spacetime singularities, holes, and extensions

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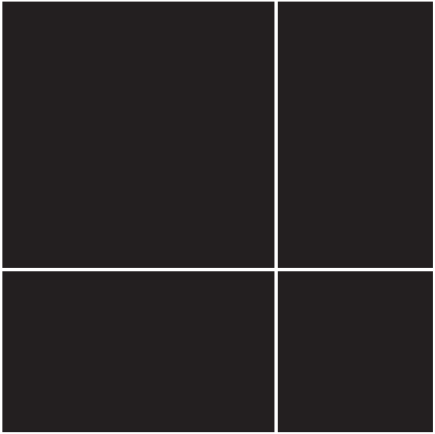
i. introduction

singularities, holes, and extensions: how are they related?

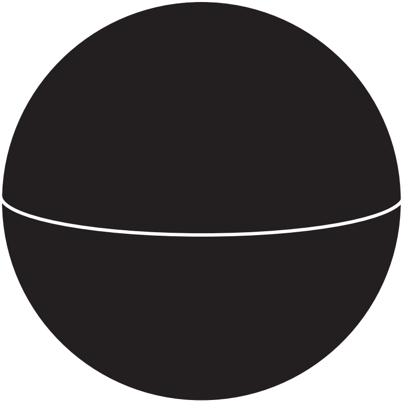
ii. preliminaries

a **spacetime** is a pair (M, g).

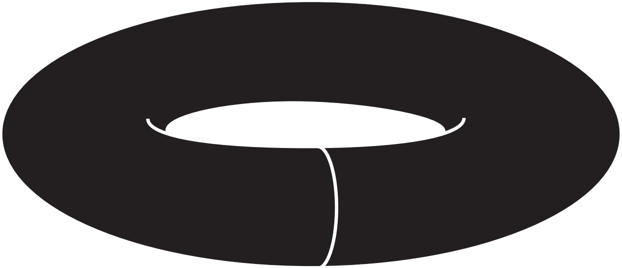
M is the spacetime **manifold**.



the plane

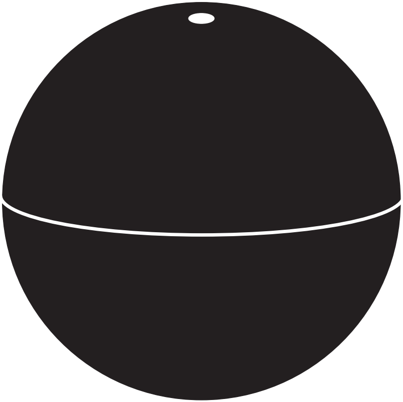


the sphere



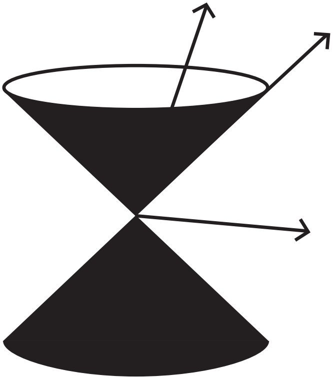
the torus

the cylinder

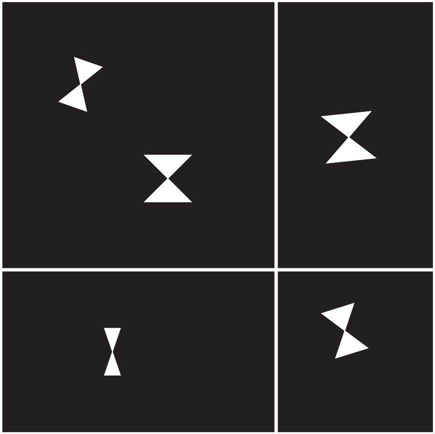


the sphere with point removed

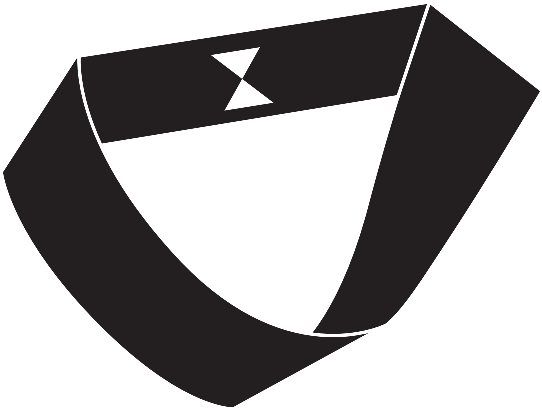
g is the spacetime **metric**.



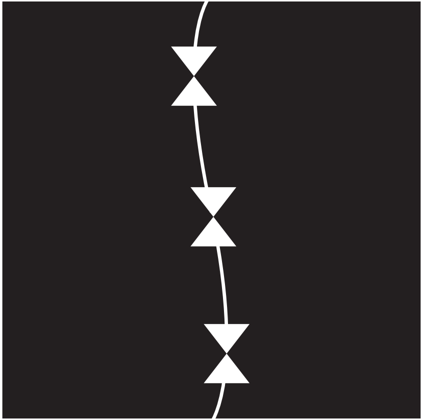
timelike, null, and spacelike vectors



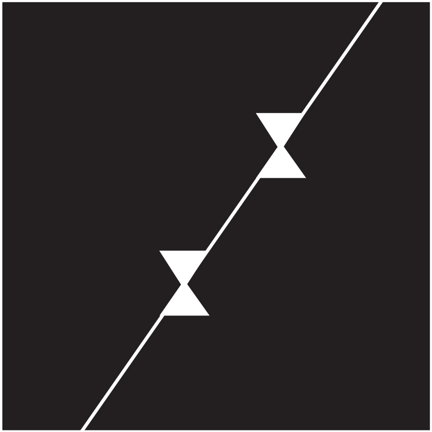
a spacetime



a spacetime which is not time-orientable a curve is **timelike** if all of its tangent vectors are timelike.

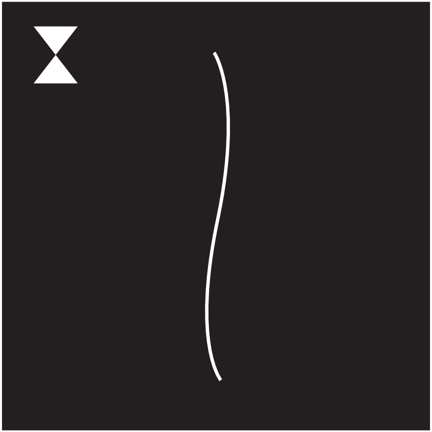


a timelike curve



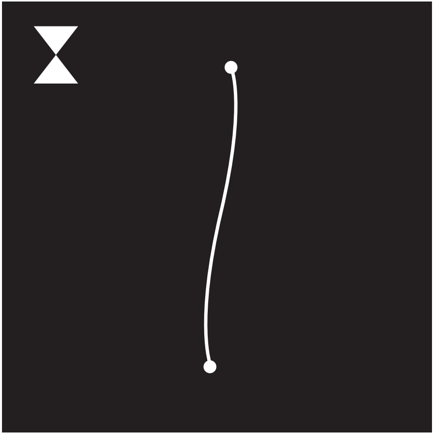
a null curve

a curve is not **maximal** if it can be smoothly extended into a larger curve.

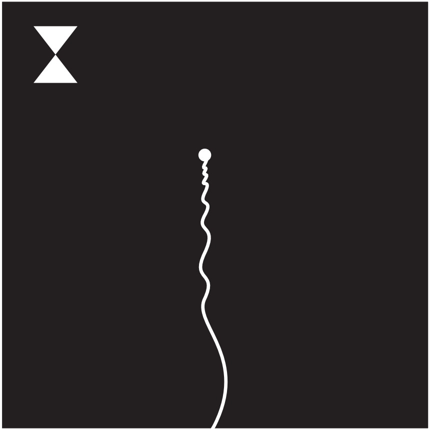


a timelike curve which is not maximal

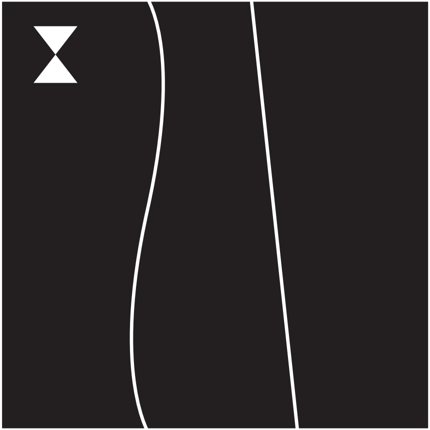
a causal curve may have **past/future endpoints**.



a timelike curve with future and past endpoints



a maximal timelike curve with future endpoint a non-accelerated curve is a **geodesic**.

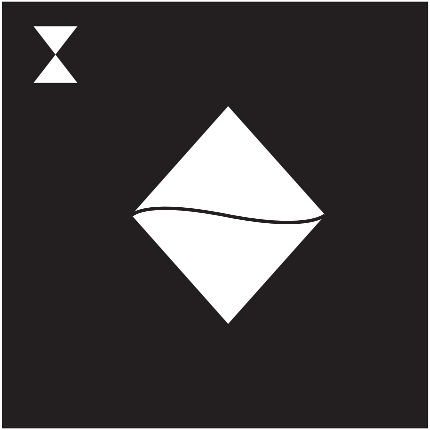


accelerated and geodesic timelike curves a surface is **spacelike** if every curve within the surface is a spacelike curve.

a set is **achronal** if it is not intersected more than once by any timelike curve.

an achronal spacelike surface

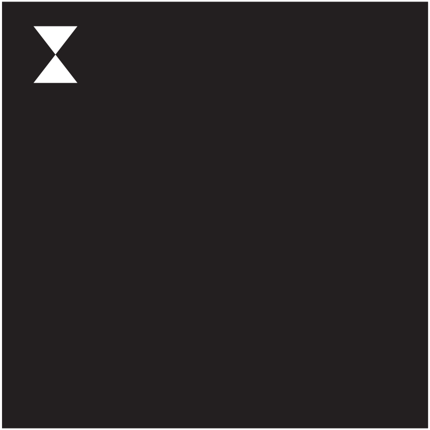
a spacelike surface which is not achronal the **domain of dependence** of a given set, is the collection of points p such that every causal curve through p without endpoint intersects the given set.



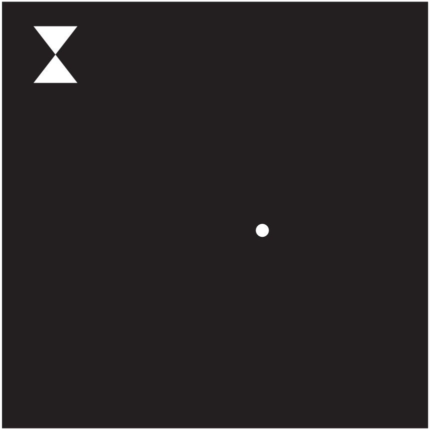
the domain of dependence of a spacelike surface

iii. singularities and holes

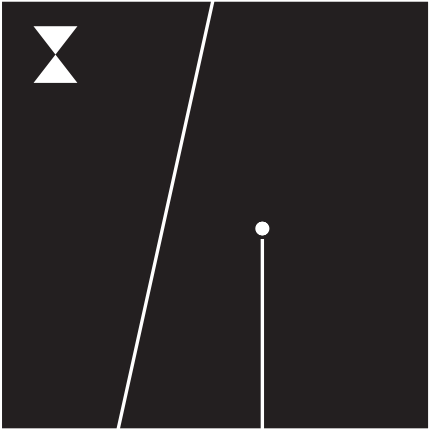
a maximal geodesic is **complete** if the parameter time goes from negative infinity to positive infinity.



minkowski spacetime



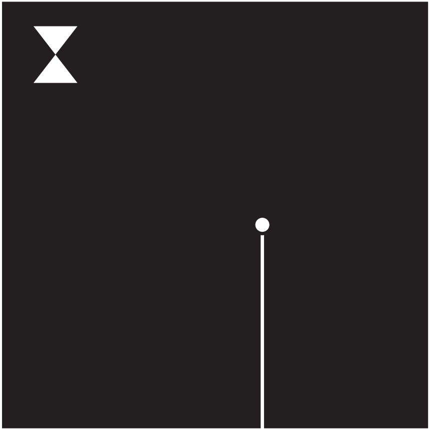
minkowski spacetime with a point removed



complete and incomplete geodesics

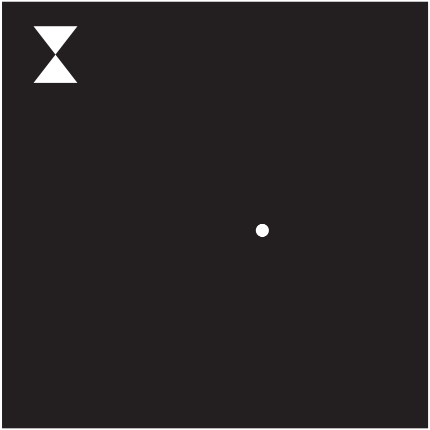
a timelike incomplete geodesic represents a freely falling observer who does not record all possible watch readings.

a causal geodesic without future endpoint is **future-incomplete** if the parameter time does not go to positive infinity.

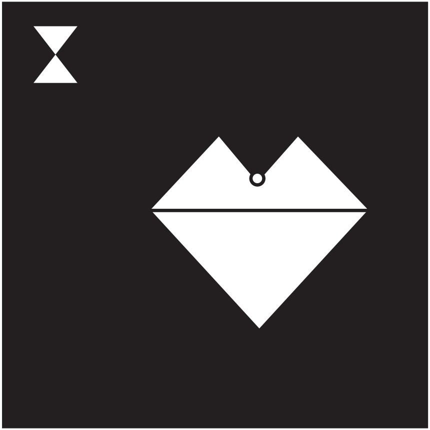


a past-complete but future-incomplete geodesic a spacetime is **geodesically complete** (gc) if it does not contain an incomplete geodesic.

(gc) is a strong condition; it seems that all physically reasonable spacetimes are geodesically incomplete.



minkowski spacetime with a point removed

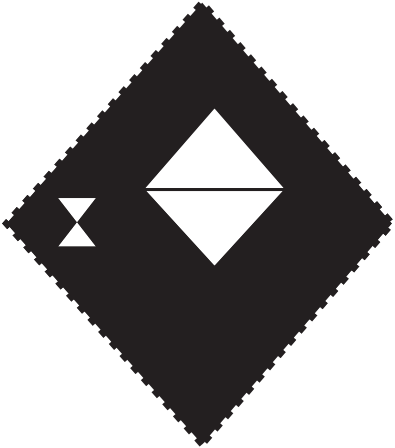


the domain of dependence of a spacelike surface a spacetime is **hole-free** (hf) if, for every spacelike surface S and every metric preserving embedding of its domain of dependence into some other spacetime, the domain of dependance of the image of S is identical to the image of the domain of dependence of S.

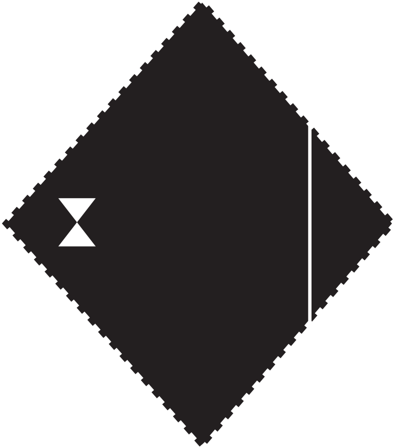
(hf) requires that the domain of dependence of every spacelike surface is “as large as it could have been”.

what is the relationship between (gc) and (hf)?

$$(hf) \not\Longrightarrow (gc)$$



a spacetime which is hole-free...

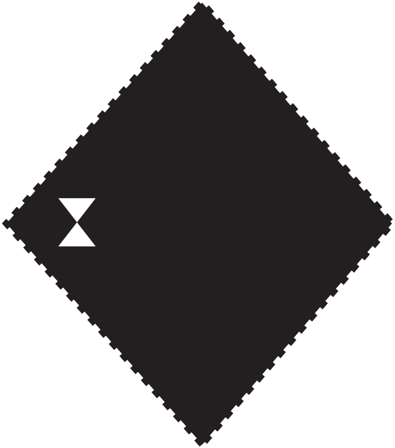


...but geodesically incomplete

$$(\mathrm{gc}) \overset{?}{\Longrightarrow} (\mathrm{hf})$$

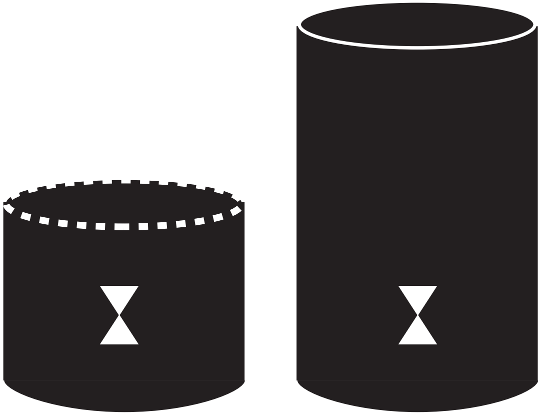
$$(gc) \Longrightarrow \; ? \; \Longrightarrow (hf)$$

iv. singularities and extensions



a hole-free spacetime

a spacetime is **inextendible** (i) if it cannot be properly embedded, while preserving all metric structure, into another spacetime.

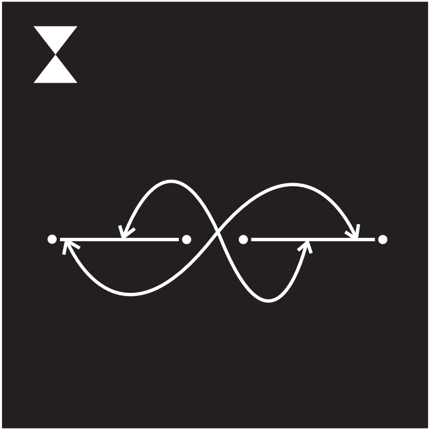


a spacetime and one of its extensions

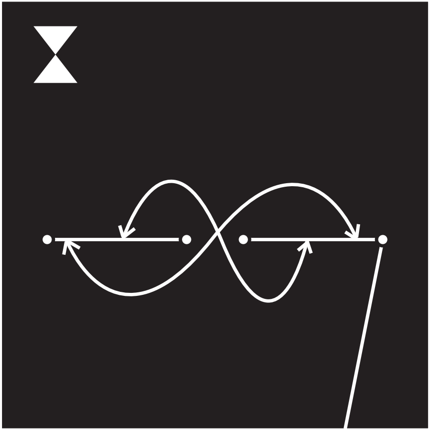
what is the relationship between (i) and (gc)?

$$\text{(gc)} \Longrightarrow \text{(i)}$$

$$(i) \not\Longrightarrow (gc)$$



a spacetime which is inextendible...

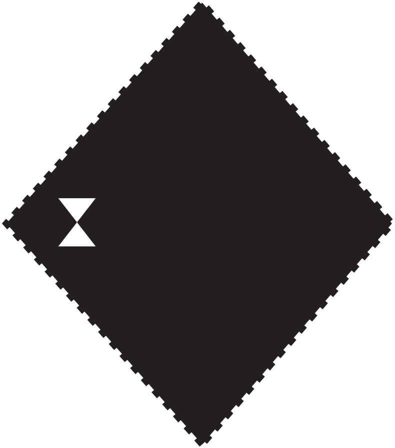


...but geodesically incomplete

$$\text{(gc)} \Longrightarrow \; ? \; \Longrightarrow \text{(i)}$$

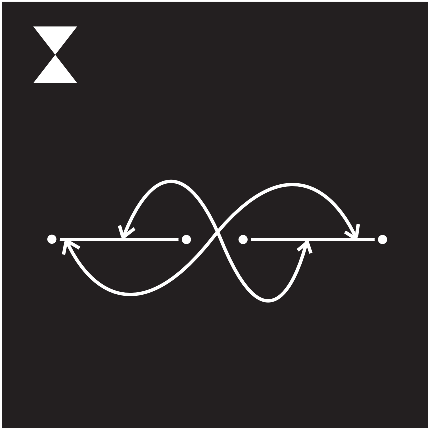
v. holes and extensions

what is the relationship between (hf) and (i)?



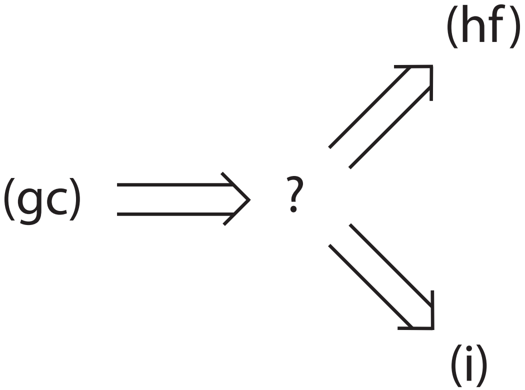
a spacetime which is hole-free but extendible

$$(hf) \not\Longrightarrow (i)$$



a spacetime which is inextendible but not hole-free

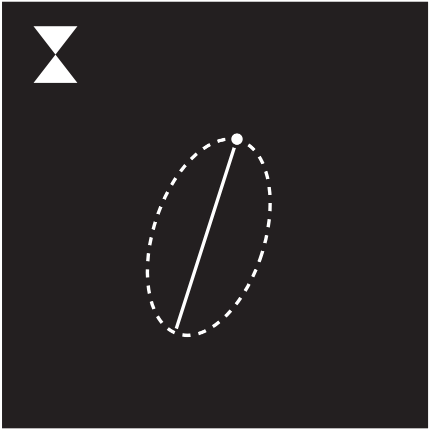
$$(\mathrm{i}) \quad \not\Longrightarrow \quad (\mathrm{hf})$$


(hf)
(gc)
?
(i)

vi. an intermediate condition

a spacetime is **effectively complete** (ec) if, for every future or past incomplete timelike geodesic, and every open set containing it, there is no metric preserving embedding of the set into some other spacetime such that the image of the curve has future and past endpoints.

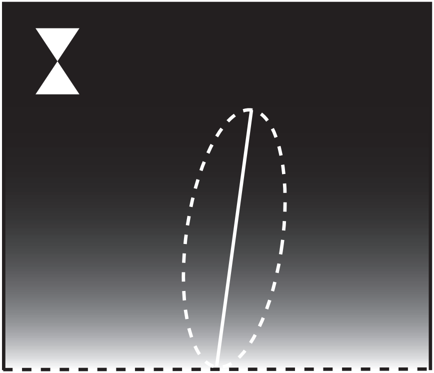
if a spacetime fails to be effectively complete, then there is a freely falling observer who never records some particular watch reading but who “could have” in the sense that nothing in the vicinity precludes it.



a spacetime which is effectively incomplete

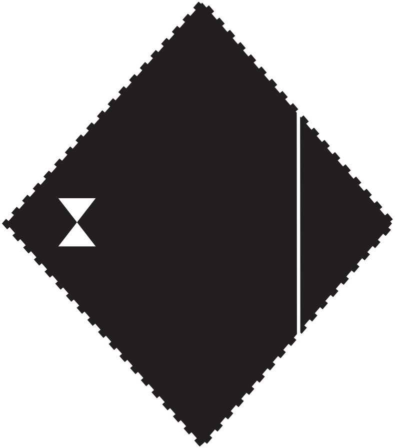
$$(\mathrm{gc}) \Longrightarrow (\mathrm{ec})$$

$$\text{(ec)} \quad \not\Longrightarrow \quad \text{(gc)}$$

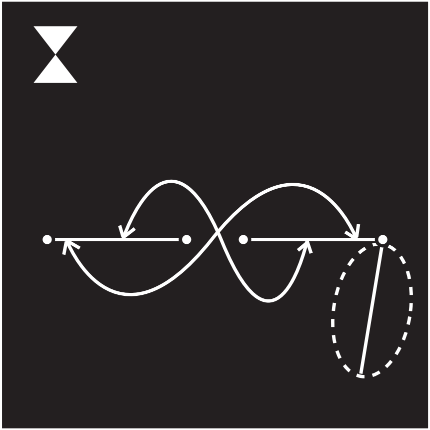


a spacetime which is effectively but not geodesically complete

$$(hf) \;\not\Longrightarrow\; (ec)$$



a spacetime which is hole-free but effectively incomplete

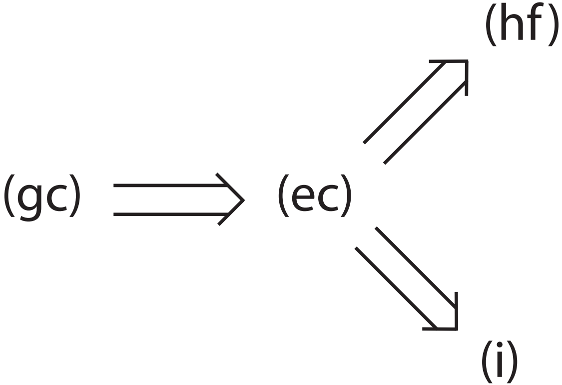


a spacetime which is inextendible but effectively incomplete

$$(\mathrm{i}) \;\not\Longrightarrow\; (\mathrm{ec})$$

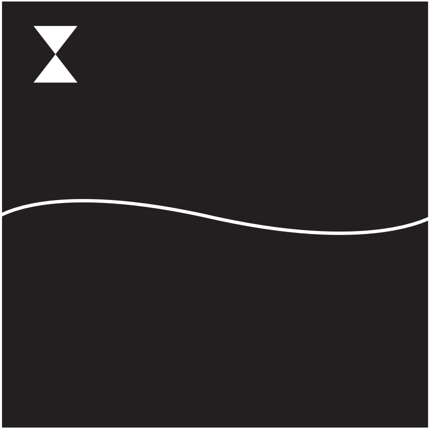
$$(ec) \Longrightarrow (i)$$

$$\text{(ec)} \Longrightarrow \text{(hf)}$$


(gc)
(ec)
(hf)
(i)

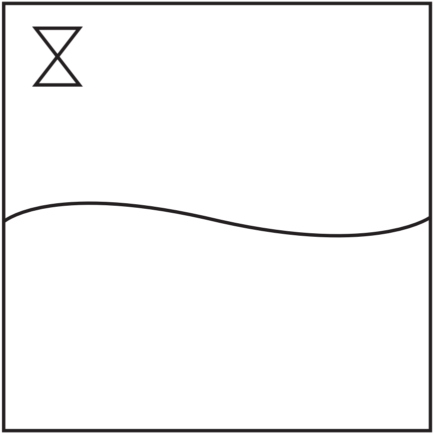
vii. conclusion

a **slice** is a set which is closed, achronal, and without edge.



a slice

a spacetime is **globally hyperbolic** if it contains a slice whose domain of dependence is identical to the manifold.

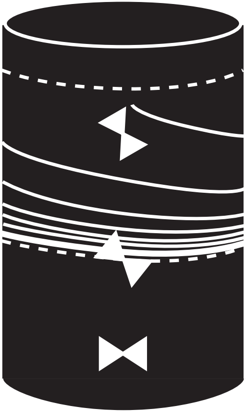


a globally hyperbolic spacetime

cosmic censorship conjecture: all physically reasonable spacetimes are globally hyperbolic.

$$(\mathrm{gc}) \Longrightarrow (\mathrm{ec}) \Longrightarrow (\mathrm{i}) \Longrightarrow (\mathrm{hf})$$

(under the assumption of global hyperbolicity) are there spacetimes which are intuitively physically reasonable but count as effectively incomplete?



a physically reasonable but effectively incomplete spacetime?

thank you.