An Infinitesimally Superluminal Neutrino is Left-Handed, Conserves Lepton Number and Solves the Autobahn Paradox

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12-SEP-2012

U. D. Jentschura
Missouri University of Science and Technology
Rolla, Missouri

Happy Birthday, Istvan Nemeti:

„Mathematics and Physics – Forever One.“

[C. N. Yang]

Experimental Physics :: Theoretical Physics :: Mathematics

The formalization and axiomatization
of physical theories (physical insight)
in the language of mathematics is of significant
general value for the scientific community.

For example:

METHODS OF MODERN MATHEMATICAL PHYSICS

IV: ANALYSIS OF OPERATORS

MICHAEL REED

Department of Mathematics
Duke University

BARRY SIMON

Departments of Mathematics
and Physics
Princeton University

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Putting quantum mechanics
on solid grounds...

“Autobahn Paradox” (Known to J.A.Wheeler Already)…

What happens if Dr. Spock overtakes a left-handed neutrino…

“Autobahn Paradox”: A left-handed, massive neutrino (traveling slower than light) appears right-handed once you pass it on a highway.

Wheeler Theorem (Proven by Himself): Nice people do not get Nobel Prizes

Wheeler said that the neutrino has to be massless,
beyond doubt, because only strictly massless spin-1/2 particles find a
convenient representation in the helicity basis (Weyl fermions).

Wheeler even discouraged experimentalists who intended to measure
the neutrino mass. The original standard model (SM) predicted massless
neutrinos; the observation of neutrino oscillations implies SM -> SM++.

Four possible ways to resolve the autobahn paradox:

[1.] The neutrino is strictly massless (Weyl fermion).
But: we have neutrino oscillations.

[2.] The neutrino is its own antiparticle (Majorana fermion).

[3.] The neutrino is an ever so slightly superluminal Dirac particle.

[4.] Exotic mechanisms (sterile neutrinos).

Answer [2.] (currently the preferred answer) would imply that the neutrino behaves differently from any other spin-1/2 particle in the Standard Model. It would force us to abandon the concept of lepton number: Muon and antimuon (negative and positive charge) are their respective antiparticles and decay into (essentially) muon neutrino and muon antineutrino. The decay products, however, would count as one and the same, identical particle. The Majorana wave functions would have to fulfill the charge conjugation invariance condition (be "real rather than complex").

Question (Answer [3.]):

Is the neutrino superluminal?

Is it possible that, although the OPERA experimental claim has been refuted, the neutrino could still be infinitesimally superluminal?

If so, which physical consequences would result?

Based on the following preprints:

arXiv:1110.4171 (relativistic quantum mechanics)
[J.Phys.A, in press]

arXiv:1201.0359 (quantized field theory)
[Eur.Phys.J C 72 (2012) 1894]

arXiv:1201.6300 (imaginary mass and helicity dependence)
[J.Mod.Phys., in press]

arXiv:1205.0145 (attempt at neutrino mass running)
[Cent.Eur.J.Phys. 10 (2012) 749]

arXiv:1205.0521v3 (generalized theory and cosmology)
[submitted]

arXiv:1206.6342 (illustrative discussion)
[abstract for the current conference]

Volume 150B, number 6 PHYSICS LETTERS 24 January 1985

THE NEUTRINO AS A TACHYON

Alan CHODOS [1], Avi I. HAUSER
Physics Department, Yale University, New Haven, CT 06511, USA

and

V. Alan KOSTELECKÝ
Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Received 30 October 1984

We investigate the hypothesis that at least one of the known neutrinos travels faster than light. The current experimental situation is examined within this purview.

Pseudo–Hermitian Quantum Dynamics of Tachyonic Spin–1/2 Particles

U. D. Jentschura and B. J. Wundt
Department of Physics, Missouri University of Science and Technology, Rolla, Missouri 65409-0640, USA

We investigate the spinor solutions, the spectrum and the symmetry properties of a matrix-valued wave equation whose plane-wave solutions satisfy the superluminal (tachyonic) dispersion relation $E^2 = \vec{p}^2 - m^2$, where E is the energy, $\vec{p}$ is the spatial momentum, and m is the mass of the particle. The equation reads $(i\gamma^\mu \partial_\mu - \gamma^5 m)\psi = 0$, where γ^5 is the fifth current. The tachyonic equation is shown to be $\mathcal{CP}$ invariant, and $\mathcal{T}$ invariant. The tachyonic Hamiltonian $H_5 = \vec{\alpha} \cdot \vec{p} + \beta \gamma^5 m$ breaks parity and is non–Hermitian but fulfills the pseudo–Hermitian property $H_5(\vec{r}) = P\, H_5^+(-\vec{r})\, P^{-1} = \mathcal{P}\, H_5^+(\vec{r})\, \mathcal{P}^{-1}$, where P is the parity matrix and $\mathcal{P}$ is the full parity transformation. The energy eigenvalues and eigenvectors describe a continuous spectrum of plane-wave solutions (which correspond to real eigenvalues for $|\vec{p}| \geq m$) and evanescent waves, which constitute resonances and antiresonances with complex-conjugate pairs of resonance eigenvalues (for $|\vec{p}| < m$) . In view of additional algebraic properties of the Hamiltonian which supplement the pseudo-Hermiticity, the existence of a resonance energy eigenvalues E implies that E^*, $-E$, and $-E^*$ also constitute resonance energies of H_5.

Accepted for publication by J. Phys. A: Math. Theor. for the special issue on "Quantum Physics with non–Hermitian Operators"

PACS numbers: 95.85.Ry, 11.10.-z, 03.70.+k

arXiv:1110.4171v3

Eur. Phys. J. C (2012) 72:1894
DOI 10.1140/epjc/s10052-012-1894-4

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

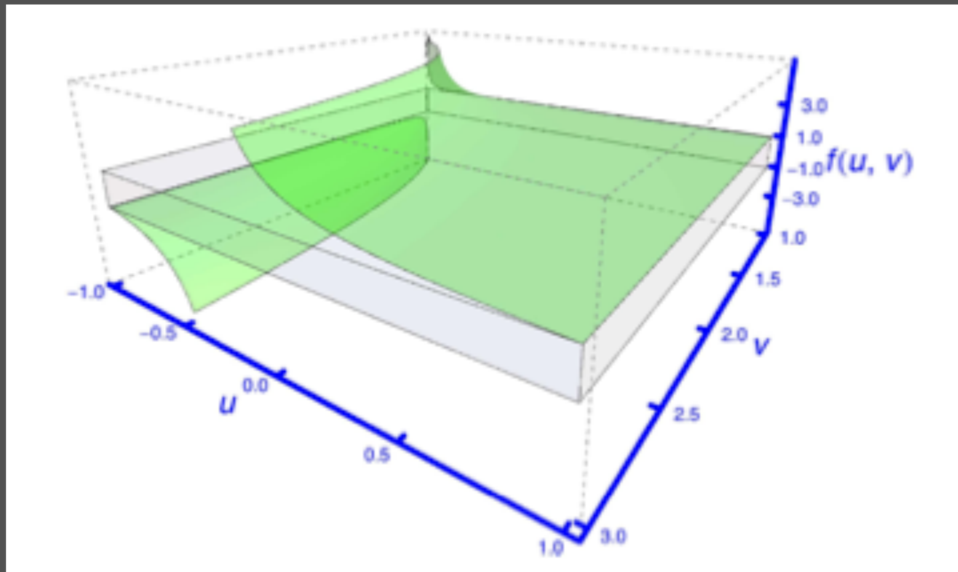
Localizability of tachyonic particles and neutrinoless double beta decay

U.D. Jentschura[1,2,a], B.J. Wundt[1]

[1]Department of Physics, Missouri University of Science and Technology, Rolla, MO 65409-0640, USA
[2]Institut für Theoretische Physik, Philosophenweg 16, 69020 Heidelberg, Germany

Received: 19 December 2011 / Revised: 9 January 2012

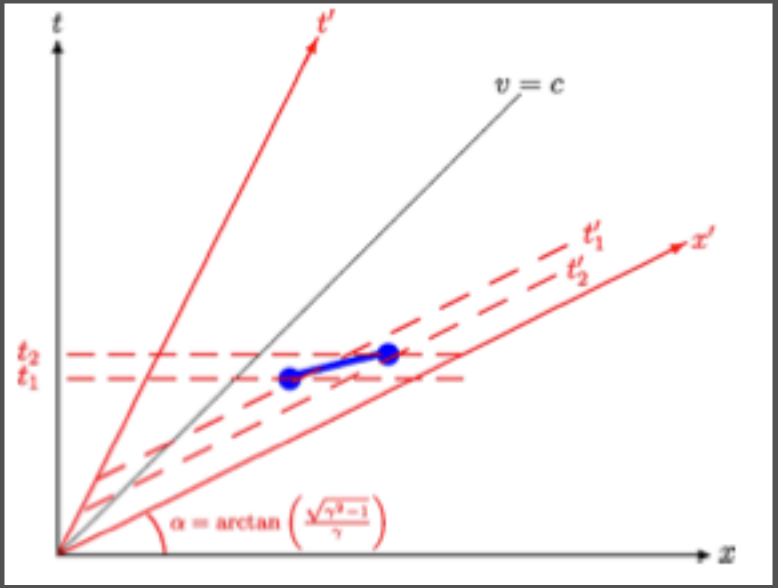
Superluminal particles remain superluminal upon Lorentz transformation (Einstein addition theorem remains valid)...



...and their existence is independent of the axioms of special relativity (G. Szekely's talk).

„Conjugate velocity" $u = -c^2/v < c$ for $v > c$.

Reinterpretation Principle for the Quantum Theory



...We always have to reinterpret antiparticle trajectories by inverting the direction of time and space, but for superluminal particles, there is an additional difficulty because the time ordering of creation and annihilation may be reversed, depending on the velocity of the observer.

Possible solution offered in arXiv:1201.0359.

Measured Mass Squares are All Negative…

Experiment	measured mass squared	formal limit	C.L.	Year
Mainz	-1.6 ± 2.5 ± 2.1	2.2	95 %	2000
Troitsk	-1.0 ± 3.0 ± 2.1 (**)	2.5	95 %	2000
Zürich	-24 ± 48 ± 61	11.7	95 %	1992
Tokyo INS	- 65 ± 85 ± 65	13.1	95%	1991
Los Alamos	- 147 ± 68 ± 41	9.3	95%	1991
Livermore	- 130 ± 20 ± 15	7.0	95%	1995
China	- 31 ± 75 ± 48	12.4	95%	1995
Average of PDG (98)	-27 ± 20	15	95 %	1998

http://cupp.oulu.fi/neutrino/nd-mass.html

Normal (Tardyonic) Dirac Equation

$$\gamma^0 = \beta = \begin{pmatrix} \mathbb{1}_{2\times 2} & 0 \\ 0 & -\mathbb{1}_{2\times 2} \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & \mathbb{1}_{2\times 2} \\ \mathbb{1}_{2\times 2} & 0 \end{pmatrix},$$

$$(\mathrm{i}\gamma^\mu \, \partial_\mu - m_1) \, \psi(x) = 0 \, .$$

$$\psi(x) = U_\pm^{(1)}(\vec{k}) \, \exp(-\mathrm{i}k \cdot x) \, , \quad \phi(x) = V_\pm^{(1)}(\vec{k}) \, \exp(\mathrm{i}k \cdot x) \, ,$$

$$E^{(1)} = \sqrt{\vec{k}^2 + m_1^2}.$$

Tachyonic Dirac Equation

$$\gamma^0 = \beta = \begin{pmatrix} \mathbb{1}_{2\times 2} & 0 \\ 0 & -\mathbb{1}_{2\times 2} \end{pmatrix}, \qquad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 0 & \mathbb{1}_{2\times 2} \\ \mathbb{1}_{2\times 2} & 0 \end{pmatrix},$$

$$\left(\mathrm{i}\gamma^\mu \partial_\mu - \gamma^5 m\right) \psi(x) = 0.$$

$$\Psi(x) = U_\pm(\vec{k})\, \mathrm{e}^{-\mathrm{i}k\cdot x} \qquad \Phi(x) = V_\pm(\vec{k})\, \mathrm{e}^{\mathrm{i}k\cdot x},$$

$$E = \sqrt{\vec{k}^2 - m^2},$$

Tachyonic and Tardyonic Dispersion Relations

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **84**, 021806(R) (2011)

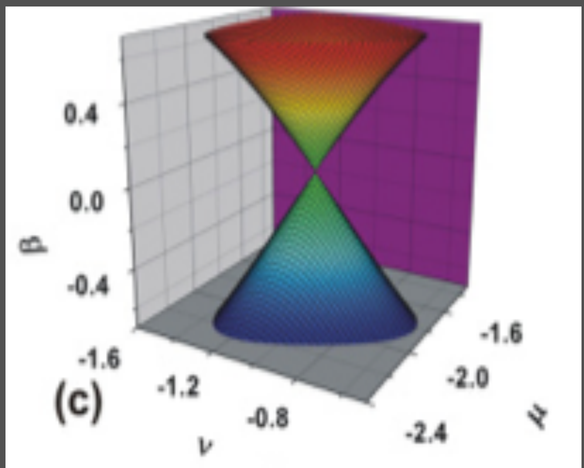
$\mathcal{PT}$-symmetry in honeycomb photonic lattices

Alexander Szameit, Mikael C. Rechtsman, Omri Bahat-Treidel, and Mordechai Segev
Physics Department and Solid State Institute, Technion, 32000 Haifa, Israel

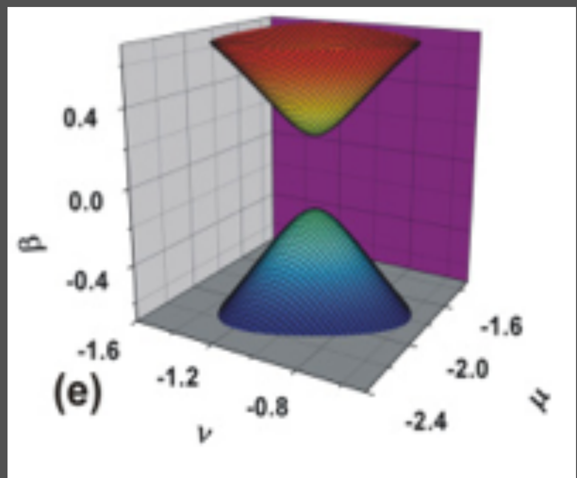
(Received 21 April 2011; published 19 August 2011)

We apply gain and loss to honeycomb photonic lattices and show that the dispersion relation is identical to tachyons—particles with imaginary mass that travel faster than the speed of light. This is accompanied by $\mathcal{PT}$-symmetry breaking in this structure. We further show that the $\mathcal{PT}$-symmetry can be restored by deforming the lattice.

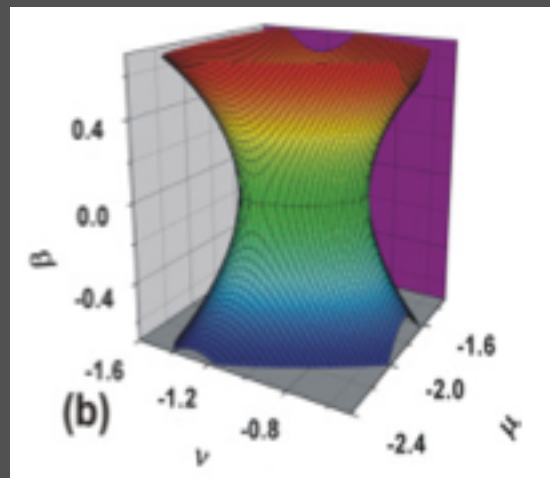
DOI: 10.1103/PhysRevA.84.021806 PACS number(s): 42.25.−p, 42.82.Et



Massless



Tardyonic



Tachyonic

Tardyonic Solutions and Sum Rules

$$U_+^{(1)}(\vec{k}) = \frac{(\not{k} + m_1)\, u_+(\vec{k})}{\sqrt{(E^{(1)} - |\vec{k}|)^2 + m_1^2}} = \begin{pmatrix} \sqrt{\frac{E^{(1)} + m_1}{2\,E^{(1)}}} a_+(\vec{k}) \\ \sqrt{\frac{E^{(1)} - m_1}{2\,E^{(1)}}} a_+(\vec{k}) \end{pmatrix},$$

$$U_-^{(1)}(\vec{k}) = \frac{(\not{k} + m_1)\, u_-(\vec{k})}{\sqrt{(E^{(1)} - |\vec{k}|)^2 + m_1^2}} = \begin{pmatrix} \sqrt{\frac{E^{(1)} + m_1}{2\,E^{(1)}}}\, a_-(\vec{k}) \\ -\sqrt{\frac{E^{(1)} - m_1}{2\,E^{(1)}}}\, a_-(\vec{k}) \end{pmatrix}.$$

$$V_+^{(1)}(\vec{k}) = \frac{(m_1 - \not{k})\, v_+(\vec{k})}{\sqrt{(E^{(1)} - |\vec{k}|)^2 + m_1^2}} = \begin{pmatrix} -\sqrt{\frac{E^{(1)} - m_1}{2\,E^{(1)}}}\, a_+(\vec{k}) \\ -\sqrt{\frac{E^{(1)} + m_1}{2\,E^{(1)}}}\, a_+(\vec{k}) \end{pmatrix}$$

$$V_-^{(1)}(\vec{k}) = \frac{(m_1 - \not{k})\, v_-(\vec{k})}{\sqrt{(E^{(1)} - |\vec{k}|)^2 + m_1^2}} = \begin{pmatrix} -\sqrt{\frac{E^{(1)} - m_1}{2\,E^{(1)}}}\, a_-(\vec{k}) \\ \sqrt{\frac{E^{(1)} + m_1}{2\,E^{(1)}}}\, a_-(\vec{k}) \end{pmatrix}$$

$$\boxed{\sum_\sigma \mathcal{U}_\sigma^{(1)}(\vec{k}) \otimes \overline{\mathcal{U}}_\sigma^{(1)}(\vec{k}) = \frac{\not{k} + m_1}{2m_1},} \qquad \boxed{\sum_\sigma \mathcal{V}_\sigma^{(1)}(\vec{k}) \otimes \overline{\mathcal{V}}_\sigma^{(1)}(\vec{k}) = \frac{\not{k} - m_1}{2m_1}.}$$

Tachyonic Solutions and Sum Rules

$$U_+(\vec{k}) = \frac{(\gamma^5 m - \not{k})\, u_+(\vec{k})}{\sqrt{(E - |\vec{k}|)^2 + m^2}} = \begin{pmatrix} \sqrt{\dfrac{|\vec{k}| + m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \\ \sqrt{\dfrac{|\vec{k}| - m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \end{pmatrix},$$

$$U_-(\vec{k}) = \frac{(\not{k} - \gamma^5 m)\, u_-(\vec{k})}{\sqrt{(E - |\vec{k}|)^2 + m^2}} = \begin{pmatrix} \sqrt{\dfrac{|\vec{k}| - m}{2\,|\vec{k}|}}\, a_-(\vec{k}) \\ -\sqrt{\dfrac{|\vec{k}| + m}{2\,|\vec{k}|}}\, a_-(\vec{k}) \end{pmatrix}.$$

$$V_+(\vec{k}) = \frac{(\gamma^5 m + \not{k})\, v_+(\vec{k})}{\sqrt{(E - |\vec{k}|)^2 + m^2}} = \begin{pmatrix} -\sqrt{\dfrac{|\vec{k}| - m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \\ -\sqrt{\dfrac{|\vec{k}| + m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \end{pmatrix},$$

$$V_-(\vec{k}) = \frac{(-\not{k} - \gamma^5 m)\, v_-(\vec{k})}{\sqrt{(E - |\vec{k}|)^2 + m^2}} = \begin{pmatrix} -\sqrt{\dfrac{|\vec{k}| + m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \\ \sqrt{\dfrac{|\vec{k}| - m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \end{pmatrix}.$$

$$\boxed{\sum_\sigma (-\sigma)\, \mathcal{U}_\sigma(\vec{k}) \otimes \overline{\mathcal{U}}_\sigma(\vec{k})\, \gamma^5 = \frac{\not{k} - \gamma^5 m}{2m}}, \qquad \boxed{\sum_\sigma (-\sigma)\, \mathcal{V}_\sigma(\vec{k}) \otimes \overline{\mathcal{V}}_\sigma(\vec{k})\, \gamma^5 = \frac{\not{k} + \gamma^5 m}{2m}}.$$

Similar Relations even hold for...

...TWO tardyonic mass terms...

$$(\mathrm{i}\gamma^\mu \partial_\mu - m_1 - \mathrm{i}\gamma^5 m_2)\ \psi(x) = 0\,.$$

$$E = \sqrt{\vec{p}^2 + m_1^2 + m_2^2}.$$

...TWO tachyonic mass terms...

$$(\mathrm{i}\gamma^\mu \partial_\mu - \mathrm{i} m_1 - \gamma^5 m_2)\ \psi(x) = 0\,.$$

$$E = \sqrt{\vec{p}^2 - m_1^2 - m_2^2}.$$

...see preprints arXiv:1206.6243 and arXiv:1205.0521v3...

Underlying Property...

...two tardyonic mass terms...
...Hermiticity...

$$H^{(t)} = \vec{\alpha} \cdot \vec{p} + \beta \, m_1 + \mathrm{i} \, \beta \, \gamma^5 \, m_2 \, .$$

...two tachyonic mass terms...
...γ^5 Hermiticity ("Pseudo-Hermiticity")...

$$H' = \vec{\alpha} \cdot \vec{p} + \mathrm{i} \, \beta \, m_1 + \beta \, \gamma^5 \, m_2 \, ,$$

$$H' = \gamma^5 H'^{+} \gamma^5 .$$

...see preprints arXiv:1206.6243 and arXiv:1205.0521v3...

REVIEWS OF
MODERN PHYSICS

VOLUME 15, NUMBER 3 JULY, 1943

On Dirac's New Method of Field Quantization*

W. PAULI

Institute for Advanced Study, Princeton, New Jersey

As a generalization of the Hermitian conjugate operator, we introduce the adjoint operator which we denote by A^*. This is given by

$$A^* = \eta^{-1} A^\dagger \eta^\dagger = \eta^{-1} A^\dagger \eta, \tag{4}$$

$$\partial\psi/\partial t = -iH\psi,$$

hence

$$(\partial\bar{\psi}/\partial t)\eta = i\bar{\psi}H^\dagger\eta = i\bar{\psi}\eta H^*, \tag{6}$$

has to be self-adjoint,

$$H^* = H.$$

$\eta = \gamma^5$

REVIEWS OF
MODERN PHYSICS
VOLUME 15, NUMBER 3 JULY, 1943

On Dirac's New Method of Field Quantization*

W. PAULI
Institute for Advanced Study, Princeton, New Jersey

$$\frac{d}{dt}\int \bar{\psi}\eta\psi dq = \frac{d}{dt}\sum_{n,m} \bar{\psi}_n \eta_{nm} \psi_m$$

$$= i\bar{\psi}\eta(H^* - H)\psi = 0;$$

Intermezzo: Pseudo-Hermiticity = PT Symmetry (At Least Approximately)

VOLUME 80, NUMBER 24 PHYSICAL REVIEW LETTERS 15 JUNE 1998

Real Spectra in Non-Hermitian Hamiltonians Having $\mathcal{PT}$ Symmetry

Carl M. Bender[1] and Stefan Boettcher[2,3]

[1]*Department of Physics, Washington University, St. Louis, Missouri 63130*

[2]*Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

[3]*CTSPS, Clark Atlanta University, Atlanta, Georgia 30314*

(Received 1 December 1997; revised manuscript received 9 April 1998)

The condition of self-adjointness ensures that the eigenvalues of a Hamiltonian are real and bounded below. Replacing this condition by the weaker condition of $\mathcal{PT}$ symmetry, one obtains new infinite classes of complex Hamiltonians whose spectra are also real and positive. These $\mathcal{PT}$ symmetric theories may be viewed as analytic continuations of conventional theories from real to complex phase space. This paper describes the unusual classical and quantum properties of these theories. [S0031-9007(98)06371-6]

Intermezzo: Pseudo-Hermiticity = PT Symmetry (At Least Approximately)

$\mathcal{PT}$-symmetric cubic anharmonic oscillator as a physical model

Ali Mostafazadeh

Department of Mathematics, Koç University, 34450 Sariyer, Istanbul, Turkey

E-mail: amostafazadeh@ku.edu.tr

Received 18 November 2004, in final form 1 June 2005
Published 6 July 2005
Online at stacks.iop.org/JPhysA/38/6557

Abstract
We perform a perturbative calculation of the physical observables, in particular, pseudo-Hermitian position and momentum operators, the equivalent Hermitian Hamiltonian operator and the classical Hamiltonian for the $\mathcal{PT}$-symmetric cubic anharmonic oscillator, $H = \frac{1}{2m}p^2 + \frac{1}{2}\mu^2x^2 + i\epsilon x^3$. Ignoring terms of order ϵ^4 and higher, we show that this system describes an ordinary quartic anharmonic oscillator with a position-dependent mass, and real and positive coupling constants. This observation elucidates the classical origin of the reality and positivity of the energy spectrum. We also discuss the quantum–classical correspondence for this $\mathcal{PT}$-symmetric system, compute the associated conserved probability density and comment on the issue of factor ordering in the pseudo-Hermitian canonical quantization of the underlying classical system.

PACS number: 03.65.−w

The Propagator in Field Theory...

...tardyonic propagator...

$$S^{(1)}(k) = \frac{1}{\not{k} - m_1 + \mathrm{i}\epsilon} = \frac{\not{k} + m_1}{k^2 - m_1^2 + \mathrm{i}\,\epsilon}\,.$$

...tachyonic propagator...

$$S_T(k) = \frac{1}{\not{k} - \gamma^5\,(m + \mathrm{i}\,\epsilon)} = \frac{\not{k} - \gamma^5\,m}{k^2 + m^2 + \mathrm{i}\,\epsilon}\,.$$

...see preprints arXiv:1201.0359 and arXiv:1205.0521v3...

The Propagator in Field Theory...

$$\langle 0 \left| T\, \psi(x)\, \overline{\psi}(y) \Gamma \right| 0 \rangle = \mathrm{i}\, S(x-y)\,,$$

$$\psi(x) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{m}{E} \sum_{\sigma=\pm} \left\{ b_\sigma(k) \mathcal{U}_\sigma(\vec{k})\, \mathrm{e}^{-\mathrm{i}\, k\cdot x} + d_\sigma^+(k)\, \mathcal{V}_\sigma(\vec{k})\, \mathrm{e}^{\mathrm{i}\, k\cdot x} \right\}\,,$$

$$\begin{aligned} \{b_\sigma(k), b_\rho(k')\} &= \{b_\sigma^+(k), b_\rho^+(k')\} = 0\,, \\ \{d_\sigma(k), d_\rho(k')\} &= \{d_\sigma^+(k), d_\rho^+(k')\} = 0\,, \end{aligned}$$

$$\begin{aligned} \{b_\sigma(k), b_\rho^+(k')\} &= f(\sigma, \vec{k})\; (2\pi)^3\, \frac{E}{m} \delta^3(\vec{k} - \vec{k}')\, \delta_{\sigma\rho}\,, \\ \{d_\sigma(k), d_\rho^+(k')\} &= g(\sigma, \vec{k})\; (2\pi)^3\, \frac{E}{m} \delta^3(\vec{k} - \vec{k}')\, \delta_{\sigma\rho}\,, \end{aligned}$$

tardyonic choice: $\quad f(\sigma, \vec{k}) = g(\sigma, \vec{k}) = 1\,, \quad \Gamma = \mathbb{1}_{4\times 4}\,,$

tachyonic choice: $\quad f(\sigma, \vec{k}) = g(\sigma, \vec{k}) = -\sigma\,, \quad \Gamma = \gamma^5\,,$

Negative Norm = Wrong Helicity...

tardyonic anticommutators:

$$\{b_\sigma(k), b^+_\rho(k')\} = (2\pi)^3 \frac{E}{m}\delta^3(\vec{k}-\vec{k}')\,\delta_{\sigma\rho}\,, \qquad \{d_\sigma(k), d^+_\rho(k')\} = (2\pi)^3 \frac{E}{m}\delta^3(\vec{k}-\vec{k}')\,\delta_{\sigma\rho}\,.$$

tachyonic anticommutators:

$$\{b_\sigma(k), b^+_\rho(k')\} = (-\sigma)\,(2\pi)^3 \frac{E}{m}\delta^3(\vec{k}-\vec{k}')\,\delta_{\sigma\rho}\,, \qquad \{d_\sigma(k), d^+_\rho(k')\} = (-\sigma)\,(2\pi)^3 \frac{E}{m}\delta^3(\vec{k}-\vec{k}')\,\delta_{\sigma\rho}\,.$$

$$|1_{k,\sigma}\rangle = b^+_\sigma(k)|0\rangle,$$

$$\langle 1_{k,\sigma}|1_{k,\sigma}\rangle = \langle 0\,|b_\sigma(k)\,b^+_\sigma(k)|\,0\rangle = \langle 0\,|\{b_\sigma(k), b^+_\sigma(k)\}|\,0\rangle = (-\sigma)\,V\,\frac{E}{m}\,,$$

Important: $\sigma = 1$ means right-handed-helicity neutrinos and left-handed antineutrinos for which the norm becomes negative...

$$\boxed{\langle\Psi|\psi_{\sigma=1}(x)|\Psi\rangle = \langle\Psi|\psi^{(-)}_{\sigma=1}(x) + \psi^{(+)}_{\sigma=1}(x)|\Psi\rangle = 0\,,}$$

Negative Norm = Wrong Helicity...

One cannot reverse the suppression of the "wrong" helicity states because this would lead to contradiction with a smooth massless limit [see arXiv:1205.0521v3 for details]

See also an illustrative explicit calculation in arXiv:1201.6300 for the Dirac equation with imaginary mass, where the effect of an inversion of the imaginary mass term is studied.

The Kicker: Gravitational Coupling…

THE ASTRONOMICAL JOURNAL, 116:1009–1038, 1998 September

OBSERVATIONAL EVIDENCE FROM SUPERNOVAE FOR AN ACCELERATING UNIVERSE AND A COSMOLOGICAL CONSTANT

ADAM G. RIESS,[1] ALEXEI V. FILIPPENKO,[1] PETER CHALLIS,[2] ALEJANDRO CLOCCHIATTI,[3] ALAN DIERCKS,[4] PETER M. GARNAVICH,[2] RON L. GILLILAND,[5] CRAIG J. HOGAN,[4] SAURABH JHA,[2] ROBERT P. KIRSHNER,[2] B. LEIBUNDGUT,[6] M. M. PHILLIPS,[7] DAVID REISS,[4] BRIAN P. SCHMIDT,[8,9] ROBERT A. SCHOMMER,[7] R. CHRIS SMITH,[7,10] J. SPYROMILIO,[6] CHRISTOPHER STUBBS,[4] NICHOLAS B. SUNTZEFF,[7] AND JOHN TONRY[11]

Received 1998 March 13; revised 1998 May 6

Some Gravity is Repulsive…

…maybe tachyons…

RIVISTA DEL NUOVO CIMENTO VOL. 4, N. 2 Aprile-Giugno 1974

Classical Theory of Tachyons (Special Relativity Extended to Superluminal Frames and Objects).

E. Recami

Istituto di Fisica Teorica dell'Università - Catania
Istituto Nazionale di Fisica Nucleare - Sezione di Catania
Centro Siciliano di Fisica Nucleare e di Struttura della Materia - Catania

R. Mignani

Istituto di Fisica dell'Università - Roma

(ricevuto il 2 Ottobre 1973)

…maybe tachyons…

We are of course *assuming* that the gravitational interaction is relativistically covariant. Equation (86) tells us that the considered bradyon suffers the gravitational 4-force:

$$F^{\mu} = - m_0 \Gamma^{\mu}_{\varrho\sigma} \frac{dx^{\varrho}}{ds} \frac{dx^{\sigma}}{ds} \qquad (\beta^2 < 1)\,. \tag{86 bis}$$

From the «tachyonization rule» (Sect. **9**), we may immediately derive the gravitational 4-force experienced by a *tachyon*, by applying a SLT (*e.g.* the transcendent transformation $\boldsymbol{K}_+$; cf. Subsect. **4**˙3) to eq. (86 *bis*). We get

$$F'^{\mu} = + m_0 \Gamma'^{\mu}_{\varrho\sigma} \frac{dx'^{\varrho}}{ds'} \frac{dx'^{\sigma}}{ds'} \qquad (\beta^2 > 1)\,, \tag{87}$$

where now m_0 is the (*real*) proper mass *of the tachyon* considered.

In other words, a tachyon is seen to experience a gravitational *repulsion* (and not attraction!). However, since the fundamental equation (79*b*) of tachyon dynamics brings about another sign change, the equations of motion *for a tachyon* in a gravitational field will *still* read

$$\frac{d^2 x'^{\mu}}{ds'^2} + \Gamma'^{\mu}_{\varrho\sigma} \frac{dx'^{\varrho}}{ds'} \frac{dx'^{\sigma}}{ds'} = 0 \qquad (ds' \text{ spacelike})\,. \tag{86'}$$

The Kicker: Gravitational Coupling…

$$S = \int \mathrm{d}^4x \, \sqrt{-\overline{g}} \, \widetilde{\psi} \, \left(\mathrm{i} \, \nabla_\mu \, \psi - \overline{\gamma}^5 \, m \right) \, \psi$$

$$\{ \overline{\gamma}^\mu(x), \overline{\gamma}^\mu(x) \} = 2 \, \overline{g}^{\mu\nu}(x) \, ,$$

$$\overline{\gamma}^5(x) = \mathrm{i} \, \overline{\gamma}^0(x) \, \overline{\gamma}^1(x) \, \overline{\gamma}^2(x) \, \overline{\gamma}^3(x)$$

$$\widetilde{\psi}(x) = \psi^\dagger \, \overline{\gamma}^0(x) \, \overline{\gamma}^5(x)$$

$$\frac{\partial \overline{\gamma}_\mu}{\partial x^\nu} - \Gamma^\rho_{\mu\nu} \overline{\gamma}_\rho - \mathrm{i} \, \overline{\gamma}_\mu \, \Gamma_\nu + \mathrm{i} \, \Gamma_\nu \, \overline{\gamma}_\mu = 0 \, ,$$

$$\Gamma_\nu = -\frac{\mathrm{i}}{2} \, \overline{\gamma}^\rho \left(\frac{\partial \overline{\gamma}_\nu}{\partial x^\rho} - \overline{\gamma}_\sigma \, \Gamma^\sigma_{\rho\nu} \right) .$$

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} \, \overline{g}^{\rho\sigma} \left(\frac{\partial \overline{g}_{\nu\sigma}}{\partial x^\mu} + \frac{\partial \overline{g}_{\mu\sigma}}{\partial x^\nu} - \frac{\partial \overline{g}_{\mu\nu}}{\partial x^\sigma} \right) ,$$

$$(\mathrm{i} \, \gamma^\mu \, \nabla_\mu - m) \, \psi(x) = [\mathrm{i} \, \gamma^\mu \, (\partial_\mu - \Gamma_\mu) - m] \, \psi(x) = 0 \, .$$

versus

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}^2 s} + \Gamma^\mu_{\rho\sigma} \, \frac{\mathrm{d}x^\rho}{\mathrm{d}s} \, \frac{\mathrm{d}x^\sigma}{\mathrm{d}s} = 0 \, ,$$

The Kicker: Gravitational Coupling…

$$\Omega = -2\frac{k_B T}{2\pi^2} V \int_0^\infty \mathrm{d}k\, k^2 \ln\left(1 + \exp\left(\frac{\mu - E(k)}{k_B T}\right)\right),$$

$$p = -\frac{\Omega}{V} = \frac{k_B T}{\pi^2} \int_0^\infty \mathrm{d}k\, k^2 \ln\left(1 + \exp\left(\frac{\mu - E(k)}{k_B T}\right)\right).$$

$$p_0 = \frac{\mathrm{i}}{3\pi^2} \int_0^m \mathrm{d}k\, k^3 \frac{\mathrm{d}\,\mathrm{Im}\, E(k)}{\mathrm{d}k} = \frac{\mathrm{i}}{3\pi^2} \int_0^m \mathrm{d}k\, \frac{k^4}{\sqrt{m^2 - k^2}} = \frac{\mathrm{i}\, m^4}{16\,\pi},$$

$$\rho_0 = \frac{\mathrm{i}}{\pi^2} \int_0^m \mathrm{d}k\, k^2\, \mathrm{Im} E(k) = -\frac{\mathrm{i}}{\pi^2} \int_0^m \mathrm{d}k\, k^2 \sqrt{m^2 - k^2} = -\frac{\mathrm{i}\, m^4}{16\,\pi} = -p_0\,.$$

$$w_0 = p_0/\rho_0 = -1.$$

If $p_0 + \rho_0 = 0$, then there is no net energy gain upon pulling on a 'piston' which contains the Universe

The Kicker: Gravitational Coupling…

$$H^2 = \left(\frac{\dot{a}}{|a|}\right)^2 = \frac{8\pi G}{3}\,(\rho_\Lambda + \rho_M + \rho_k) \approx \frac{8\pi G}{3}\,(\rho_\Lambda + \rho_M)\,,$$

$$\frac{\ddot{a}}{|a|} = \frac{8\pi G}{3}\,\rho_\Lambda - \frac{4\pi G}{3}\,(\rho_M + 3p_M + \rho_k + 3p_k) \approx \frac{4\pi G}{3}\,(2\rho_\Lambda - \rho_M)\,.$$

$$\rho'_\Lambda \approx \rho_0 = -\frac{\mathrm{i}\,m^4}{16\,\pi}\,, \qquad |2\rho'_\Lambda - \rho_M| = \sqrt{4\,|\rho'_\Lambda|^2 + \rho_M^2} \overset{!}{=} 2\rho_\Lambda - \rho_M \approx 1.19\,\rho_{\rm crit}\,.$$

Solving the evolution equations of the Universe by complex scaling:

$$(5.79 \times 10^{-45}\,\mathrm{GeV}^4) \sim |\rho_0| = \frac{m^4}{16\pi} \qquad \Rightarrow \qquad \boxed{m \sim 0.0232\,\mathrm{eV}\,.}$$

This tachyonic mass is not excluded by any terrestrial experiments…
[see arXiv:1205.0521v3 for details]

Conclusions

Conclusions

(From arXiv:1205.0521v3)

On the other hand, if we assume that the neutrino is described by the tachyonic Dirac equation, then the following statements are valid:

- Statement #1: We can properly assign lepton number and use plane-wave eigenstates for incoming and outgoing particles, while allowing for nonvanishing mass terms and thus, mass square differences among the neutrino mass (not flavor) eigenstates.
- Statement #2: There is a natural resolution for the 'autobahn paradox' because a left-handed spacelike neutrino always remains spacelike upon Lorentz transformation and cannot be overtaken.
- Statement #3: The right-handed particle and left-handed antiparticle states are suppressed due to negative Fock-space norm.
- Statement #4: At least qualitatively, tachyonic neutrinos could yield an explanation for a repulsive force on intergalactic distance scales as they are repulsed, like all tachyons, by gravitational interactions ('dark energy').

A superluminal neutrino could appear to
solve at least as many problems as it raises.

Conclusions

Inspiration for mathematical study:

Spectral properties of the superluminal Hamiltonians (pseudo-Hermitian Hamiltonians), analytic structure of the S matrix and necessary extensions of the axiomatic formulation into the quantum and superluminal domain (if possible).

Conclusions

What happens if Dr. Spock overtakes a left-handed neutrino...

Spinor Sum for Positive-Energy States:

$$\sum_{\sigma}(-\sigma)\,\mathcal{U}_{\sigma}(\vec{k})\otimes\overline{\mathcal{U}}_{\sigma}(\vec{k})\,\gamma^{5}=\frac{\not{k}-\gamma^{5}\,m}{2m}$$

Spinor Sum for Negative-Energy States:

$$\sum_{\sigma}(-\sigma)\,\mathcal{V}_{\sigma}(\vec{k})\otimes\overline{\mathcal{V}}_{\sigma}(\vec{k})\,\gamma^{5}=\frac{\not{k}+\gamma^{5}\,m}{2m}$$