On The Notion of Possibility in Relativity Theory

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Reminder

The summary of the 1st Tutorial of yesterday morning:

Special Relativity can be axiomatized in a classical first order language.

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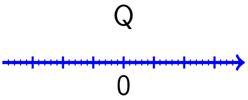
1. We will examine this extensional theory from the aspect of how it uses the notion of possibility.
2. We will present a simple modal system to solve some problems of its interpretation of possibility.
3. We will present a complex modal system to achieve goals what an extensional theory never could.

SpecRelKin



- One sort for Mathematics: $\langle Q, +, \cdot, \leq \rangle$
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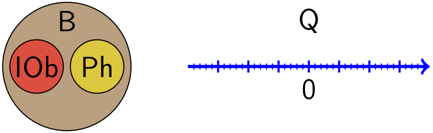

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  - $Ph(p)$: „p is a photon”
  - $IOb(k)$: „k is an inertial observer”

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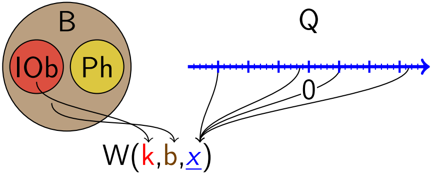

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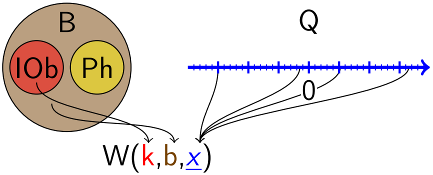
  - $IOb(k)$: „k is an inertial observer"
- Connection:
  - $W(k, b, \underline{x})$ is the world-view relation. „k sees b at the coordinate point $\underline{x}$. (k must be IOb).

SpecRelDyn



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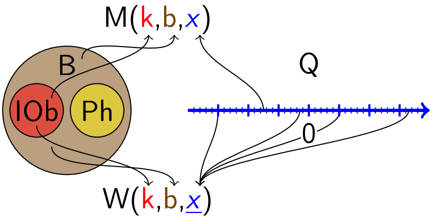


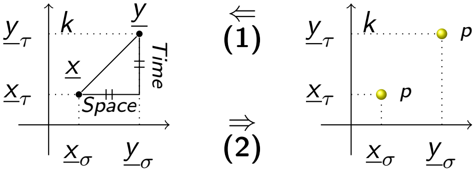
- One more connection:
  - $M(k, b, x)$ is the mass relation. „The mass of b is x according to k". ($k \in IOb$) and M is a function.

Axioms

Kinematics

Axiom of Photons

AxPh:
(1) Every observer sees the world-lines of photons as of slope 1.
(2) In every direction it is possible to send out a photon.



$$(\forall k \in IOb)(\forall \underline{x}, \underline{y} \in Q^d)$$

$$\left(\frac{\mathrm{Space}(\underline{x}, \underline{y})}{\mathrm{Time}(\underline{x}, \underline{y})} = 1 \leftrightarrow (\exists p \in Ph)(W(k, p, \underline{x}) \land W(k, p, \underline{y}))\right)$$

where $\mathrm{Space}(\underline{x}, \underline{y})$ and $\mathrm{Time}(\underline{x}, \underline{y})$ are spatial and temporal distance of the coordinate points.

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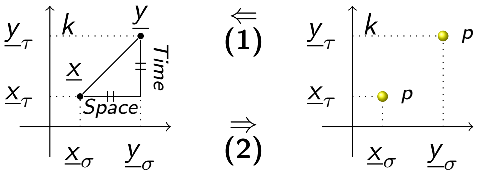

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where $\mathrm{Space}(\underline{x},\underline{y})$ and $\mathrm{Time}(\underline{x},\underline{y})$ are spatial and temporal distance of the coordinate points.

$\exists$ plays the role of possibility!

World-view relation

An *event* in $\underline{x}$ according to k:

The set of bodies occuring there.

$$\mathrm{ev}_k(\underline{x}) \underset{\text{def.}}{=} \{b \in B : W(k, b, \underline{x})\}$$

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The world-view relation $w_{kh}(\underline{x}, \underline{y})$:
$\underline{y}$ is a coordinate point where h sees what k sees at $\underline{x}$.

$$w_{kh}(\underline{x}, \underline{y}) \quad \Longleftrightarrow_{\text{def.}} \quad \mathrm{ev}_k(\underline{x}) = \mathrm{ev}_h(\underline{y})$$

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Assume AxPh. $w_{kh}(\underline{x}, \underline{y})$ is an injective function.

Axiom of Events

AxEv: *Every inertial observer coordinatizes the same events,*

i.e. *World-view function is total and bijective.*

$$(\forall k, h \in IOb)(\forall \underline{x} \in Q^d)(\exists \underline{y} \in Q)[ev_k(\underline{x}) = ev_h(\underline{y})]$$

or

$$(\forall k, h \in IOb)(\forall \underline{x} \in Q^d)(\exists \underline{y} \in Q)w_{kh}(\underline{x}) = (\underline{y})$$

Axioms to Coordinatize

AxSelf: Every inertial observer sees itself at the origin.
AxSym: Every inertial observer uses the same measure system.

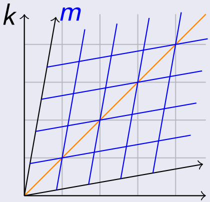
Thus Kinematics:

$$\text{SpecRelKin} = \{\text{AxPh}, \text{AxEv}, \text{AxSelf}, \text{AxSym}\}$$

Theorems

Theorem

w_{kh} is a Poincaré-transformation.



Theorem

NoFTL: There are no faster than light observers.

However „Kinematics is ready”, there is another kinematical axiom which is important for Dynamics.

Axiom of Thought Experiment

AxThExp:
For each observer, in every direction, with any speed less than that of light, it is possible to send out an inertial observer. The time of this observer flows „forwards".

$$(\forall k \in IOb)(\forall \underline{x}, \underline{y} \in Q^d)$$

$$\begin{pmatrix} \frac{\mathrm{Space}(\underline{x},\underline{y})}{\mathrm{Time}(\underline{x},\underline{y})} < 1 \\ \underline{x}_\tau < \underline{y}_\tau \end{pmatrix} \rightarrow (\exists h \in IOb) \begin{pmatrix} \underline{x}, \underline{y} \in wl_k(h) \\ w_{kh}(\underline{x})_\tau \leq w_{kh}(\underline{y})_\tau \end{pmatrix}$$

Where the world-line of b: $\underline{y} \in \mathrm{wl}_k(b) \iff_{\text{def.}} W(k, b, \underline{y})$

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Where the world-line of b: $\underline{y} \in \mathrm{wl}_k(b) \iff_{\text{def.}} W(k, b, \underline{y})$

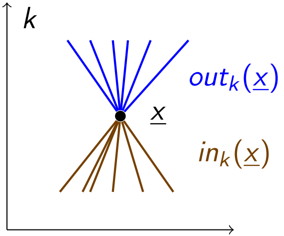
Again: $\exists$ plays the role of possibility!

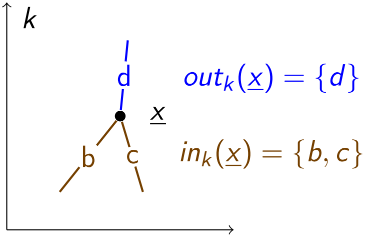
Dynamics

Definitions

Inelastic Collision

There is an inelastic collision exactly where two worldlines end and one begins.





$$\mathrm{in}_k(\underline{x}) \underset{\text{def.}}{=} \{b \in B : b \in \mathrm{ev}_k(\underline{x}) \wedge (\forall \underline{y} \in \mathrm{wl}_k(b))\underline{y}_\tau < \underline{x}_\tau \vee \underline{y} = \underline{x}\}$$

$$\mathrm{out}_k(\underline{x}) \underset{\text{def.}}{=} \{b \in B : b \in \mathrm{ev}_k(\underline{x}) \wedge (\forall \underline{y} \in \mathrm{wl}_k(b))\underline{y}_\tau > \underline{x}_\tau \vee \underline{y} = \underline{x}\}$$

Inelastic Collision

$$\mathrm{inecoll}_{k,\underline{x}}(b, c : d) \underset{\text{def.}}{\Longleftrightarrow} \begin{pmatrix} b \neq c \\ \mathrm{in}_k(\underline{x}) = \{b, c\} \\ \mathrm{out}_k(\underline{x}) = \{d\} \end{pmatrix}$$

This definition can be expressed in mere Kinematics.

But note that if $\mathrm{inecoll}_{k,\underline{x}}(b, c : d)$ holds, then $\underline{x}$ is **reserved**.

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$\Rightarrow$ this notion cannot be a notion of possible collision.

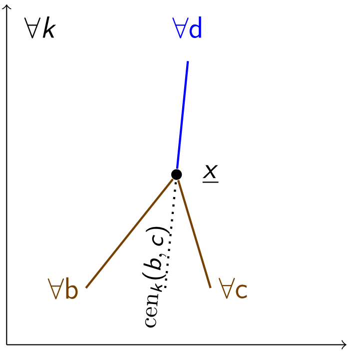
Anyway, it seems to work...

Axioms

Dynamics

AxCenter

AxCenter: *The world-line of inertial bodies involved in an inelastic collision is the continuation of the center of their mass.*



$$(\forall k \in IOb)(\forall b, c, d \in Ib)$$
$$[\mathrm{inecoll}(b, c : d) \to \mathrm{cen}_k(b, c) \cup wl_k(d) \subseteq \ell \text{ for some line } \ell]$$

Ib: inertial bodies, $\mathrm{cen}_k(b,c)$: centerline of mass.

AxSpeed

AxSpeed: *The rest mass and the speed determines the relativistic mass uniquely.*

$$(\forall k \in IOb)(\forall b, c \in Ib)\begin{pmatrix} m_0(b) = m_0(c) \\ v_k(b) = v_k(c) \end{pmatrix} \rightarrow m_k(b) = m_k(c)$$

where $m_0(b)$ is the rest mass, i.e. relativistic mass according to the observers seeing b at rest.

Collision-placing Axiom

Ax∀inecoll: Every possible collision is realizable: Every observer can observe an inelastic collision (somewhere) which is realized by arbitrary inertial bodies having arbitrary rest mass:

$$(\forall k \in IOb)(\forall \underline{v}_1, \underline{v}_2 \in Q^{d-1})(\forall m_1, m_2 \in Q)$$

$$\begin{pmatrix} |\underline{v}_1| < 1 \\ |\underline{v}_2| < 1 \\ m_1 > 0 \\ m_2 > 0 \end{pmatrix} \to (\exists b, c, d \in Ib) \left[\mathrm{inecoll}_k(b, c : d) \land \begin{pmatrix} \underline{v}_k(b) = \underline{v}_1 \\ \underline{v}_k(c) = \underline{v}_2 \\ m_0(b) = m_1 \\ m_0(c) = m_2 \end{pmatrix} \right]$$

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We are not allowed to choose the location!

Because two different inecolls cannot happen in the same space-time location...

Theorems

$$\mathsf{SpecRelDyn} = \mathsf{SpecRelKin} \cup \left\{ \begin{array}{l} \mathsf{AxThExp} \\ \mathsf{AxCenter} \\ \mathsf{AxSpeed} \\ \mathsf{Ax\forall inecoll} \end{array} \right\}$$

Theorems

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Theorem

$$m_0(b) = \sqrt{1 - v_k(b)^2} \cdot m_k(b)$$

Problems

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- **Possible bodies:** We could explain, that a possible body is possible because it does not exist, but there exists a specific way to make it exist. For different possible bodies, different ways. In a extensional framework such distinctions of ways are merged.

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- **Inelastic Collisions:** This merging became a real problem, because this is the reason why are we incapable to define the possible inelastic collisions.
- There are notions unavailable in the extensional setting: **the operational definition of mass**.

Why should we switch to intensional logic?

Operational definition of mass: We can define the atomic relation M with W and a relation E for etalons.

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Operational definition of mass: We can define the atomic relation M with W and a relation E for etalons.

So (with the very little help of E) we could use the language of Kinematics to describe Dynamics.

Idea

The concept of $\Diamond$

The informal description of axioms of SpecRel contained the word „possibility" were:

- „it is possible to send out a photon" (AxPh)
- „it is possible to send out an inertial observer" (AxThExp)
- „every possible collision is realizable" (Ax∀inecoll)

These are expressions of experimentation.

The concept of $\Diamond$

The intended meaning of the modal operators:

- $\Diamond\varphi$: *There is a situation accessible with an experiment such that* φ.
- $\Box\varphi$: *It does not matter, what experiment we make, the state will be that* φ.

The intended meanings of the elements of modal semantics:

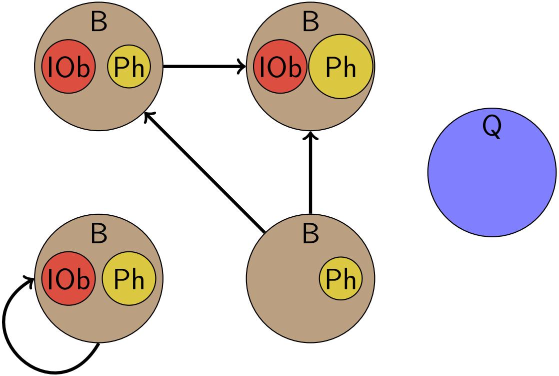
- Possible worlds: Space-times.
- Alternative relation: Experimentation.

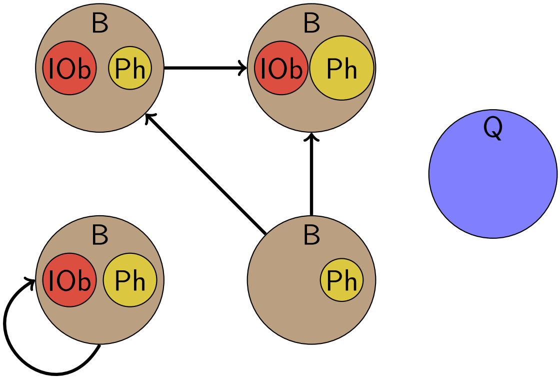
Plan

$$\text{SpecRel} \xrightarrow{\text{1. step}} \Diamond\text{SpecRel}_B \xrightarrow{\text{2. step}} \Diamond\text{SpecRel}_{BW}$$

$\Diamond\text{SpecRel}_B$	$\Diamond\text{SpecRel}_{BW}$
Only B changes	Only B and W change
Collision-placing axioms	Colliding axioms
„Unpacked SpecRel"	Operational def. of Mass
Simple translation available from/to SpecRel	

$\Diamond$SpecRel$_B$





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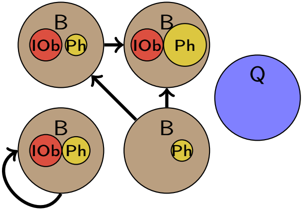

- One modal sort for Physics: $\langle S, R, B, IOb, Ph \rangle$
  - S is the set of (the names for) possible worlds (Space-times)
  - R is the relation connecting the accessible possible worlds.
  - B_w is the existing bodies in w.
    And the domain of quantification as well.
  - Ph_w is the set of photons in w.
  - IOb_w is the set of inertial observer in w.

$\Diamond SpecRel_B$

- And the connecting relations: $\langle W, M \rangle$
  - $W(k, b, \underline{x})_w$ is the world-view relation in w. $k \in IOb$.
  - $M(k, b, x)_w$ is the mass relation in w. $k \in IOb$ and M is a function.

Restrictions of the Atomic Predicates

$$
\begin{array}{rrcl}
\forall k & IOb(k) & \rightarrow & \Box IOb(k) \\
\forall p & Ph(p) & \rightarrow & \Box Ph(p) \\
\forall b & B(b) & \rightarrow & \Box B(b) \\
\forall kb\underline{x} & W(k,b,\underline{x}) & \rightarrow & \Box W(k,b,\underline{x}) \\
\forall kb\underline{x} & \neg W(k,b,\underline{x}) & \rightarrow & \Box \neg W(k,b,\underline{x}) \\
\forall kbx & M(k,b,x) & \rightarrow & \Box M(k,b,x) \\
 & \mu & \leftrightarrow & \Box \mu \\
 & t \neq s & \rightarrow & \Box (t \neq s) \\
 & (t = s & \rightarrow & \Box (t = s))
\end{array}
$$

where μ is a pure mathematical formula

Notes on the changes of Logic:

- Classical Tautologies
- The weakest modal logic (**K**) axioms:

$$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi) \qquad\qquad \frac{\varphi}{\Box\varphi}$$

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This is strong enough, but we could make this stronger:

- $A \to \Diamond A$ defines reflexivity:

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This is strong enough, but we could make this stronger:

- $A \to \Diamond A$ defines reflexivity:
  We could call it an experiment when we do nothing. („Empty experiment")
- $\Diamond\Diamond A \to \Diamond A$ defines transitivity:
  We could call the composition of experiments an experiment too.

But these are not necessary.

- Mathematics: Classic First Order Logic
- Physics: Classic First Order Logic except Substitution, which we replace with:

$$(\forall b \in B)((\forall c \in B)\varphi(c) \to \varphi(b/c))$$

i.e. we are able to substitute only the actually existing entities

$$\mathbf{B}(t) \to (\forall b \in B)(\varphi(t) \to \varphi(t/b))$$

$$\mathbf{B}(t) \to (\varphi(t/b) \to (\exists b \in B)\varphi(b))$$

Axioms

Rearrangement of Axioms

Axiom of Photon $\longrightarrow$ Axiom of Photon-Observation

Axiom of Photon $\xrightarrow{+\Diamond}$ Modal Axiom of Photon-Sending

Axiom of Event $\xrightarrow{+\boxdot}$ Modal Axiom of Event

Axiom of Symmetry $\xrightarrow{+\boxdot}$ Modal Axiom of Symmetry

Axiom of Thought Experiment $\xrightarrow{+\Diamond}$ Modal Axiom of Thought Experiment

Axiom of Inelastic Collisions $\xrightarrow{+\Diamond}$ Modal Axiom of Inelastic Collisions

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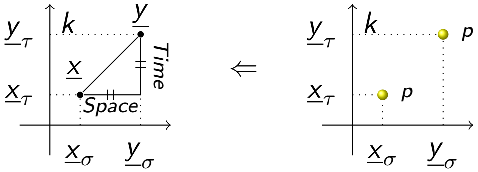

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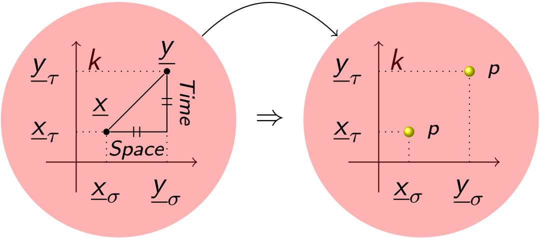
$$\left(\frac{\mathrm{Space}(\underline{x}, \underline{y})}{\mathrm{Time}(\underline{x}, \underline{y})} = 1 \leftarrow (\exists p \in Ph)(\underline{x}, \underline{y} \in wl_k(p)) \right)$$

AxPhExp

Axiom of Sending Out a Light Signal

In every direction it is possible to send out a photon.



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AxPh and the Notion of Event

In a modal setting, the formula

$$w_{kh}(\underline{x}, \underline{y}) \underset{?}{\Longleftrightarrow} \mathrm{ev}_k(\underline{x}) = \mathrm{ev}_h(\underline{y})$$

no longer defines a function.

Possible bodies made this definition to an injective function, so if we want to keep it, we have to go for them for the accessible possible worlds too...

$$w_{kh}(\underline{x}, \underline{y}) \underset{\text{def.}}{\Longleftrightarrow} \boxed{\boxdot}\,\mathrm{ev}_k(\underline{x}) = \mathrm{ev}_h(\underline{y})$$

where $\boxdot$ is the reflexive closure of $\Box$:

$$\boxdot\varphi \underset{\text{def.}}{\Longleftrightarrow} \Box\varphi \wedge \varphi$$

Ax$\boxdot$Ev

Ax$\boxdot$Ev: *The world-view function is total and bijective.*

$$(\forall k, h \in IOb)(\forall \underline{x} \in Q^d)(\exists \underline{y} \in Q^d)[\boxdot ev_k(\underline{x}) = ev_h(\underline{y})]$$

or

$$(\forall k, h \in IOb)(\forall \underline{x} \in Q^d)(\exists \underline{y} \in Q^d) w_{kh}(\underline{x}) = (\underline{y})$$

Axiom of Thought experiment

AxThExp: *For each observer, in every direction, with any speed less than that of light, it is possible to send out an inertial observer. The time of this observer flows „forwards"*

$$(\forall k \in IOb)(\forall \underline{x}, \underline{y} \in Q^d)$$

$$\left(\begin{array}{c} \frac{\mathrm{Space}(\underline{x},\underline{y})}{\mathrm{Time}(\underline{x},\underline{y})} < 1 \\ \underline{x}_\tau < \underline{y}_\tau \end{array} \right) \rightarrow \Diamond(\exists h \in IOb) \left(\begin{array}{c} \underline{x}, \underline{y} \in wl_k(h) \\ w_{kh}(\underline{x})_\tau \leq w_{kh}(\underline{y})_\tau \end{array} \right)$$

Ax∀inecoll: Every possible collision is realizable: Every observer can observe an inelastic collision (somewhere) which is realized by arbitrary inertial bodies having arbitrary rest mass:

$$(\forall k \in IOb)(\forall \underline{v}_1, \underline{v}_2 \in Q^{d-1})(\forall m_1, m_2 \in Q)(\forall \underline{x} \in Q^d)$$

$$\begin{pmatrix} |\underline{v}_1| < 1 \\ |\underline{v}_2| < 1 \\ m_1 > 0 \\ m_2 > 0 \\ \neg inecoll_{k,\underline{x}} \end{pmatrix} \rightarrow \Diamond(\exists b, c, d \in Ib)\left[\mathrm{inecoll}_{k,\underline{x}}(b, c : d) \wedge \begin{pmatrix} \underline{v}_k(b) = \underline{v}_1 \\ \underline{v}_k(c) = \underline{v}_2 \\ m_0(b) = m_1 \\ m_0(c) = m_2 \end{pmatrix}\right]$$

Theorems

Axiom system

$$\underbrace{\underbrace{\left\{\begin{array}{l} \mathsf{AxPhObs}\\ \mathsf{Ax}\Diamond\mathsf{PhExp}\\ \mathsf{Ax}\boxdot\mathsf{Ev}\\ \mathsf{AxSelf}\\ \mathsf{Ax}\boxdot\mathsf{Sym}\end{array}\right\}}_{\Diamond\mathsf{SpecRelKin}_B} \cup \left\{\begin{array}{l} \mathsf{Ax}\Diamond\mathsf{ThExp}\\ \mathsf{AxCenter}\\ \mathsf{AxSpeed}\\ \mathsf{Ax}\forall\Diamond\mathsf{inecoll}\end{array}\right\}}_{\Diamond\mathsf{SpecRelDyn}_B}$$

Theorems

Theorem (NoFTL)

There is no faster than light observer.

Theorem (Poincaré)

w_{kh} *is a Poincaré transformation.*

Theorem

$$m_0(b) = \sqrt{1 - v_k(b)^2} \cdot m_k(b)$$

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These proofs are so straightforward that the question arises: How could we systematize proving?

Systematize Proving

The idea behind this axiom system is the following „unpacking" translation:

$$
\begin{aligned}
m(A) &= A \\
m(\neg\varphi) &= \neg m(\varphi) \\
m(\varphi \supset \psi) &= m(\varphi) \supset m(\psi) \\
m(\exists x \varphi) &= \exists x\, m(\varphi) \\
m(\exists b \varphi) &= \boxed{\Diamond\!\!\cdot}\,\exists b\, m(\varphi) \\
&= \Diamond \exists b\, m(\varphi) \vee \exists b\, m(\varphi)
\end{aligned}
$$

Where A is an atomic formula.

$$
\left(
\begin{aligned}
m(\forall b \varphi) &= \boxed{\cdot}\,\forall b\, m(\varphi) \\
&= \Box \forall b\, m(\varphi) \wedge \forall b\, m(\varphi)
\end{aligned}
\right)
$$

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Strengthening the Logic

Strengthening to K4
i.e.
make R transitive

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Weakening the Translation

Introduce a transitive closure $\Diamond^{*}$

$$\Diamond\varphi \to \Diamond^{*}\varphi$$

$$\Diamond\Diamond^{*}\varphi \to \Diamond^{*}\varphi$$

$$(\Box\varphi \wedge \Box^{*}(\varphi \to \Box\varphi)) \to \Box^{*}\varphi \ ?$$

$$\neg B(x) \to \Diamond^{*}B(x)$$

replace the modality of m.

Systematize Proving

Either way we choose:

Theorem

$$SpecRelKin \vdash \varphi \Longrightarrow \Diamond SpecRelKin_B^* \vdash m(\varphi)$$

Systematize Proving

But we don't have

$$\mathsf{SpecRelDyn} \vdash \varphi \Longrightarrow \Diamond\mathsf{SpecRelDyn}^*_B \vdash m(\varphi)$$

because $\mathsf{Ax}\forall\Diamond\mathrm{inecoll}$ needs free space to place the collision.
So if we accept the assumption that there is a Free Coordinate for Collision

$$\mathbf{FCC}{:}(\exists k \in IOb)(\exists \underline{x} \in Q^d)\neg(\exists b,c,d \in Ib)\mathrm{inecoll}_{k,\underline{x}}(b,c:d)$$

then we have

Theorem

$$\mathit{SpecRelDyn} \vdash \varphi \Longrightarrow \Diamond \mathit{SpecRelDyn}^*_B \cup \{\mathbf{FCC}\} \vdash m(\varphi)$$

The other way – Consistency

The „merging”/„collapting”/„packing” translation:

$$
\begin{aligned}
m^-(A) &= A \\
m^-(\neg\varphi) &= \neg m^-(\varphi) \\
m^-(\varphi \supset \psi) &= m^-(\varphi) \supset m^-(\psi) \\
m^-(\exists x \varphi) &= \exists x\, m^-(\varphi) \\
m^-(\exists b \varphi) &= \exists b\, m^-(\varphi) \\
m^-(\Diamond\varphi) &= m^-(\varphi) \\
m^-(\Box\varphi) &= m^-(\varphi)
\end{aligned}
$$

Where A is an atomic formula.

$$\left(\ m^-(\Box\varphi) \;=\; m^-(\varphi)\ \right)$$

The other way – Consistency

Theorem

$$\Diamond SpecRelKin_B \vdash \varphi \Longrightarrow SpecRelKin \vdash m^-(\varphi)$$

But again, we do not have

$$\Diamond \mathrm{SpecRelDyn}_B \vdash \varphi \Longrightarrow \mathrm{SpecRelDyn} \vdash m^-(\varphi)$$

because Ax$\forall\Diamond$inecoll again. It fills an empty universe with „starting bodies''. It ensures that in every space-time location there is at least two body starting there.

The other way – Consistency

But with the old Ax∀inecoll we can get

Theorem

$\Diamond SpecRelDyn_B \cup \{Ax\forall inecoll\} \vdash \varphi \Longrightarrow SpecRelDyn \vdash m^-(\varphi)$

This means that we have two more theorem too:

Theorem

$\Diamond SpecRelDyn_B \cup \{Ax\forall inecoll\}$ *is consistent.*

Theorem

$\Diamond SpecRelDyn_B$ *is consistent.*

Colliding

Idea

How do we collide?

i.e.

How do we postulate a new body and a new world where the all necessary inelastic collision caused by it happened already?

Colliding

Start with:

- Delete

$$W(k, b, \underline{x}) \to \Box W(k, b, \underline{x})$$

We will replace this with the Colliding axioms.

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  For each observer, in every direction, with any mass according to that observer, with any speed less than that of light, it is possible to send out a body.

Colliding

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  $$W(k, b, \underline{x}) \rightarrow \Box W(k, b, \underline{x})$$

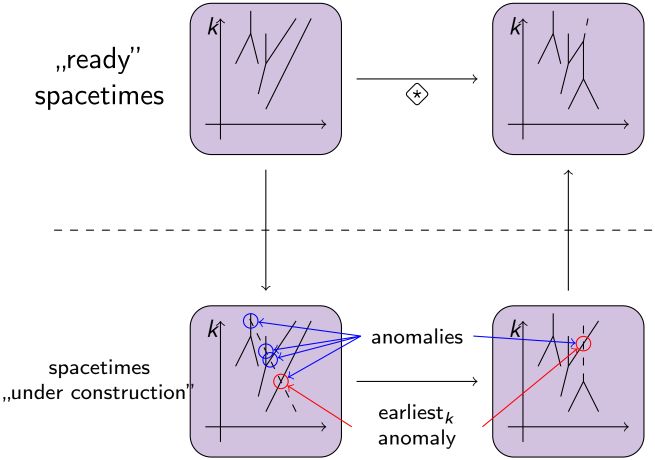
  We will replace this with the Colliding axioms.
- Replace Ax$\forall\Diamond$inecoll, with AxBSend:
  For each observer, in every direction, with any mass according to that observer, with any speed less than that of light, it is possible to send out a body.
- Restrict Ax$\Diamond$PhExp, Ax$\Diamond$ThExp and AxBExp^- for space-times free from anomalies:
  Two bodies do not share the same space-time location without colliding.

Colliding

So start with the physical axioms

$$\underbrace{\underbrace{\left\{\begin{array}{l}\mathrm{AxPhObs}\\ \mathrm{Ax}\Diamond\mathrm{PhExp}^-\\ \mathrm{Ax}\boxdot\mathrm{Ev}\\ \mathrm{AxSelf}\\ \mathrm{Ax}\boxdot\mathrm{Sym}\end{array}\right\}}_{\Diamond\mathrm{SpecRelKin}_B} \cup \left\{\begin{array}{l}\mathrm{AxThExp}^-\\ \mathrm{AxCenter}\\ \mathrm{AxSpeed}\\ \mathrm{AxBExp}^-\end{array}\right\}}_{\Diamond\mathrm{SpecRelDyn}_B}$$

Plan
The Big Picture



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Manual for colliding:

1. We pin an observer, whom perspective we will do the colliding.
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   This may crossing through bodies without colliding.
   We will use the word anomaly for the phenomenon when two body share the same space-time location without collision
3. For erasing/solving such anomalies, we use a While-do algorythm:
   `While` there is anomaly, postulate a world, where:

Colliding

While-do

Colliding
While-do

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Colliding

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  1. the world-lines of bodies NOT involved in the anomaly are preserved, and where

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While-do

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  1. the world-lines of bodies NOT involved in the anomaly are preserved, and where
  2. the world-lines of bodies involved in the anomaly are
     1. deleted if it is coordinatized later than the observer measures the anomaly.

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While-do

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     2. preserved if it is coordinatized sooner than or it is at the same when the observer measures the anomaly.
  3. there exists a body whose worldline begins at the coordinates of the anomaly (of the previous world). (AxCenter still works)
  4. there are no other new body.

The real $\Diamond$ of $\Diamond\mathsf{SpecRel}_{BW}$

With this solution, we will have two type of worlds:

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- The type we want: Space-times without anomalies

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With this solution, we will have two type of worlds:

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The real $\Diamond$ of $\Diamond$SpecRel$_{BW}$

With this solution, we will have two type of worlds:

- The type we want: Space-times without anomalies
- The type we have to use: Space-times with anomalies.

We could take the transitive closure of the accessibility relation:

$$\Diamond\varphi \rightarrow \circledast\varphi$$
$$\Diamond\circledast\varphi \rightarrow \circledast\varphi$$

$$\blacklozenge A \underset{\text{def.}}{\Longleftrightarrow} \text{NoAnomaly} \wedge \circledast(A \wedge \text{NoAnomaly})$$

$\blacklozenge$ will be the the main modal operator in this system.

Outlines of the Formalism

$$\mathrm{AnomalyPos}_k(\underline{x}) \iff_{\text{def.}} (\exists b, c \in CB)\begin{pmatrix} b \neq c \\ b, c \in \mathrm{in}_k(\underline{x}) \\ b, c \in \mathrm{out}_k(\underline{x}) \end{pmatrix}$$

$$\mathrm{MinAnomalyPos}_k =_{\text{def.}} \min(\mathrm{AnomalyPos}_k)$$

$$\mathrm{MinAnomalyPart}_k(b) \iff_{\text{def.}} \begin{pmatrix} b \in \mathrm{in}_k(\min(\mathrm{Anomaly}_k)) \\ b \in \mathrm{out}_k(\min_k(\mathrm{Anomaly}_k)) \end{pmatrix}$$

$$\mathrm{NoAnomaly} \iff_{\text{def.}} \neg(\exists k \in IOb)(\exists \underline{x} \in Q^d)\ \mathrm{AnomalyPos}_k(\underline{x})$$

Axioms:

$$\begin{aligned} \mathsf{AxPhExp}^- &\iff \mathsf{noanomaly} \rightarrow \mathsf{AxPhExp} \\ \mathsf{AxThExp}^- &\iff \mathsf{noanomaly} \rightarrow \mathsf{AxThExp} \\ \mathsf{AxSendBody}^- &\iff \mathsf{noanomaly} \rightarrow \mathsf{AxThExp} \end{aligned}$$

Space-time locations preserved:

$$(\forall b \in B)(\forall k \in IOb)(\forall \underline{x} \in Q^d)$$

$$\begin{pmatrix} \mathrm{owner}(b) = k \\ (\underline{x}_t < \mathrm{MinAnomalyPos}_{kt} \vee \neg \mathrm{MinAnomalyPart_k(b)}) \\ W(k, b, \underline{x}) \end{pmatrix} \rightarrow \Box W(k, b, \underline{x})$$

Space-time locations replaced:

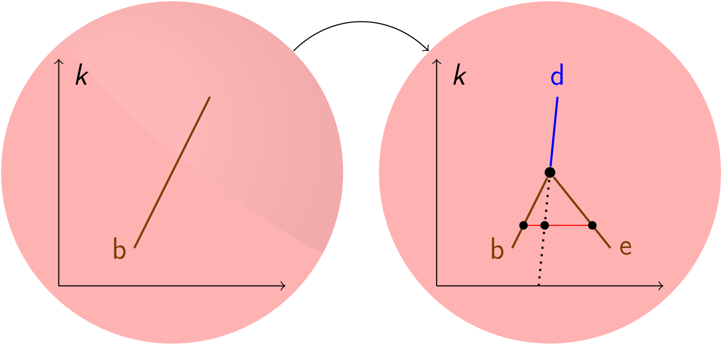
$$(\forall b \in B)(\forall k \in IOb)(\forall \underline{x} \in Q^d)(\forall \underline{y} \in Q^d)$$

$$\begin{pmatrix} \mathrm{owner}(b) = k \\ \underline{y} = \mathrm{MinAnomalyPos}_k \\ \underline{x}_t > \mathrm{MinAnomalyPos}_{kt} \\ \mathrm{MinAnomalyPart_k(b)} \\ W(k, b, \underline{x}) \end{pmatrix} \rightarrow \begin{pmatrix} \Box \neg W(k, b, \underline{x}) \\ \Diamond (\exists c \in B) \mathrm{min}(wl_k(c)) = \underline{y} \end{pmatrix}$$

Upper bound for the growing of Domain:

$$BF^{-1} : (\exists b \in B)(\forall c \in B)[b \neq c \wedge \Diamond E(c)] \rightarrow E(c))$$

Operational definition of mass



$$m_k(b) \underset{\text{def.}}{=} \begin{cases} 1, \text{ if } b \in E_k \\ m, \text{ if } \blacklozenge(\exists e \in E_k)(\exists d \in B)[\mathrm{inecoll}_k(b, e : d) \wedge \\ \quad (\exists t < \mathrm{inecoll}_k(b, e : d)_\tau)\ m = \frac{|wl_k(e,t) - \overline{wl_k(d,t)}|}{|wl_k(b,t) - \overline{wl_k(d,t)}|}\Big] \end{cases}$$

Where $E(k, b)$: *b is an etalon body of the observer k.*

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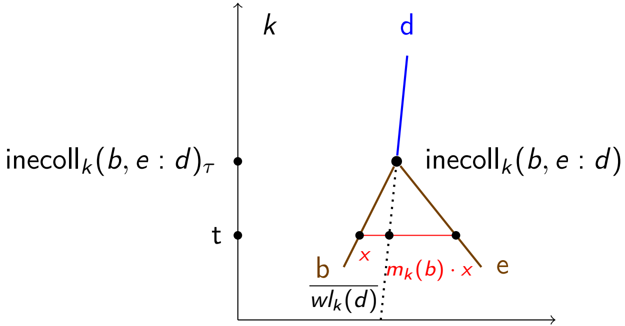

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Fin

Thank you for your attention.