can't just make signals of arbitrarily low frequency
   - ⇒ Németi-Dávid: put computer on a spacecraft and send it away from black hole (possibly drawn by other black hole) so that emanated signal is redshifted to exactly counterbalance blueshift
     - Problem with this solution: potentially need arbitrarily large amount of energy to accelerate spacecraft or another conveniently located black hole, and a perfect synchronization of the whole procedure...

2. The blueshift problem II continued:
    - ⇒ computer hands letter stating that inconsistency has been found to Hungarian post, which delivers it to programmer by sending a mailman after him (cf. figure)
        - Problem with this solution: even if Hungarian post delivers with lightning speed (well, not literally), the message will be received perhaps very much later
    - ⇒ If the expansion of the universe does not continue to accelerate forever, the computer might 'feed' the black hole with matter (also to avoid its evaporation), and change the feeding patterns or change the black hole's angular momentum or its charge, to signal to the programmer that it found an inconsistency.
        - Problem with this solution: acceleration of the universe's expansion needs to slow down eventually, for otherwise the computer may no longer have the means to execute the change when it finds the inconsistency; even so, implementation of a protocol like this requires quite a bit of engineering...

# The promise of a more elegant solution:
## Use non-local quantum correlations

- In fact, Németi and Dávid allude to such a solution:

> *It is an interesting future research possibility to solve the communication problem between $\gamma$ and $\gamma_p$ by some quantum-information theoretic methods. (132)*

- vantage point: (anti-)correlations between observed properties of entangled pairs of particles
- Idealization: correlations are assumed to be perfect
- important point: these correlations obtain non-locally, i.e. for spacelike separated measurement events
- 'quantum connection' between entangled particles is unattenuated, discriminating, and instantaneous (whatever that means in the present context)

# A first (Chinese) stab...

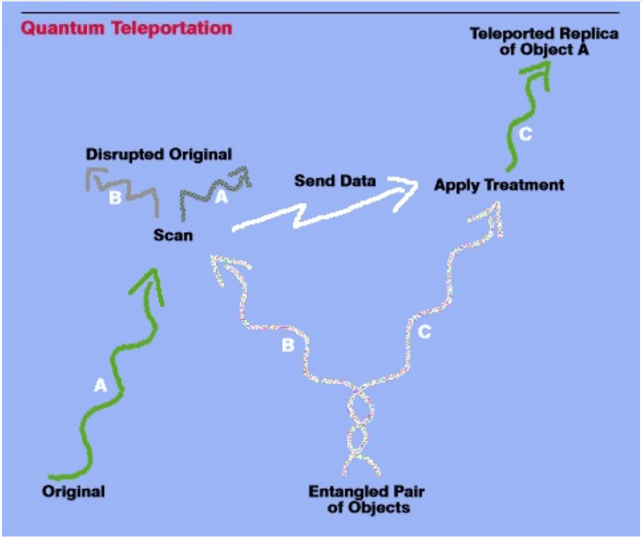
Ge Xian-Hui and Shen You-Gen, 'Quantum teleportation and Kerr-Newman spacetime', *Chinese Physics* **14** (2005): 1512-1516.

- Xian-Hui and You-Gen (2005): they consider quantum teleportation in Kerr-Newman spacetime, but let 'Bob' only hover over the event horizon, 'Alice' stays in asymptotic region
- Then they do QFT on a fixed, but curved background spacetime in order to compute how high 'fidelity' is.

$\Rightarrow$ at best first-order approximation–is this good enough in the strong gravitational fields that the programmer will encounter?

- result: for slow, massive black holes, the fidelity is high (and so it is for extremal black holes)
- Perhaps their idea could be modified to let Bob fall into the black hole...

# Quantum teleportation



# Quantum teleportation

- still needs classical communication channel

⇒ back to the photon problem we tried to solve...

⇒ useless (except if the goal is to transmit large amounts of information, e.g. the Encyclopedia Britannica of the year 3000 or the complete Springer series of Graduate Texts in Mathematics up to that year...

- but none of this is going to tell you if ZFC is consistent

⇒ Question: could we do it without any channel? After all, (theoretically) we only need to transmit one bit of information!

- Simple idea: if computer finds inconsistency, it makes a measurement (or some local operation) on its part of the entangled system; if not, it doesn't
- Question: could the programmer somehow find out on his entangled particle that a measurement has been performed on the entangled counterpart?

# No-signalling theorems in QM

**Theorem (No-signalling in ordinary QM, rough statement)**

*There can be no superluminal information transmission ('signalling') between spacelike-related regions even if they share an entangled pair of quantum systems.*

Some comments:

- Usually thought to protect integrity of relativity in the face of non-local Bell correlations.
- You might think that these theorems (there are many), which are all stated in terms of spacelike relations between the regions do not concern the present problem, which is to try and use entanglement to signal between timelike- or null-related regions.
- But we can't that easily evade the strictures of these theorems because they make no assumption concerning the spatiotemporal separation of the subsystems:

# No-signalling theorems in QM

## Typical premises of these theorems:

1. tensor product space: $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
2. density operator of system: state is given by density operator on $\mathcal{H}$, i.e. $\rho = \sum_i A_i \otimes B_i$, where $A_i$ and $B_i$ are operators on $\mathcal{H}_A$ and $\mathcal{H}_B$, resp. (need not be states on subsystems)
3. restricted action of operators: for instance, $\hat{O} = \hat{O}_A \otimes \hat{I}_B$, where $\hat{I}_B$ is the identity operator on $\mathcal{H}_B$

# Some comments

- From this, it can be derived that the statistics in Bob's wing do not change under local operations by Alice.

$\Rightarrow$ So whatever Alice does to her particle, Bob has no way of detecting this–in fact, even whether she did anything at all!

- time is not assumed; instead it is assumed that the local operations don't change the state of the distant subsystem

$\Rightarrow$ theorems question-begging (Kennedy 1995, Peacock 2009)

- You shouldn't conclude that signalling is possible, or even easy. Still, timelike signalling may not face the same restrictions as spacelike signalling.

John Bernard 'Jay' Kennedy, Jr., 'On the empirical foundations of the quantum no-signalling proofs', *Philosophy of Science* **62** (1995): 543-560.

Kent A Peacock, 'The no-signalling theorems: A nitpicking distinction', Manuscript 2009.

# Relativize to first approximation: QFT

- All these results, whatever their merits, are from ordinary QM augmented by the premises of the theorems (which are not axioms of ordinary QM).
- ordinary QM will not be true in strong gravitational fields

$\Rightarrow$ Let's move to QFT on curved spacetime as a first step towards a quantum theory of gravity;

- in fact if we do that, there are (perhaps unsurprisingly) some indications that spacelike and timelike separations indeed differ relevantly...
- In axiomatic QFT, 'locality' of operations is built into axioms: 'microcausality' or 'locality', which assume that, roughly, spacelike-separated operators commute

# Locality in axiomatic QFT

In an axiomatic formulation of QFT, a 'local action' principle is usually observed,

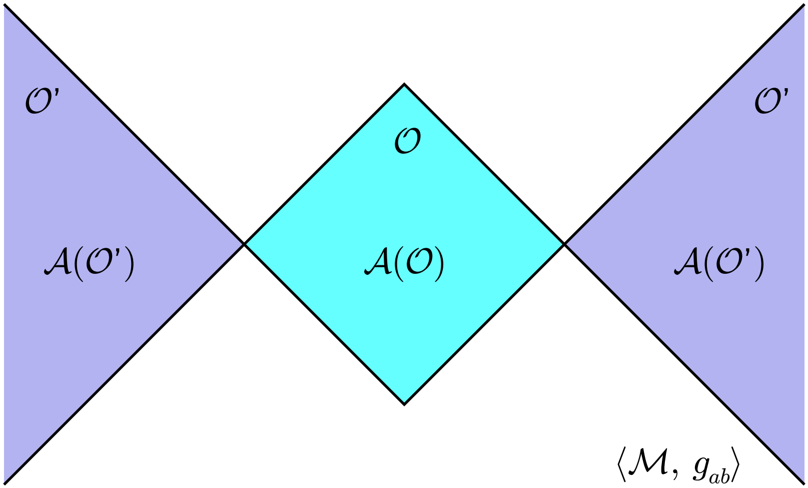
- where 'local action' means something like 'no physical action can propagate faster than the speed of light'.
- More technically, e.g., Locality in the Haag-Kastler axioms means that algebras living in spacelike separated regions commute, as follows:

**Axiom (Haag-Kastler Locality)**

*Given an algebra $\mathcal{A}(\mathcal{O})$ defined over a spacetime region $\mathcal{O} \subset \mathcal{M}$, with $\mathcal{O}' \subset \mathcal{M}$ denoting the set of spacetime points spacelike separated from every point in $\mathcal{O}$ and $\mathcal{A}'$ the set of operators that commute with every operator in $\mathcal{A}$ (the 'commutant' of $\mathcal{A}$), then*

$$\mathcal{A}(\mathcal{O}') \subseteq \mathcal{A}(\mathcal{O})'.$$

# Algebra of complement as subalgebra of commutant



# Where does this get us?

- Since timelike-separated operators may not commute, it is at least not prohibited that there can be timelike signalling through entanglement only.

$\Rightarrow$ We shouldn't consider the no-signalling theorems no-go theorems for a purely entanglement-based communication between computer and programmer.

- But it's not more than a possibility; we need a concrete protocol and a concrete understanding of how this would work!
- Towards the latter: it turns out that on some interpretations of non-relativistic QM, superluminal signalling is permitted (cf. Berkovitz 2008, §7).

Joseph Berkovitz, 'Action at a distance in quantum mechanics', in E. Zalta (ed.), *Stanford Encyclopedia of Philosophy*, Winter 2008, URL = <http://plato.stanford.edu/archives/win2008/entries/qm-action-distance/>.

# Action at a distance in quantum physics

At least for a large class of theories, Berkovitz (2008, §7.1) argues that the following two conditions are individually necessary and jointly sufficient for superluminal signalling:

## Necessary and sufficient conditions for superluminal signalling

1. "Controllable probabilistic dependence: The probabilities of distant measurement outcomes depend on some nearby controllable physical quantity.
2. "$\lambda$-distribution: There can be in theory an ensemble of particle pairs the states of which deviate from the quantum-equilibrium distribution; where the quantum-equilibrium distribution of pairs' states is the distribution that reproduces the predictions of orthodox quantum mechanics."

- I take it that since we are only looking for timelike signalling, they will be jointly sufficient, but may not be necessary.

# Interpretations that permit superluminal signalling

- Bohmian mechanics: involves parameter dependence and therefore controllable probabilistic dependence; if quantum statistics is regarded not as lawlike, but only as contingently true, then there may be (atypical?) worlds in which $\lambda$-distribution holds
- Collapse theories: perhaps in some, such as e.g. the so-called 'non-linear Continuous Stochastic Localization models', the two conditions may obtain if we can very finely control some degrees of freedom (note that in this case the statistics would be different from orthodox QM) (cf. Butterfield et al. 1993)

Butterfield, J. N., Fleming, G. N., Ghirardi, G. C. and Grassi, R., 'Parameter dependence in dynamical models for state reductions', *International Journal of Theoretical Physics* **32** (1993): 2287-2303.

# ‘Protective measurements’

- Protective measurements (not technically an interpretation): Aharonov et al. 2004 proposed signalling protocol based on the idea of ‘protective measurements’ which supposedly permit the extraction of information from an entangled state without ‘disturbing’ it (cf. also Peacock 2009)

Aharonov, Y., Anandan, J., Maclay, G.J., and Suzuki, J.. ‘Model for entangled states with spin-spin interaction’, *Physical Review* A**70** (2004): 052114.

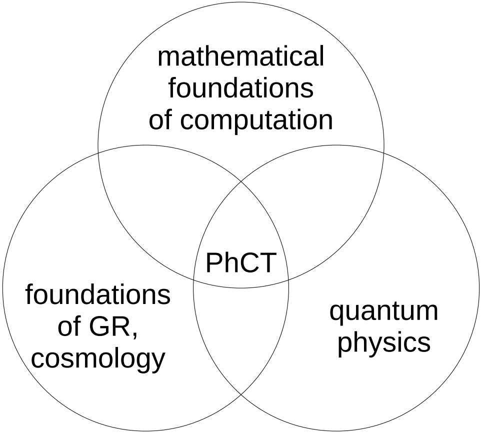
# Concluding:
## The prospects of entanglement-based signalling

- A fair assessment of the prospects of a communication between the computer and its programmer based on shared quantum entanglement requires a resolution of a number of issues at the forefront of understanding quantum physics.
- (In fact, there is much more than I had time to mention: entanglement may dwindle if programmer falls into the black hole (Fuentes-Schuller and Mann 2005), the potential quantum instability of the Cauchy horizon may interfere with the entanglement (and, of course, with the survival of the programmer!)...)
- Ultimately, of course, we will need a quantum theory of gravity to accurately describe how quantum systems and strong gravitational fields interact.

# The bigger issue:
## How to combine gravity and the quantum

- But that is already true for the purely photonic protocol: photons are quantum systems, and to consider classical test particles amounts to a bet that this will give ‘sufficiently true’ results, for which there is no guarantee...
- The central lesson, I take it, is that studying PhCT not only ties computational questions of CT thesis with foundational questions in GR (stability of horizons, cosmic censorship hypothesis) and cosmology (acceleration of expansion, cosmological standard model), but also with quantum physics and hence quantum gravity

# The really big issue:
## PhCT as an intersection



# So is hypercomputation physically realizable?

- Well, who knows.
- But we should be grateful to István who has been so instrumental in opening up this area for fruitful research!