Our Beloved Leon Henkin

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1. Life

2. Two of Henkin's contributions

3. Offsprings of Henkin's papers

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- I am presenting this paper here because Henkin acts as an emotional bond between Istvan and me. Henkin was the first person to introduce Istvan's and Hajnal's work to me in 1982

He was conscious that we live in history and can hardly escape. This is quoted from a thought-provoking paper on the history of mathematical education:

> *"Waves of history wash over our nation, stirring up our society and our institutions.*
>
> *Soon we see changes in the way that all of us do things, including our mathematics and our teaching. These changes form themselves into rivulets and streams that merge at various angles with those arising in parts of our society quite different from education, mathematics, or science. Rivers are formed, contributing powerful currents that will produce future waves of history.*
>
> *The Great Depression and World War II formed the background of my years of study; the Cold War and the Civil Rights Movement were the backdrop against which I began my career as a research mathematicians, and later began to involve myself with mathematics education."*

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- At Princeton University he completed his Ph.D program in mathematics, interrupted by four years of work as a mathematician in the famous Manhattan Project —the period May, 1942-March,1946—. *The Completeness of Formal Systems* is the title of his dissertation written under Alonzo Church that was defended in 1947 at Princeton University

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- He joined the maths department at the University of Southern California in 1949 and UC Berkeley in 1953. It was Alfred Tarski, the founder in 1942 of the center for the study of logic and foundations who called Henkin

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     - or what Henkin termed spoons, having a cycle and a handle.
- The reason being that induction axiom always drag along either the first or the second Peano axioms for Arithmetic

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  1. **Completeness in the theory of types**, of 1950
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- From 1: we learn that a modification of the semantics can adapt validities (in the new semantics) to logical theorems
- From 2: many-sorted logic plays an important role

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- The new logic will also be complete

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Offsprings of Henkin's papers

Hybrid Type Theory: A Quartet in Four Movements. Areces, Blackburn, Huertas & Manzano

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  4. Henkin's completeness




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  2. $\mathcal{A}, w \Vdash @_i\varphi$ iff $\mathcal{A}, v \Vdash \varphi$ v being the unique element of W where i is true

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4. $\delta :=$ *Things existed at the dawn of time.*

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  2. in $\mathcal{HTT}$ we use **rigid terms**

Thanks, Leon

- Would you like to participate in a book about Leon Henkin?