# Do faster than light particles violate special relativity?

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- We will investigate this question within a first-order logic frame.

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- We will build up relativistic dynamics in first-order logic, obtaining axiom system SRdyn and we will see that the existence of FTL particles is independent of SRdyn.

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- Axiom system SR for kinematics contains Einstein's two original postulates and some auxiliary axioms implicitly assumed by Einstein.
- Axiom system SRdyn for dynamics consists of SR and the axioms for dynamics of relativity theory, which are some natural assumptions on collision of particles. The axioms concern FTL particles, too.

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To axiomatize relativity theory within logic, we need a language (set of basic concepts).

Language: $\{B, \quad Q, \quad \}$


B $\leftrightsquigarrow$ Bodies (things that move)

Q $\leftrightsquigarrow$ Quantities

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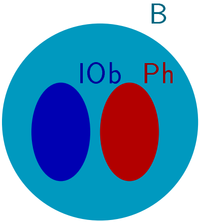
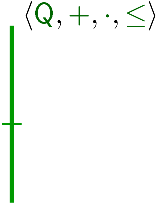
Language: $\{\mathsf{B}, \mathsf{IOb}, \mathsf{Ph}, \mathsf{Q}, +, \cdot, \leq, \qquad\qquad \}$

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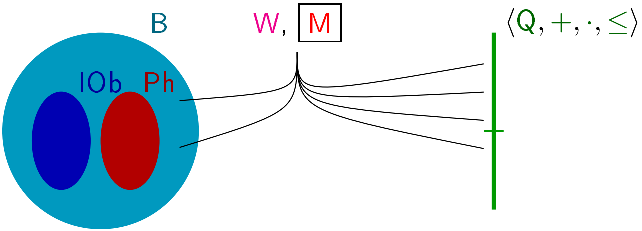
$+$, $\cdot$ and $\leq$ $\leftrightsquigarrow$ field operations and ordering

Language: $\{ B, IOb, Ph, Q, +, \cdot, \leq, W, \boxed{M} \}$

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IOb ⇝ Inertial Observers    Ph ⇝ Photons (light signals)

Q ⇝ Quantities

$+$, $\cdot$ and $\leq$ ⇝ field operations and ordering

W ⇝ Worldview (a 6-ary relation of type BBQQQQ )

M ⇝ Mass function,    $M : B \times B \longrightarrow Q$

$\mathsf{W} \subseteq \mathsf{IOb} \times \mathsf{B} \times \mathsf{Q}^4$ $\qquad$ $\mathsf{Q}^4$ is the coordinate system.

$\mathsf{W}(i, b, \bar{x}) \leftrightsquigarrow$

" Inertial observer $i$ coordinatizes body $b$ at spacetime location $\bar{x}$."

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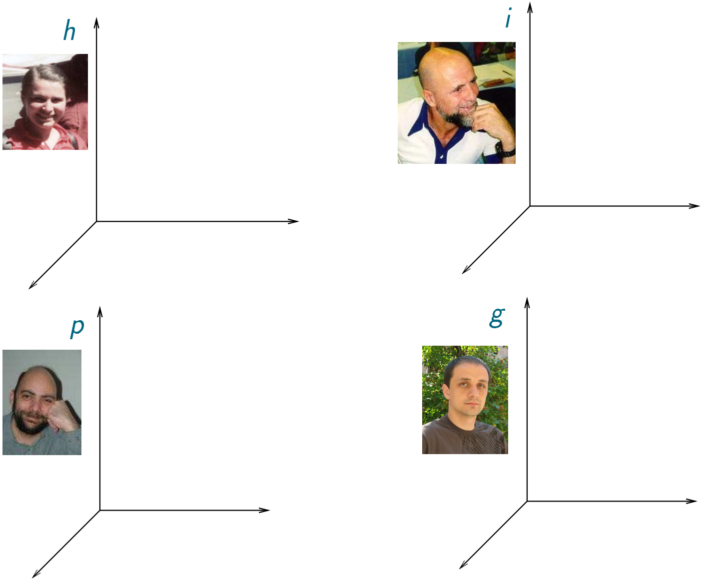
$\mathsf{M} : \mathsf{IOb} \times \mathsf{B} \longrightarrow \mathsf{Q}$.

$\mathsf{m}_i(b) := \mathsf{M}(i, b)$ is the relativistic mass of body $b$ according to observer $i$.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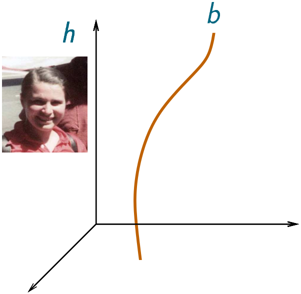
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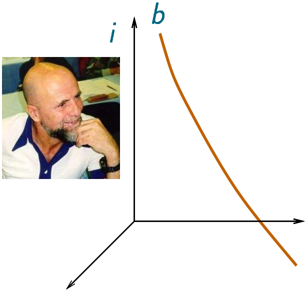
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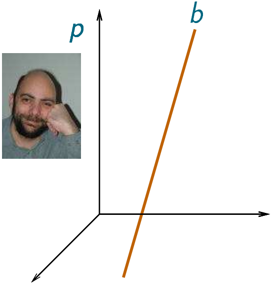
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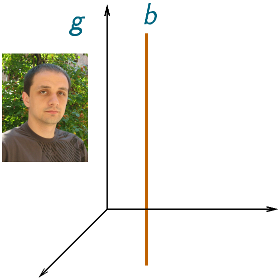
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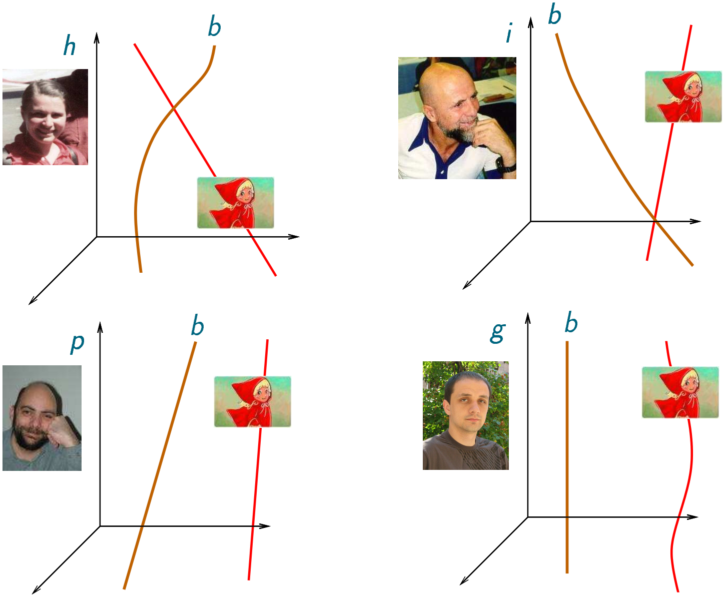
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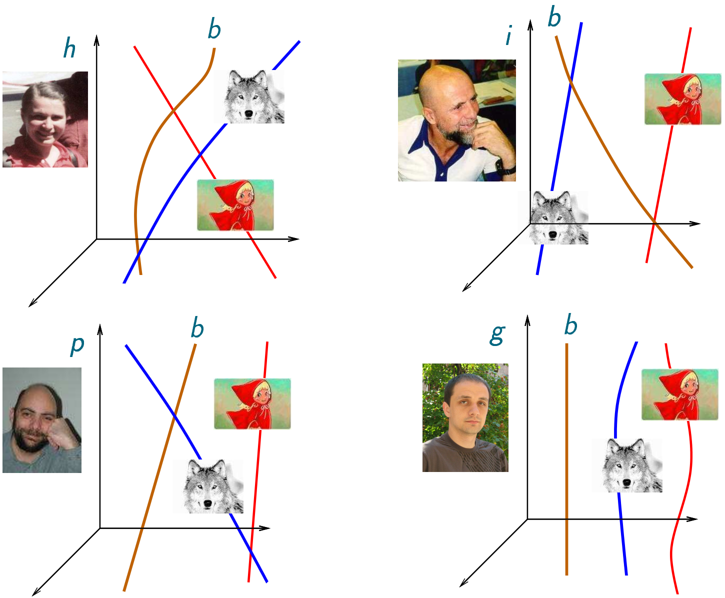
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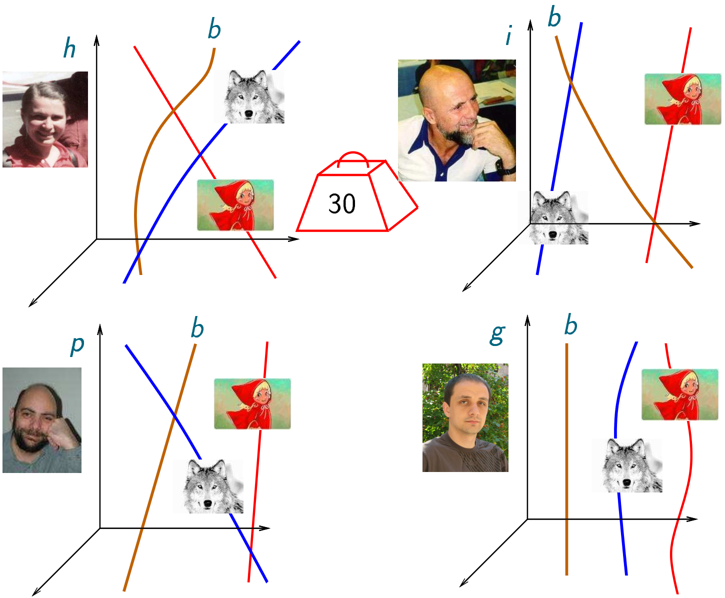
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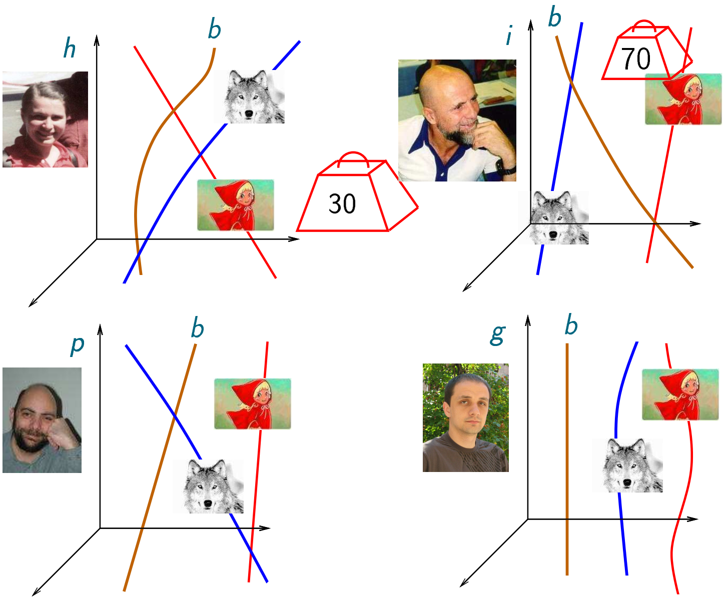
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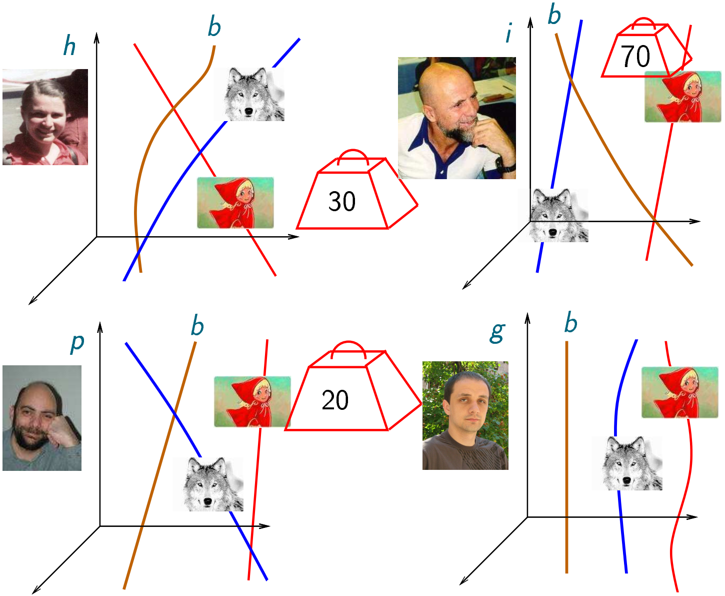
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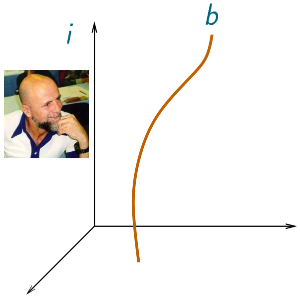
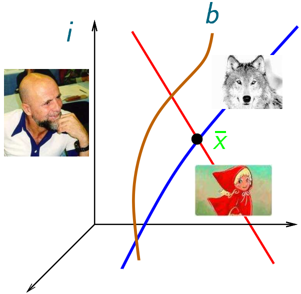



world-line of body $b$ as “seen” by inertial observer $i$
is the set of coordinate-points where $i$ “sees” $b$ by W:
$wline_i(b) := \{\, \bar{x} \in \mathrm{Q}^4 \;:\; \mathrm{W}(i, b, \bar{x}) \,\}$

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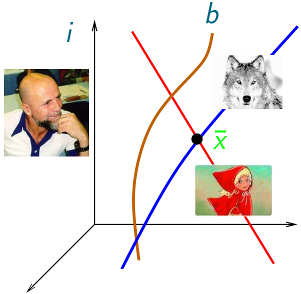


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world-view transformation $w_{hi}$ between inertial observers $h$ and $i$ is the set of pairs of coordinate-points $\langle \bar{x}, \bar{y} \rangle$ such that $h$ and $i$ see the same event in $\bar{x}$ and $\bar{y}$, respectively:
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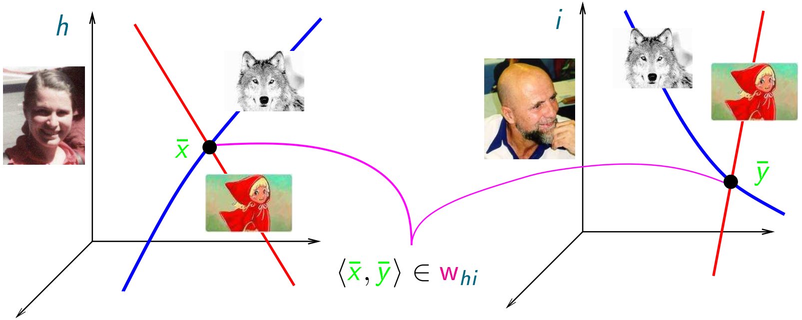

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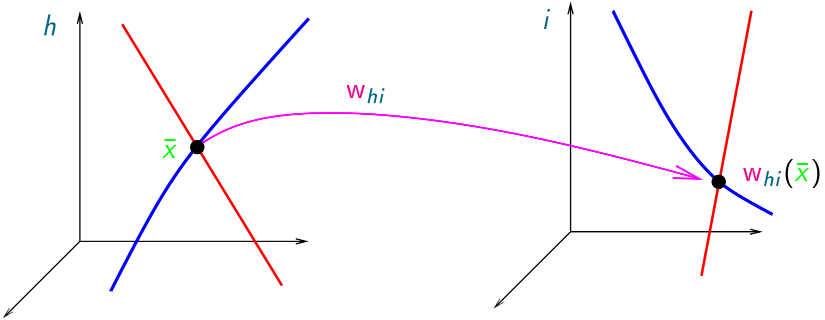
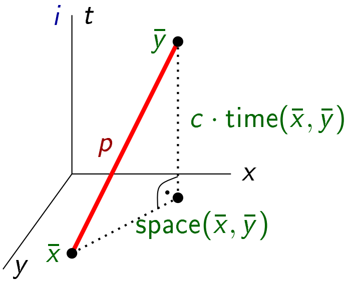



Under mild assumptions $w_{hi} : Q^4 \to Q^4$ is a bijective function.

## AxLight :

*There is an inertial observer, according to whom, any light signal moves with the same velocity c in every direction.*




$$\exists i c \Big[ \mathsf{IOb}(i) \wedge c > 0 \wedge \forall \bar{x}\bar{y} \Big( \exists p \big[ \mathsf{Ph}(p) \wedge \mathsf{W}(i,p,\bar{x}) \wedge \mathsf{W}(i,p,\bar{y}) \big] \leftrightarrow \mathrm{space}(\bar{x},\bar{y}) = c \cdot \mathrm{time}(\bar{x},\bar{y}) \Big) \Big]$$

Let $\mathcal{F}$ be the set of **potential laws of physics**.

$\mathsf{SPR}_{\mathcal{F}}$ :

*Every $\varphi \in \mathcal{F}$ potential law of physics either holds for every inertial observer or none of them.*

$$\left\{ \mathsf{IOb}(m) \wedge \mathsf{IOb}(k) \rightarrow \left[\varphi(m, \bar{x}) \leftrightarrow \varphi(k, \bar{x})\right] \ : \ \varphi \in \mathcal{F} \right\}.$$

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$\mathsf{SPR}^{+}$: when $\mathcal{F}$ contains all the formulas having only 1 free variable of type bodies.

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*Inertial observers agree as to the spatial distance between two events if these two events are simultaneous for both of them. Furthermore, the speed of light is* $1$.

Axiom system for kinematics of special relativity theory

$$\mathsf{SR} := \mathsf{AxLight} + \mathsf{SPR}^+ + \mathsf{Auxiliary\ Axioms}.$$

Some theorems of SR

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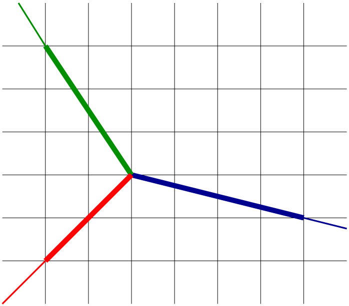
SR $\vdash$ *"The worldview transformations between inertial observers are Poincaré transformations."*

# Dynamics

Collisions of particles

An anomaly to be solved:



inelastic collision

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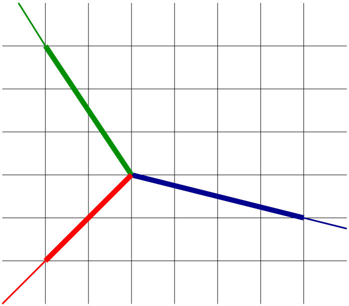

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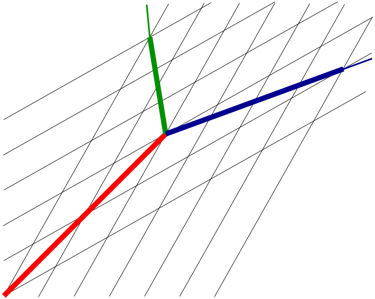

decay

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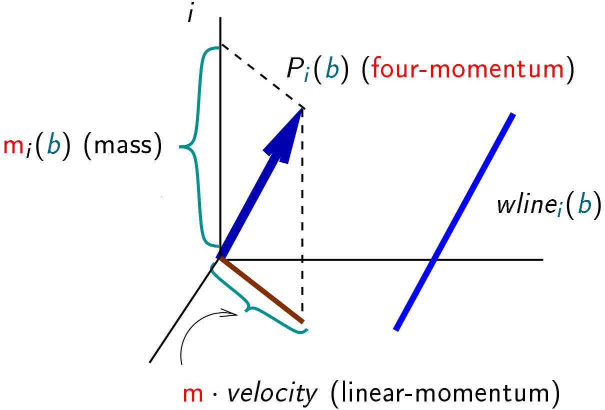
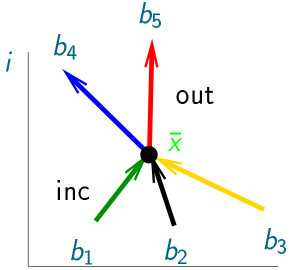
$b$ is inertial particle iff it is an inertial body and for every inertial observer $i$:

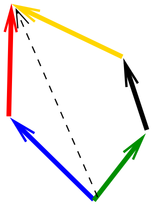
$$\mathrm{m}_i(b) \neq 0 \qquad \text{if} \qquad speed_i(b) < \infty.$$

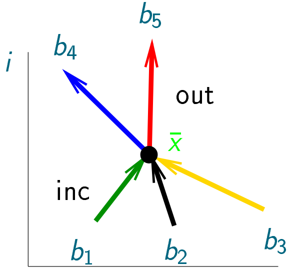


$i$
$P_i(b)$ (four-momentum)
$m_i(b)$ (mass)
$wline_i(b)$
$m \cdot velocity$ (linear-momentum)




$coll_i(b_1 \ldots b_5)$



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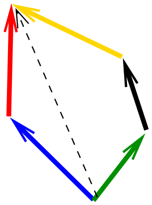

$$coll(b_1 \ldots b_n) \iff \exists \bar{x} \bigwedge_{j=1}^{n} [inc_i(b_j, \bar{x}) \vee out_i(b_j, \bar{x})] \wedge$$

(conservation of mass and momentum:) $\sum_{inc} P_i(b_j) = \sum_{out} P_i(b_j)$

## Axioms for dynamics

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*Possible collisions do not depend on the observer.*

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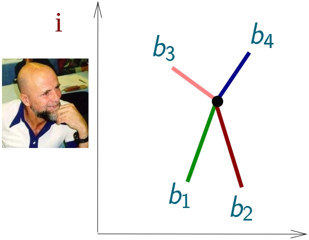
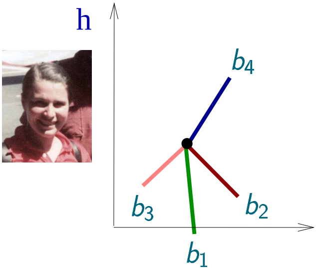
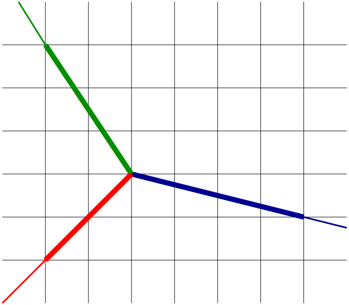
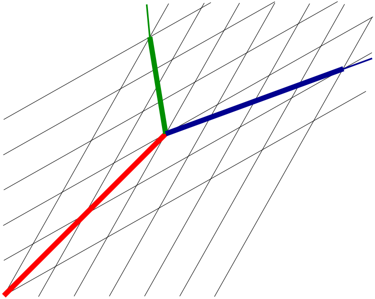



$$coll_i(b_1 \dots b_n) \wedge \bigwedge speed_h(b_j) < \infty \implies coll_h(b_1 \dots b_n)$$



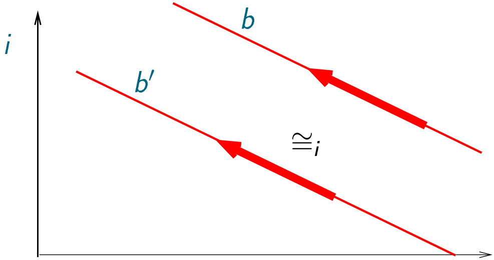
inelastic collision



decay

Def:

$$b \cong_i b' \iff \mathsf{m}_i(b) = \mathsf{m}_i(b') \wedge velocity_i(b) = velocity_i(b')$$
$$\iff P_i(b) = P_i(b').$$



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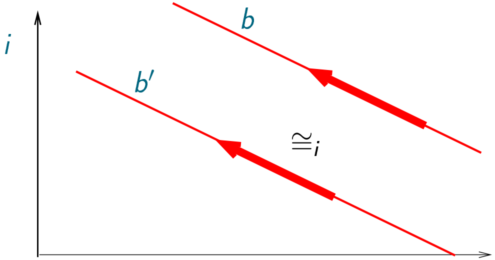


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*Congruence $\cong_i$ does not depend on the observer.*

**Ax∀Inecoll :**

*Any two particles can be collided inelastically at any coordinate point according to any observer.*

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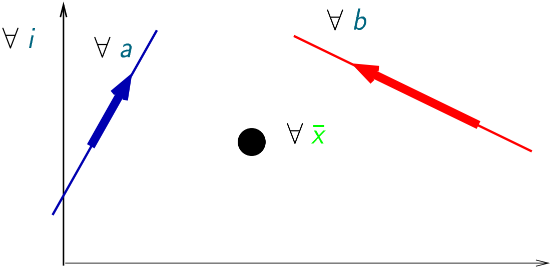


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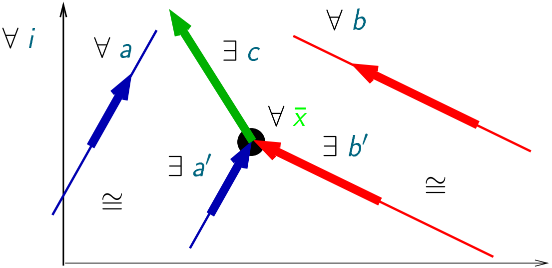


$$\forall\ a\ b\ \bar{x}\ i \qquad \mathsf{m}(a) + \mathsf{m}(b) \neq 0 \qquad \Longrightarrow$$

$$\exists\ a'\ b'\ c\ (a' \cong a \wedge b' \cong b \wedge coll(a'b'c) \wedge inc(a') \wedge inc(b') \wedge out(c))$$

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$$\mathsf{SRdyn} := \mathsf{SR} + \mathsf{AxColl} + \mathsf{Ax\forall Inecoll} + \text{Auxiliary Axioms}.$$

$\exists$FTLip there is a faster that light inertial particle.

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**Theorem:**

*$\exists$FTLip is independent of SRdyn, that is*

$$\mathsf{SRdyn} \nvDash \exists\mathsf{FTLip} \qquad \mathsf{SRdyn} \nvDash \neg\exists\mathsf{FTLip}.$$

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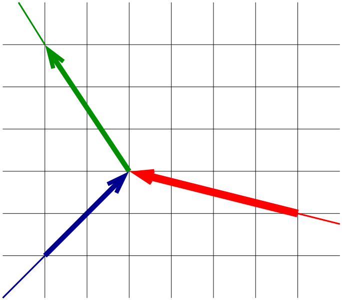
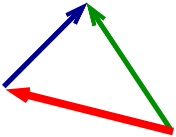
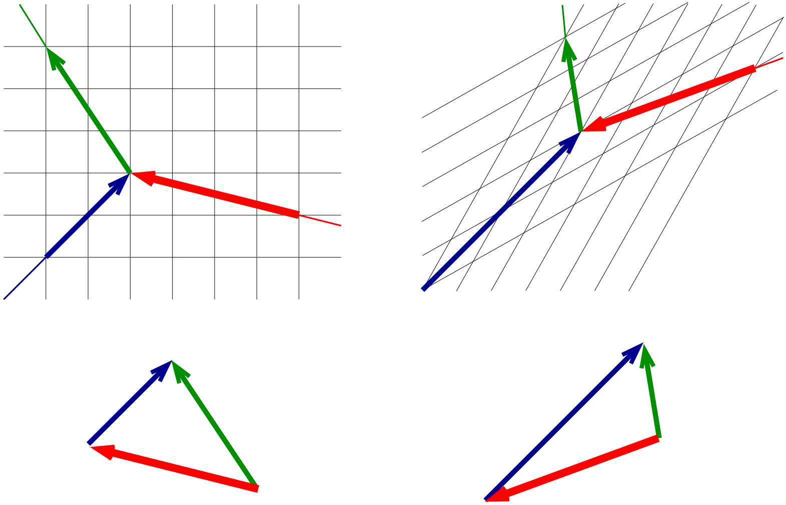
*$\exists$FTLip is independent of SRdyn, that is*

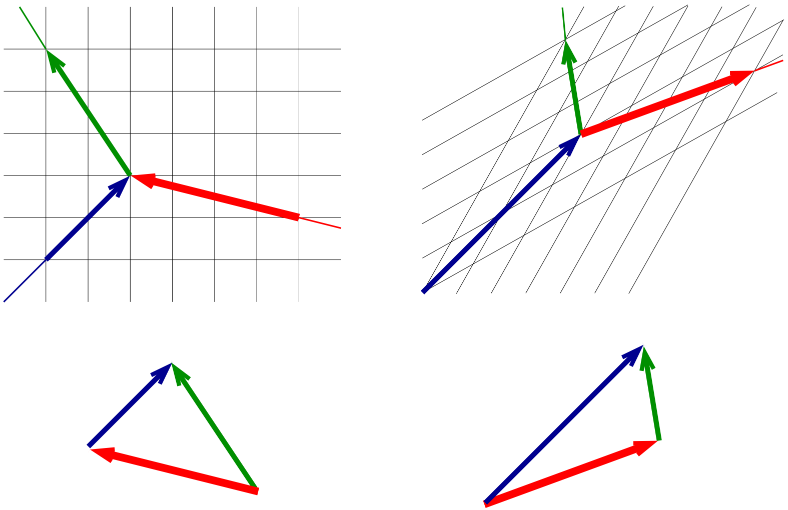
$$\text{SRdyn} \not\models \exists\text{FTLip} \qquad \text{SRdyn} \not\models \neg\exists\text{FTLip}.$$

Proof: $\mathfrak{M}_1 \models \exists\text{FTLip}$ and $\mathfrak{M}_2 \models \neg\exists\text{FTLip}$.









**Theorem:**

*Assume* SRdyn$^{-}$ *and that masses are positive.*

$$\mathsf{m}_k(b) \cdot \sqrt{|1 - speed_k^2(b)|} = \mathsf{m}_h(b) \cdot \sqrt{|1 - speed_h^2(b)|}.$$