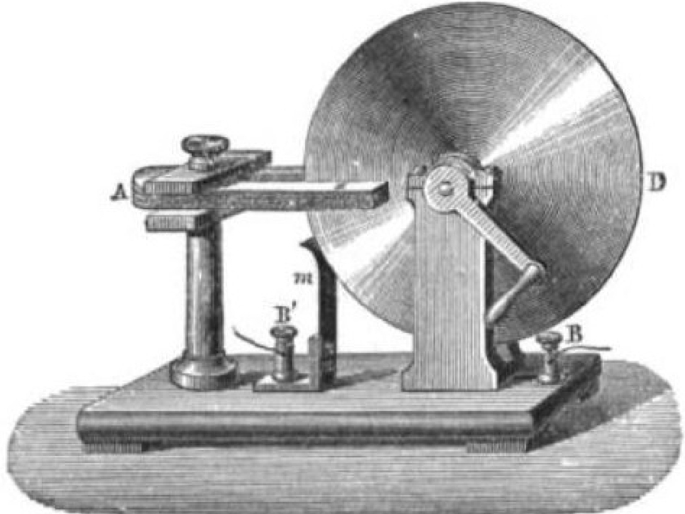
Gandy's Thesis and Relativistic Computation; in honour of István Németi's 70'th Birthday

P.D.Welch, September 10, 2012.



Gandy's Thesis and Relativistic Computation

- I Introduction
- II Gandy's *Principles for Mechanisms*
- III The Nemeti/Malament-Hogarth Context

I Introduction

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• This allows, via appropriate signalling, results of what would normally take an infinite amount of time to decide, such as a universally quantified $\forall k P(k)$ with $P(k)$ a recursive predicate, to be decided, by searching through $P(0), P(1), \ldots$ for a counterexample. If and when one is found, for r_0 may signal 'ahead' to r_1 that such had been found. The infinite proper time of r_0's world line might correspond to lunch-time for r_1, and after lunch r_1 may have received no signal, and thus know that there is no counterexample to $\forall k P(k)$.

• Hogarth [1] argued that one might stack up Turing machines $\langle T_k \mid k < n \rangle$ in a particular Riemannian manifold where each machine T_k was headed off to a singularity along a path of infinite proper time length in an open region O_k, but which path was completely contained in the causal past of an observer p_k - also in O_k. This was called an "*SAD*-region") . Just as on the last slide, this allows p_k toknow after some finite measure of its time, whether a search that took an infinite amount of T_k's time was or was not successful. Then p_k might signal forwards to the next observer controlling T_{k+1} to initiate a subsequent computation.

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• He further suggested stacking infinitely many such regions to calculate any arithmetic question ("*AD*-regions").

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• We argued that this can be carried further[2]. We make two assumptions:

Assumption 1 *The open regions O_j are disjoint*;
Assumption 2 ("No swamping") *No observer or part of the machinery of the system has to send or receive infinitely many signals.*

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In the recursive case, this would allow one for any *hyperarithmetic* predicate $Q(n)$ to devise an MH-spacetime where it could be decided. (Such predicates can be thought of as Σ^0_α predicates for recursive ordinals α.)

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A universal constant $w(\mathcal{M})$ for such calculations

Definition

Let $\mathcal{M} = (M, g_{\mathrm{ab}})$ be a spacetime. We define $w(\mathcal{M})$ to be the least ordinal η so that $\mathcal{M}$ contains no SAD region whose underlying tree structure has ordinal rank η.

• Note that $0 \leq w(\mathcal{M}) \leq \omega_1$
($0 = w(\mathcal{M})$ implies that $\mathcal{M}$ contains no SAD regions whatsoever, that is, is not MH);
• The upper bound is for the trivial reason that every finite path tree is a countable object and so cannot have uncountable ordinal rank.

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Proposition

For any spacetime $\mathcal{M}$, $w(\mathcal{M}) < \omega_1$.

• Our universe $\mathcal{M}_0$ then may or may not have $0 < w(\mathcal{M}_0)$.

Question: Can we argue that any *means of computing in such a spacetime can only answer questions at the* $\Sigma^0_{w(\mathcal{M})}$*-level?*

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Question: In particular can we argue that any *means of computing in such a hyperarithmetically deciding spacetime can only answer hyperarithmetic questions?*

(In the latter spacetimes $w(\mathcal{M}) = \omega_1^{ck}$ - the first non-recursive ordinal.)

II Gandy's Thesis P_0

We model an answer on an argument of Gandy's. He there suggested a mechanistic thesis:

Thesis M_0: What can be calculated by a machine is computable.

This is too general stated, hence some assumptions are needed:

II Gandy's Thesis P_0

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Thesis M_0: What can be calculated by a machine is computable.

This is too general stated, hence some assumptions are needed:

(1) essentially analogue machines are excluded;
(2) the progress of calculation by a mechanical device is assumed to be discrete;
(3) the device is deterministic.

He articulates then four *Principles for Mechanisms* $(I) - (IV)$, and then:

Thesis P_0*: A discrete deterministic device satisfies (I)-(IV).*

and will argue for:

Theorem (Gandy)

What can be calculated by a device satisfying (I)-(IV) is Turing computable.

III The Németi\MH-context

If we fix an $\mathcal{M}$ with $w(\mathcal{M}) = \eta$ we may likewise formulate Principles $(1)_\eta$-$(4)_\eta$ by analogy to his (I)-(IV), and state:

Thesis P_η: A discrete deterministic device operating in an manifold $\mathcal{M}$ with $w(\mathcal{M}) = \eta$, satisfies $(1)_\eta$-$(4)_\eta$..

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Theorem

What can be calculated by a device satisfying $(1)_\eta$-$(4)_\eta$ is computable by Turing machines in a generalised SAD_η region of $\mathcal{M}$ where $\eta < w(\mathcal{M})$.

State Descriptions

- Fix an $\mathcal{M}$ with $w(\mathcal{M}) = \eta = \omega_1^{ck}$ - the first non-recursive ordinal - as a starting example. (The account below works for α any countable *admissible ordinal*.)

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• For Gandy a state as a finite description reflects the 'actual, concrete structure of the device in a given state'.
• G. gave a schematic description of states of the machine in terms of *labels* that might label cogs, beads, electrodes, cells of a Turing tape or whatever the device consisted of. Labels, from a potentially infinite set L, might be built up into hereditary finite sets to form *structures*.

Structures

Definition

$\mathrm{HF}_0 =_{\mathrm{df}} \varnothing; \mathrm{HF}_{n+1} =_{\mathrm{df}} \mathcal{P}_{\mathrm{fin}}(\,\mathrm{HF}_n \cup L); \mathrm{HF} = \bigcup\{\mathrm{HF}_n \mid n \in \mathbb{N}\} \cup \{\varnothing\}.$

Here $\mathcal{P}_{\mathrm{fin}}(X)$ is the set of finite subsets of X. A set s in some HF_n is thought of as a *structure*, and variables $r, s, t, u, \ldots$ vary over HF or subsets of HF. Variables $a, b, c, l, \ldots$ refer to labels.

We shall be taking *metafinite descriptions* and structures, and be assuming a label space as follows:

Definition

We let $L = \{l_\alpha \mid \alpha < \omega_1\}$ be a set of *indexed labels* with $l_\alpha =_{\text{df}} \langle 0, \alpha \rangle$.
(ii) A *metafinite set of labels* is a set $K \in \mathcal{P}(L) \cap L_{\omega_1}$.
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For any metafinite x, we define $\mathrm{supp}(x) =_{\mathrm{df}} \{l \mid l \in x \vee \exists y \in x(l \in \mathrm{supp}(y))\}$.

This is a definition by transfinite recursion; $\mathrm{supp}(x)$ is then also metafinite and we define the *rank* of x, $\mathrm{rk}(x) =_{\mathrm{df}} \mathrm{rk}(\mathrm{supp}(x))$.

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Definition

If $A \subseteq L$ then $x \upharpoonright A =_{\text{df}} (x \cap A) \cup \{y \upharpoonright A \mid y \in x \wedge \text{supp}(y) \cap A \neq \varnothing\}$.

• The *next state* of a machine is determined by a *transition function* F which determines the description $F(x)$ from the previous state x. Such transition functions may require the use of new labels (for example when a TM requires a new cell on the tape, or a cellular automaton builds a clutch of new cells). *However no physical significance is attributed to the new labels.* We require that objects in the state x, if they persist into the next state $F(x)$, will retain the same labels.

Structures and Stereotypes

(1) We let $\pi : L \longrightarrow L$ be any permutation (any meta-recursive permutation) of L. The effect of π on a structure a is defined by $a^\pi =_{\mathrm{df}} \pi(a)$ for $a \in L$; for x a structure $x^\pi =_{\mathrm{df}} \{y^\pi \mid y \in z\} \cup \{a^\pi \mid a \in x\}$.

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(2) Two structures are *isomorphic over a set A of labels, if:*

$$x \simeq_A y \Longleftrightarrow_{\mathrm{df}} \exists\pi[\pi \restriction A = \mathrm{id} \restriction A \wedge x^\pi = y].$$

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(5) A function $F : \mathrm{HF} \longrightarrow \mathrm{HF}$ $(F : \mathrm{MF} \longrightarrow \mathrm{MF})$ is structural *iff for all π*

$$(F(x))^\pi \simeq_{x^\pi} F(x^\pi).$$

Principle (1) (and $(1)_\eta$)

Principle (1) (and (1)) $_\eta$ *Any machine M can be described by giving a structural set $S_M \subseteq \mathrm{HF}$ (MF) of state descriptions together with a (meta-)recursive structural function $F : S_M \longrightarrow S_M$. If $x_0 \in S_M$ describes an initial state then $F(x_0), F(F(x_0)), F(F(F(x_0))), \ldots, F^{(k)}(x_0), \ldots$ describes the successive states for $k < \omega$.*

Boundedness Conditions

Principle (2) (and $(2)_\eta$) *The set $S = S_M$ of state descriptions is contained in HF_k for some $k < \omega$ (is metafinite).*

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• The Principle (3) (and $(3)_\eta$) will also be a boundedness condition and will have the effect that any device can be assembled from a (meta)finite set of parts of (meta)finite size, and that the parts can be labelled so that there is a unique way of putting them together. First we have to define *parts* of a device, and say what it means for a device to be *reassembled* from those parts.

Parts of a device

Definition

Let $P \subseteq \mathrm{HF}\,(\mathrm{MF}) \cup L$.

(i) The *set of parts of x from the list P,* $\mathrm{Part}(x, P)$ is defined by:

$$\begin{aligned} \mathrm{Part}(x, P) &= \{\{x\}\} \text{if } x \in P \\ &= \bigcup\{\mathrm{Part}\{y, P\} \mid y \in x\} \cup (x \cap P \cap L) \text{ otherwise.} \end{aligned}$$

(ii) The *restriction of x to the list of parts P*, $x \restriction P$, is defined as follows:

$$\begin{aligned} x \restriction P &= x \;\text{ if } x \in P \\ &= \{y \restriction P \mid y \in x \wedge \mathrm{Part}(y, P) \neq \varnothing\} \cup (x \cap P \cap L) \text{ otherwise.} \end{aligned}$$

(iii) The list P *covers* x if $x \restriction P = x$; if additionally $P \subseteq \mathrm{TC}(\{x\})$ then P is a *set of parts for x.*

Principle (3) (and $(3)_\eta$)

Definition

Let $Q \subseteq \mathcal{P}(\mathrm{TC}(x))$. The structure x can be *uniquely reassembled* from the set Q of sub-assemblies iff x is the unique object y satisfying
(i) $y \in \mathrm{HF}\ (\mathrm{MF})$; (ii) $\bigcup Q$ covers y; (iii) $\forall T \in Q(x \upharpoonright T = y \upharpoonright T)$.

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Principle (3) (and $(3)_\eta$) *There is a bound* $\chi < \omega\ (\omega_1)$ *and for each* $x \in S$ *a (meta)finite set* $Q \subseteq \mathcal{P}(\mathrm{TC}(x))$ *from which* x *can be uniquely reassembled, and such that* $\mathrm{rk}(T) < \chi$ *for each* $T \in Q$.

Principle of Local Causation

Principle (4) (and $(4)_\eta$) (Approximate Version) *The next state, F(x), of a machine can be reassembled from its restrictions to overlapping "regions" s, and these restrictions are locally caused. That is for each* "determined region" *s of* $F(x)$ *there is a* "causal neighbourhood" $t \subseteq \mathrm{TC}(x)$ *of bounded size such that* $F(x) \upharpoonright s$ *depends only on* $x \upharpoonright t$.

This splits into cases:
Case 1: $supp(F(x)) \subseteq supp(x)$
Case 2: Otherwise.
Thus depending on whether the transition function requires new labels for its description or not.

• One needs to define *"causal neighbourhoods"* of x and *"determined regions"* of $F(x)$, and in particular deal with overlapping determined regions of $F(x)$.

The final conclusions

- A key Lemma shows that if the structural function fixing determined regions requires only boundedly many (metafinitely many) new labels, then the stereotypes of the determined region structures are unique.

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- A key Lemma shows that if the structural function fixing determined regions requires only boundedly many (metafinitely many) new labels, then the stereotypes of the determined region structures are unique.

- This allows Gandy to conclude that the calculation the device is performing amount to just bounded searches that Turing computable.

- For the our case the searches are metafinitely bounded, hence are essentially hyperarithmetic questions, and thus we know can be decided in any spacetime with $\mathcal{M} = \omega_1$.