Proof Verification and Proof Discovery for Relativity

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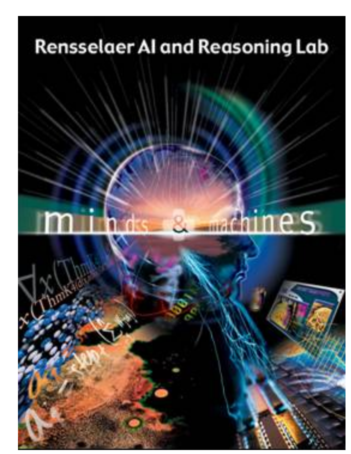

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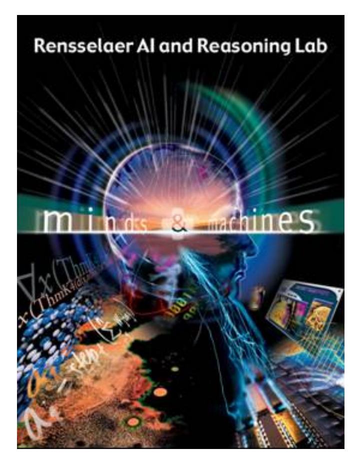

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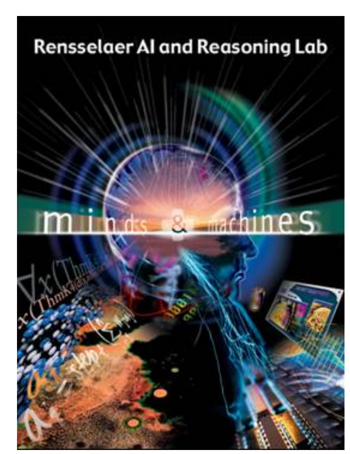

We are deeply grateful for support received from both the John Templeton Foundation and the Air Force Office of Scientific Research, and to all those who made it possible for Selmer to present at the Rényi Institute.

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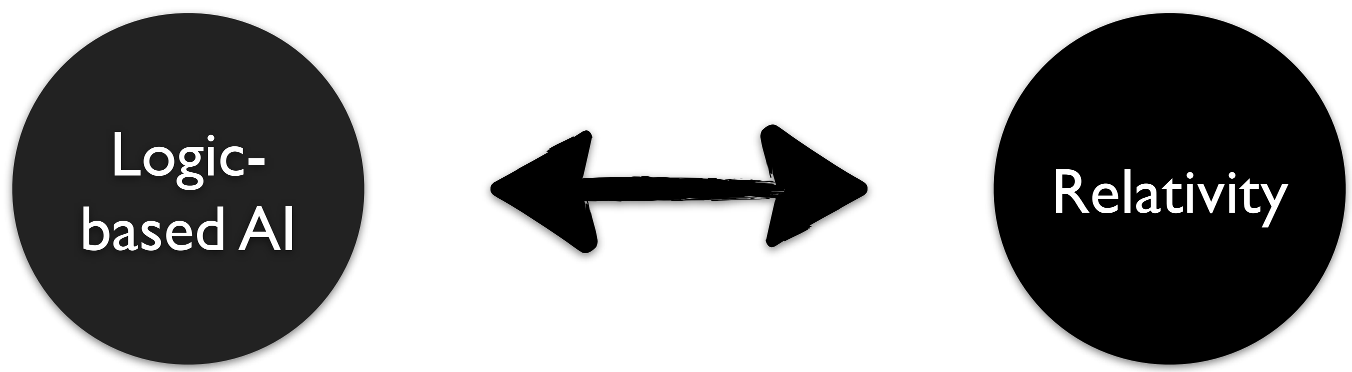
Logic (formal)   Physics


Logic-
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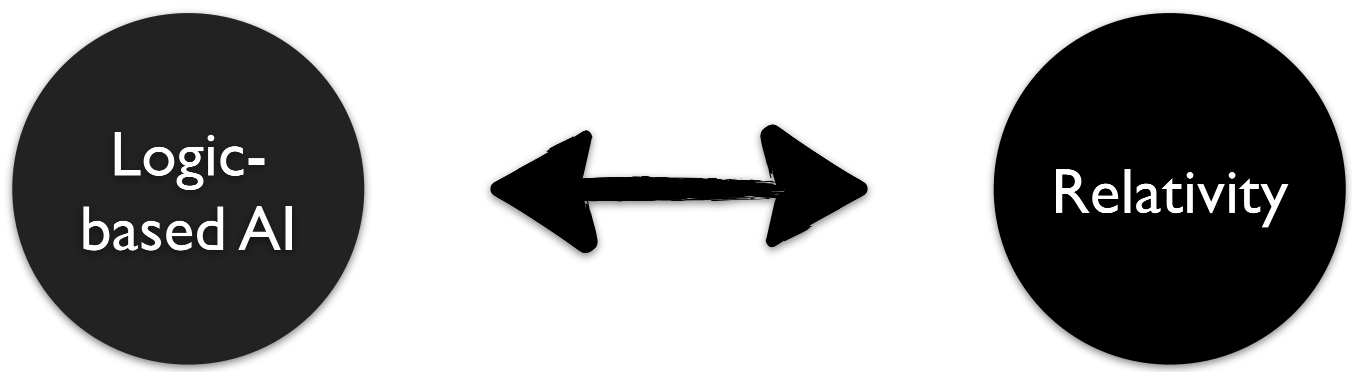

Bringsjord, S. (2008) "The Logicist Manifesto: At Long Last Let Logic-Based AI Become a Field Unto Itself" *Journal of Applied Logic* **6.4**: 502–525.

Preprint: http://kryten.mm.rpi.edu/SB_LAI_Manifesto_091808.pdf

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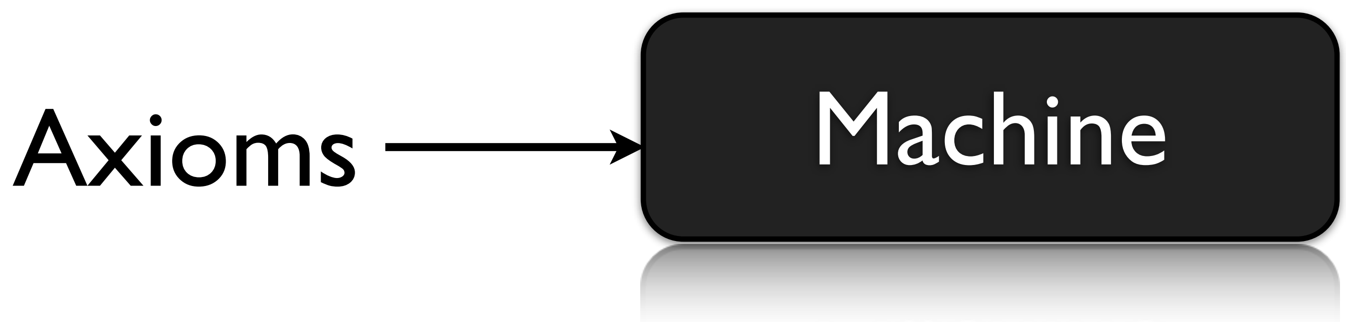
The Vision, Overall ...

Machine

Axioms

Machine


Axioms
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Axioms → Machine → Theorems

Examples of Machine-Assisted Formal Knowledge Discovery

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  - Semi-formally: biological ontologies.

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Triadic Background ...

Proof Verification Given a proof ρ, a formula ϕ, and a set of axioms Γ, verify that ρ represents a proof of ϕ from Γ. Expressed in query form: Is it true that $\Gamma \vdash_{\rho} \phi$?

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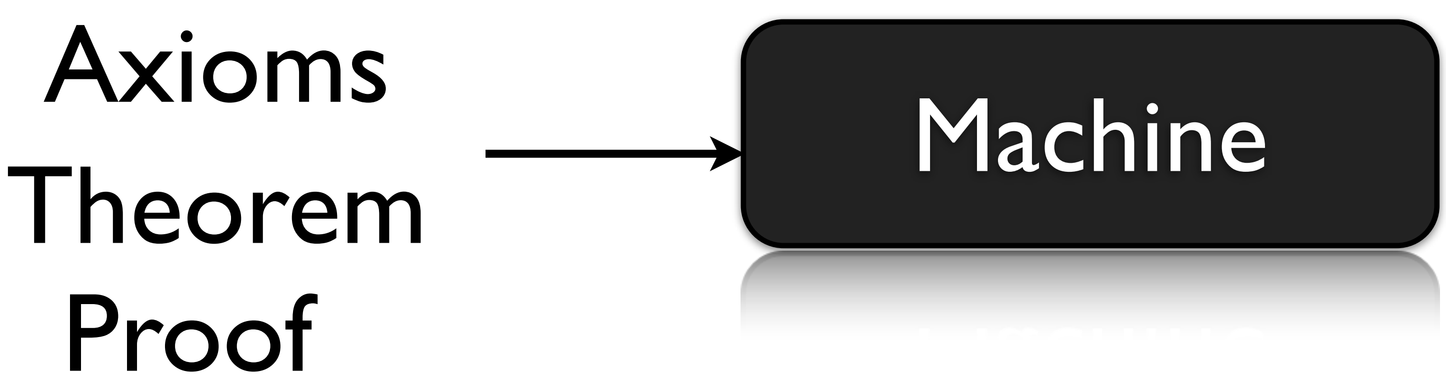

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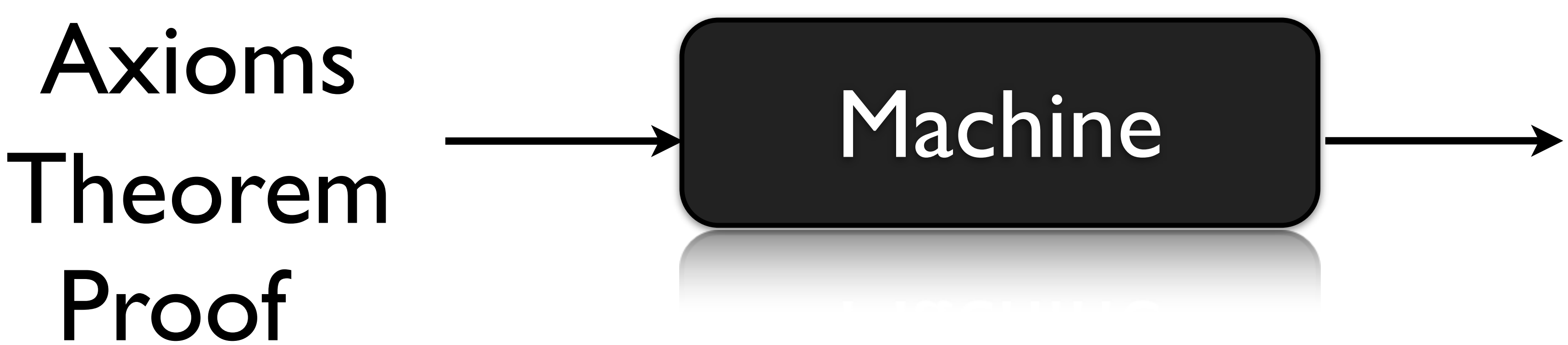

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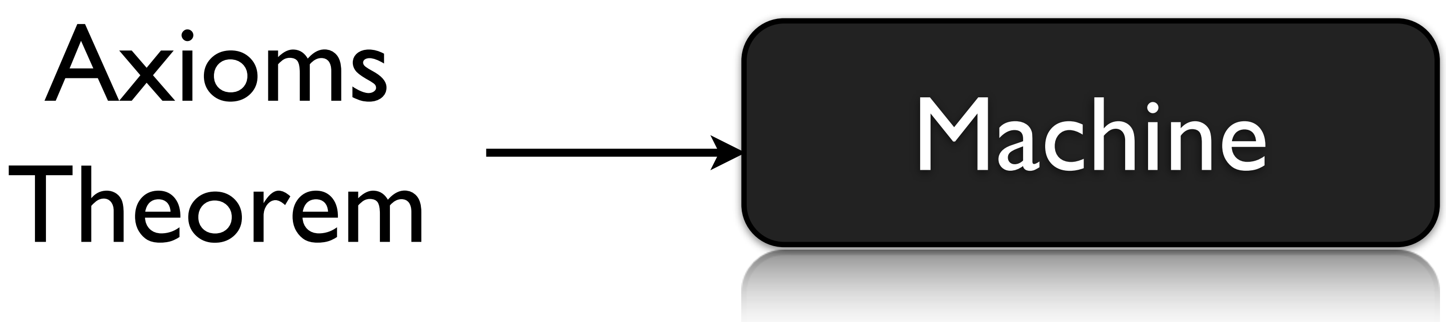

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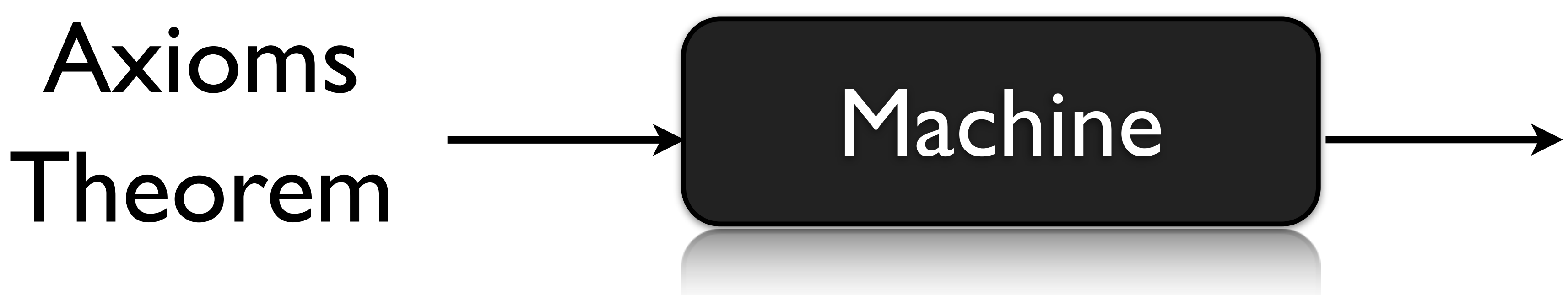

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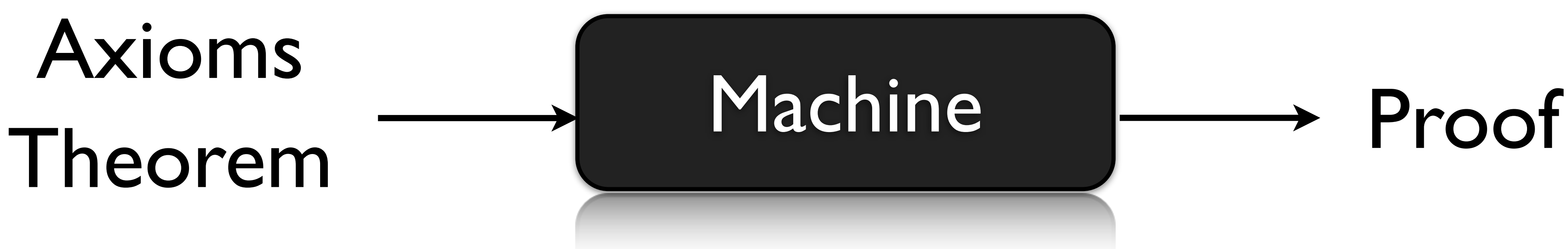

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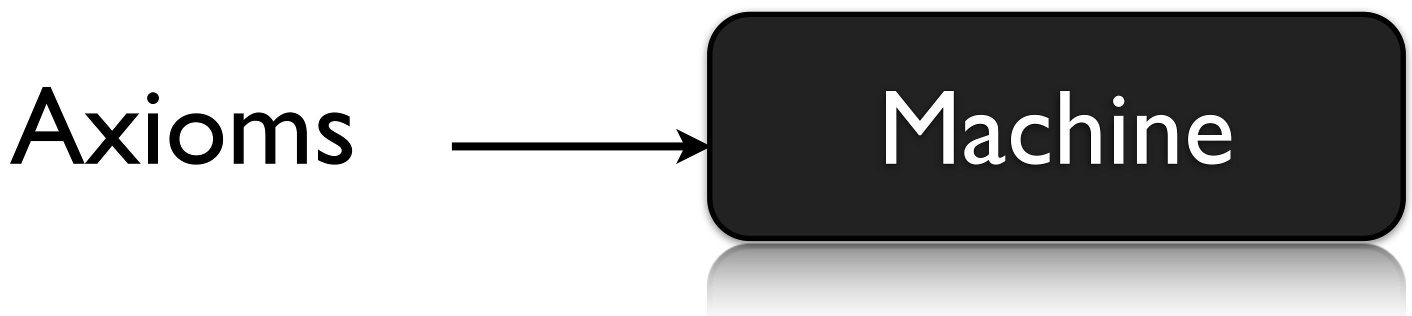

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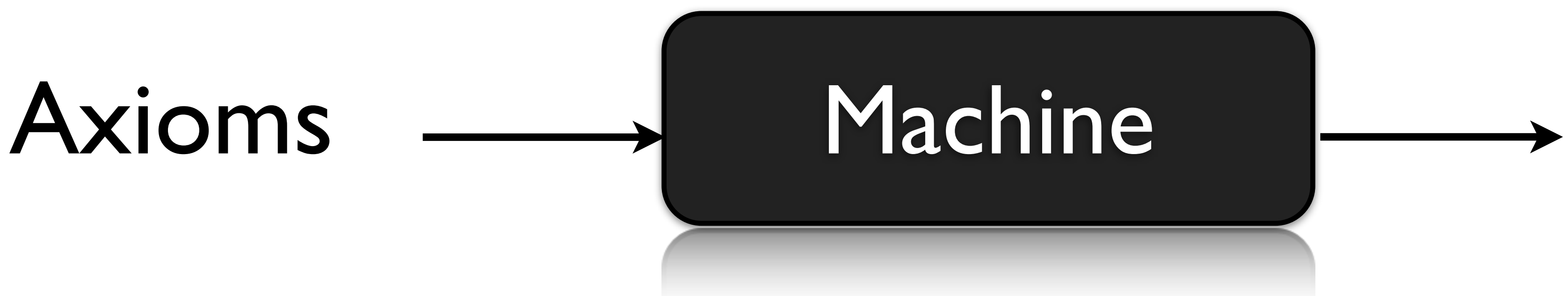

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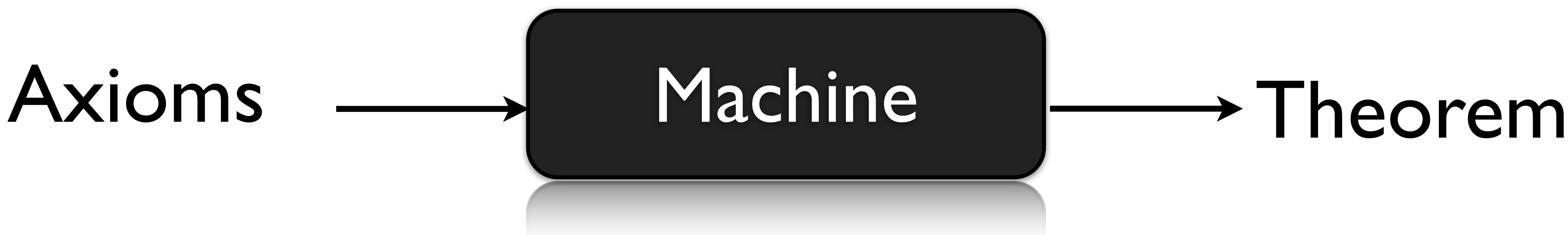
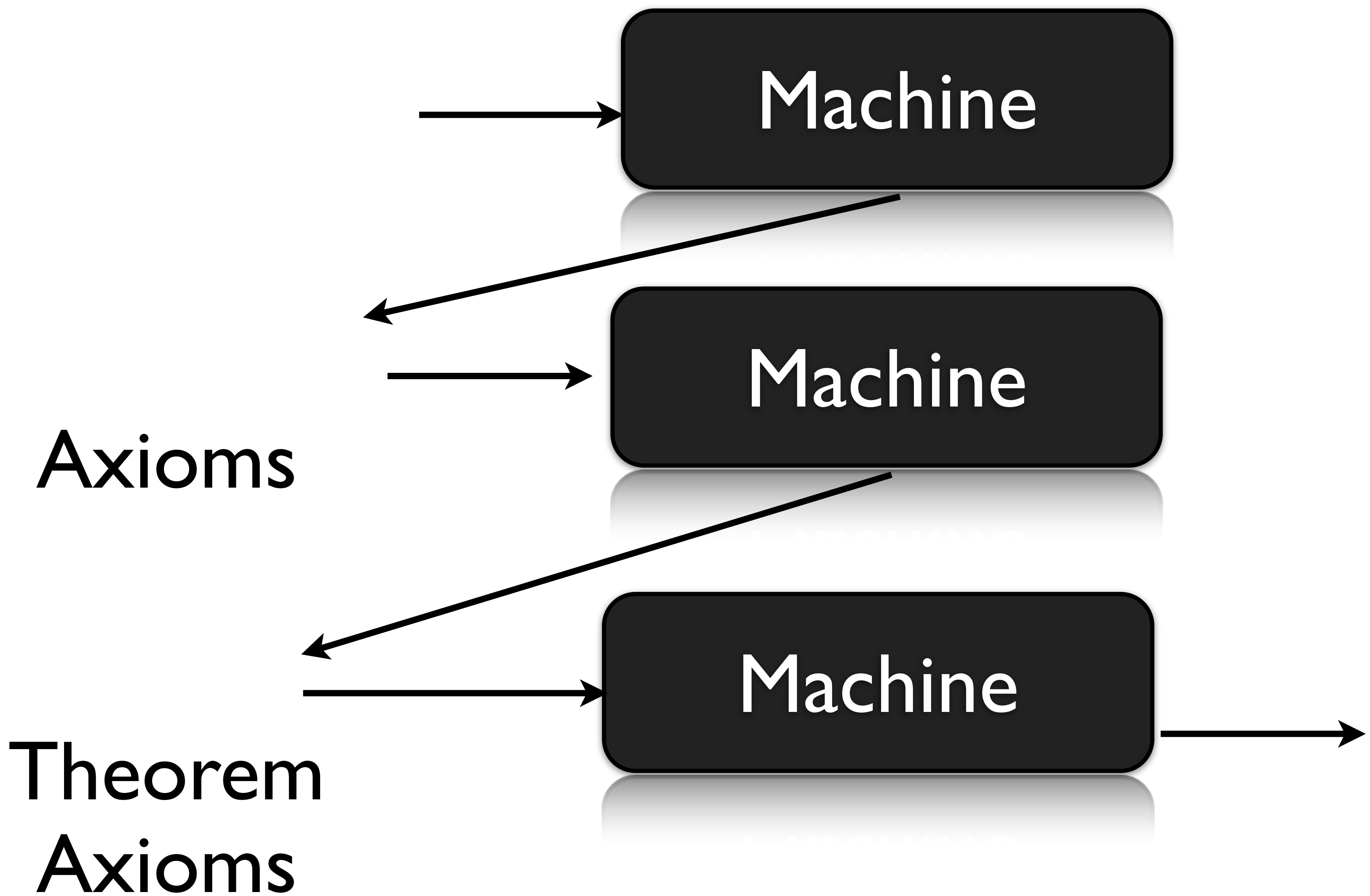


The Master Process: Combine All Three



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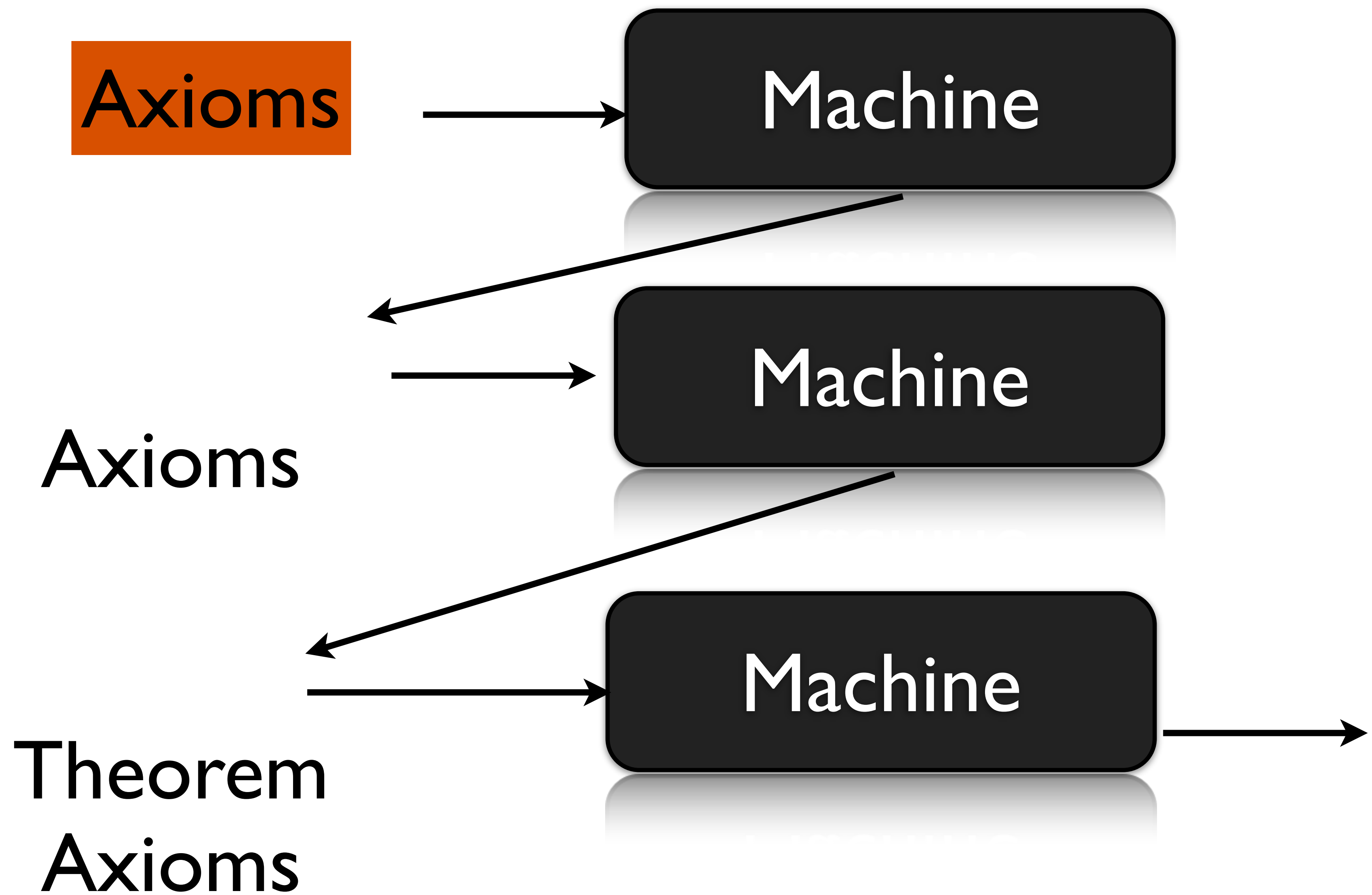

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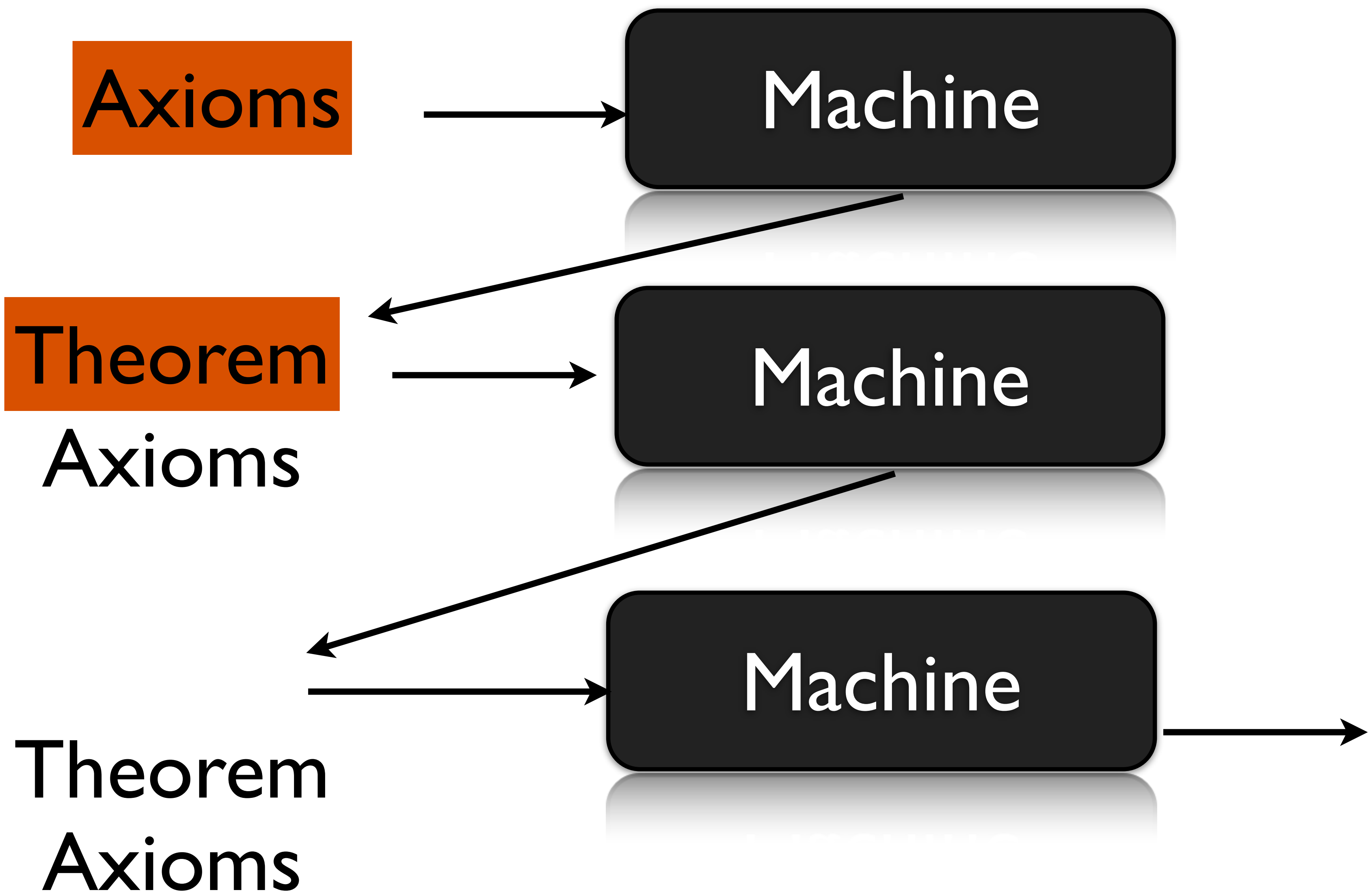

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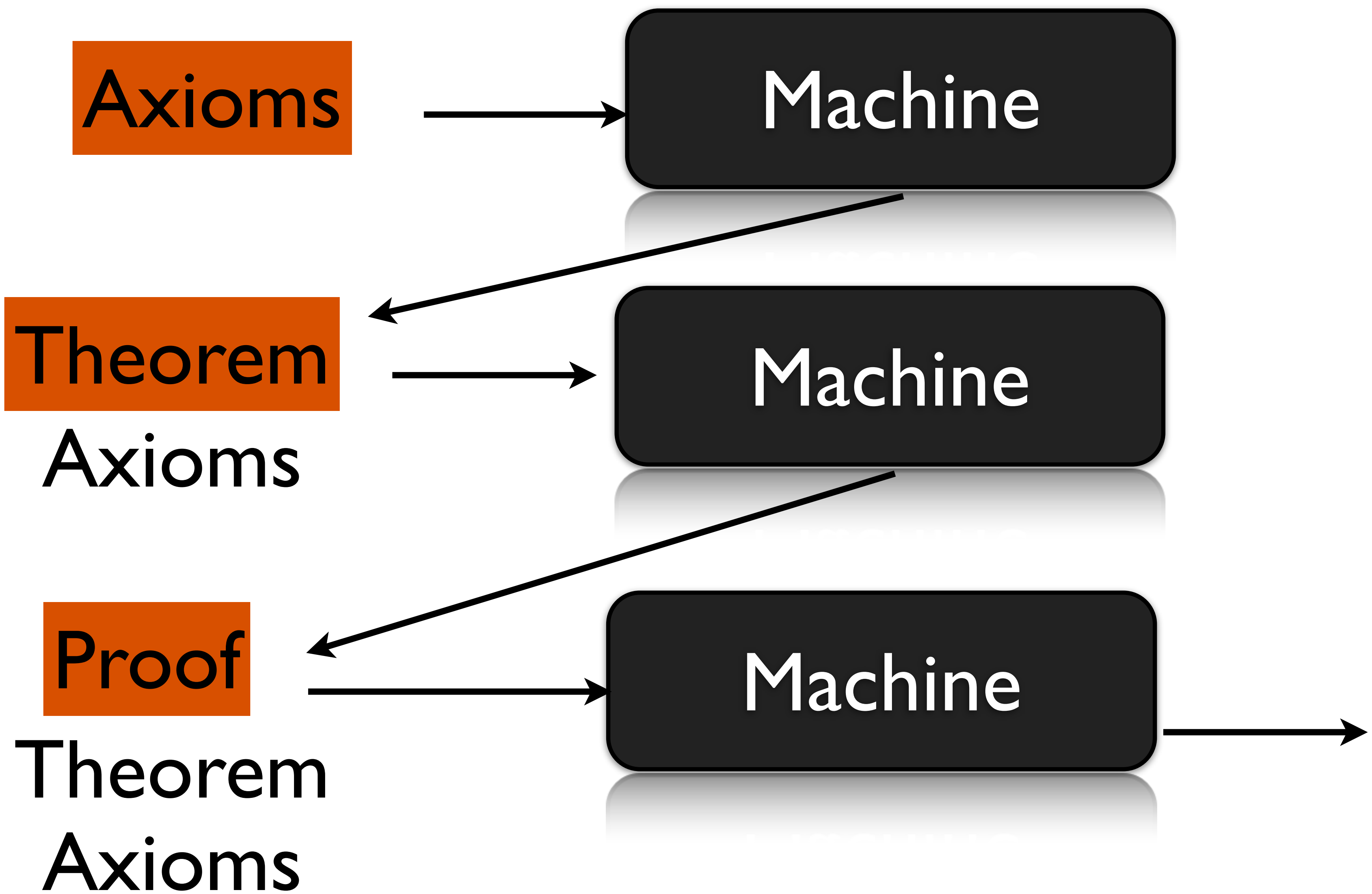

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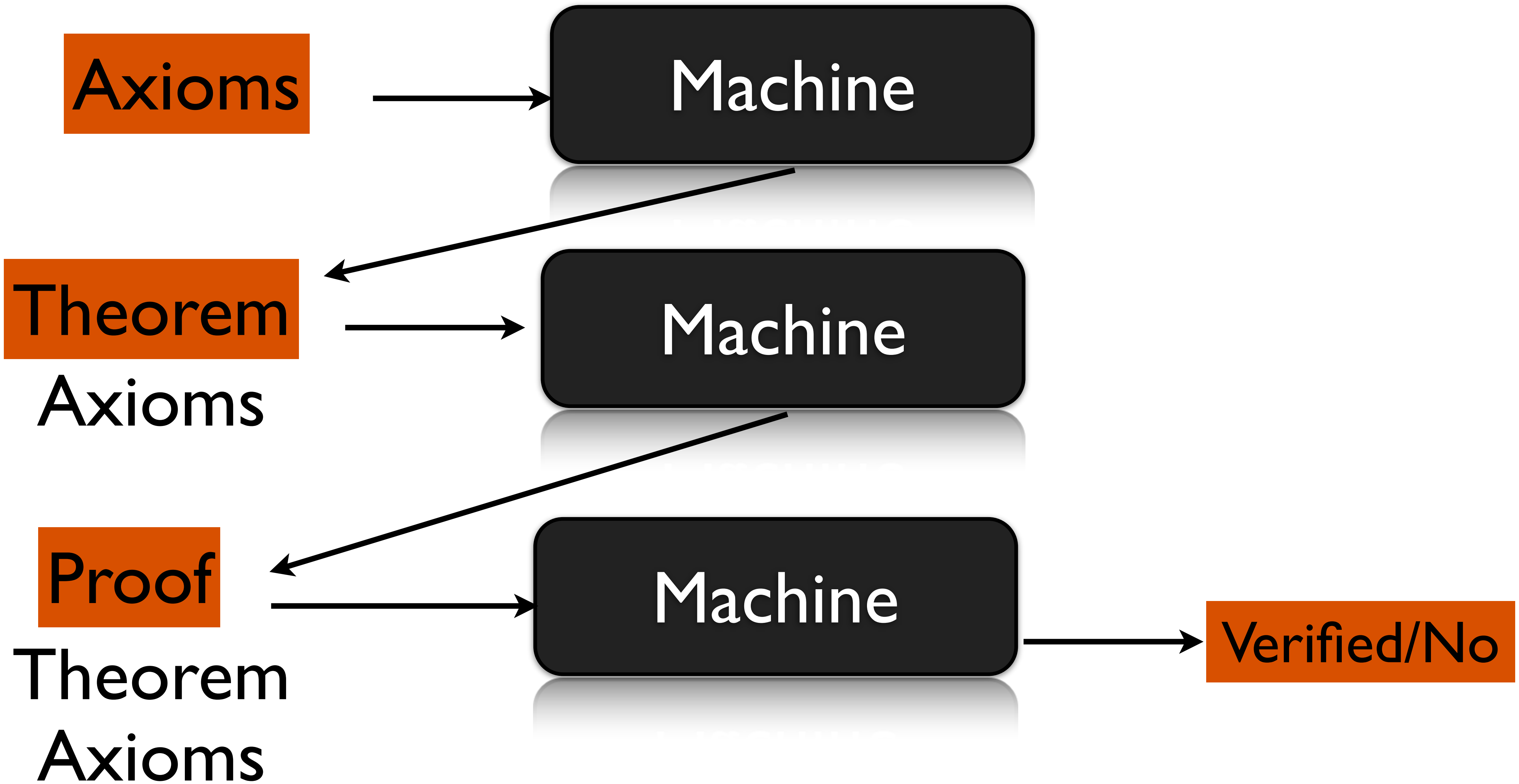

Systems for Proof Verification and Proof Discovery ...

Proof Verification Using Denotational Proof Languages

Proof Verification Using Denotational Proof Languages

sentences

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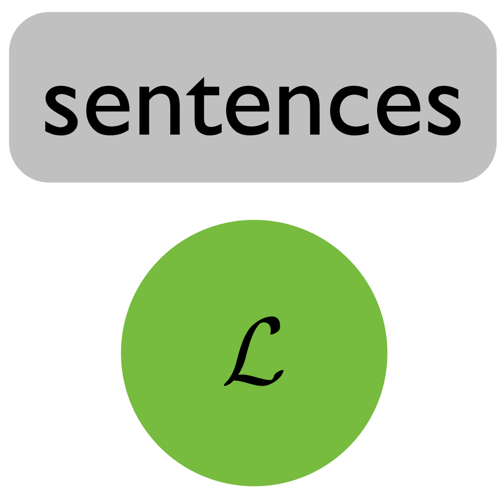

Proof Verification Using Denotational Proof Languages

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Proof Verification Using Denotational Proof Languages

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λ-calculus

Proof Verification Using Denotational Proof Languages

expression

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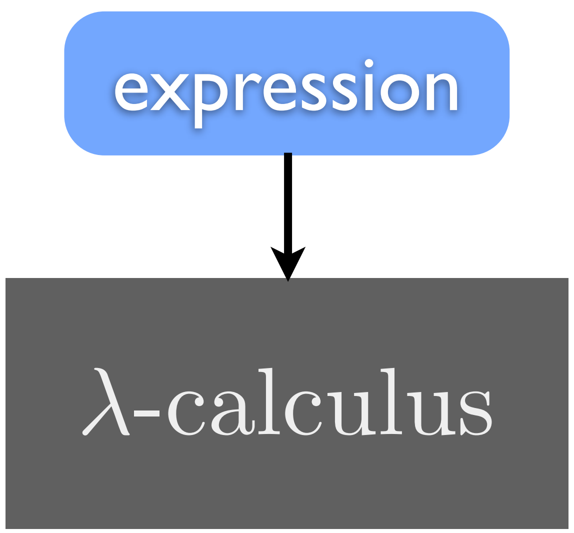

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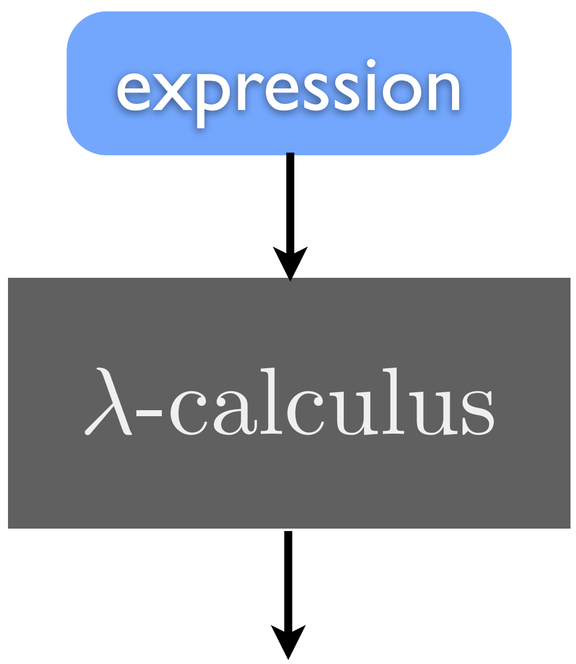

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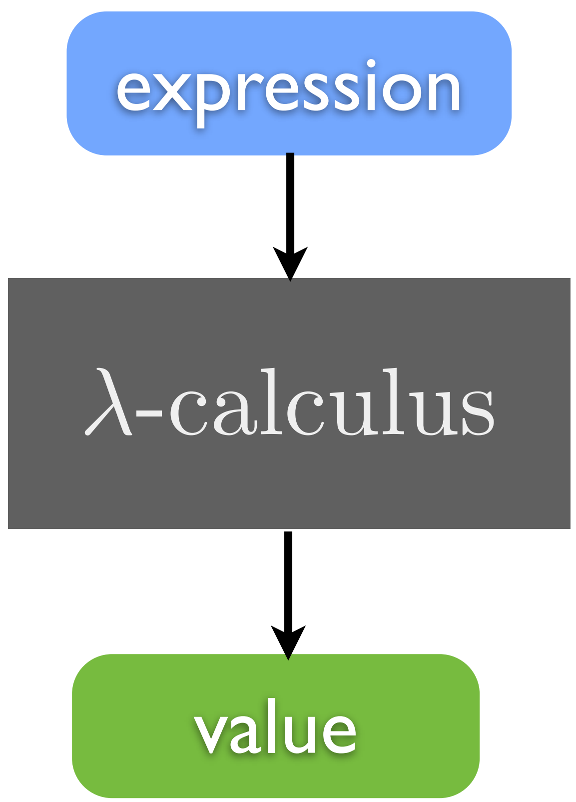

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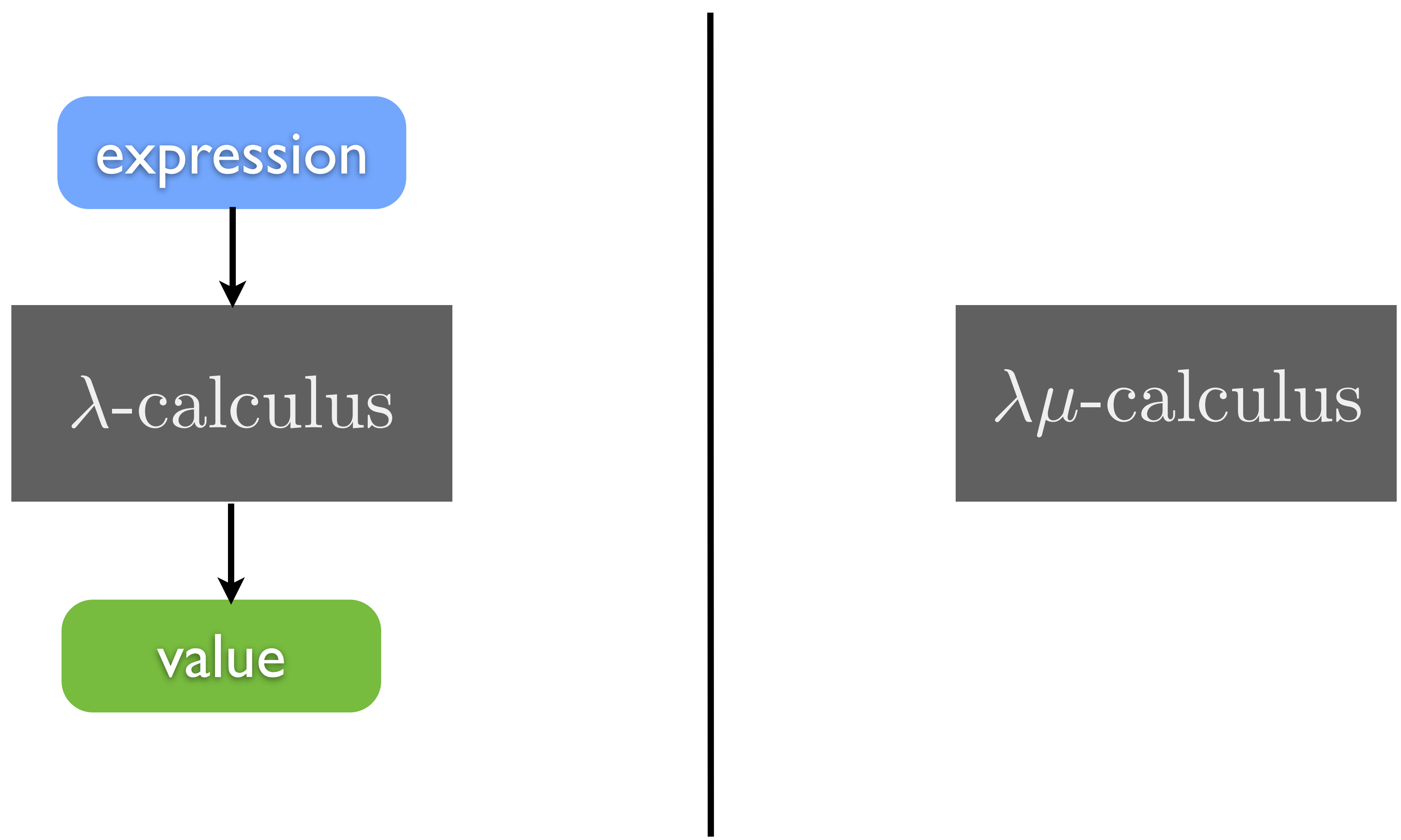

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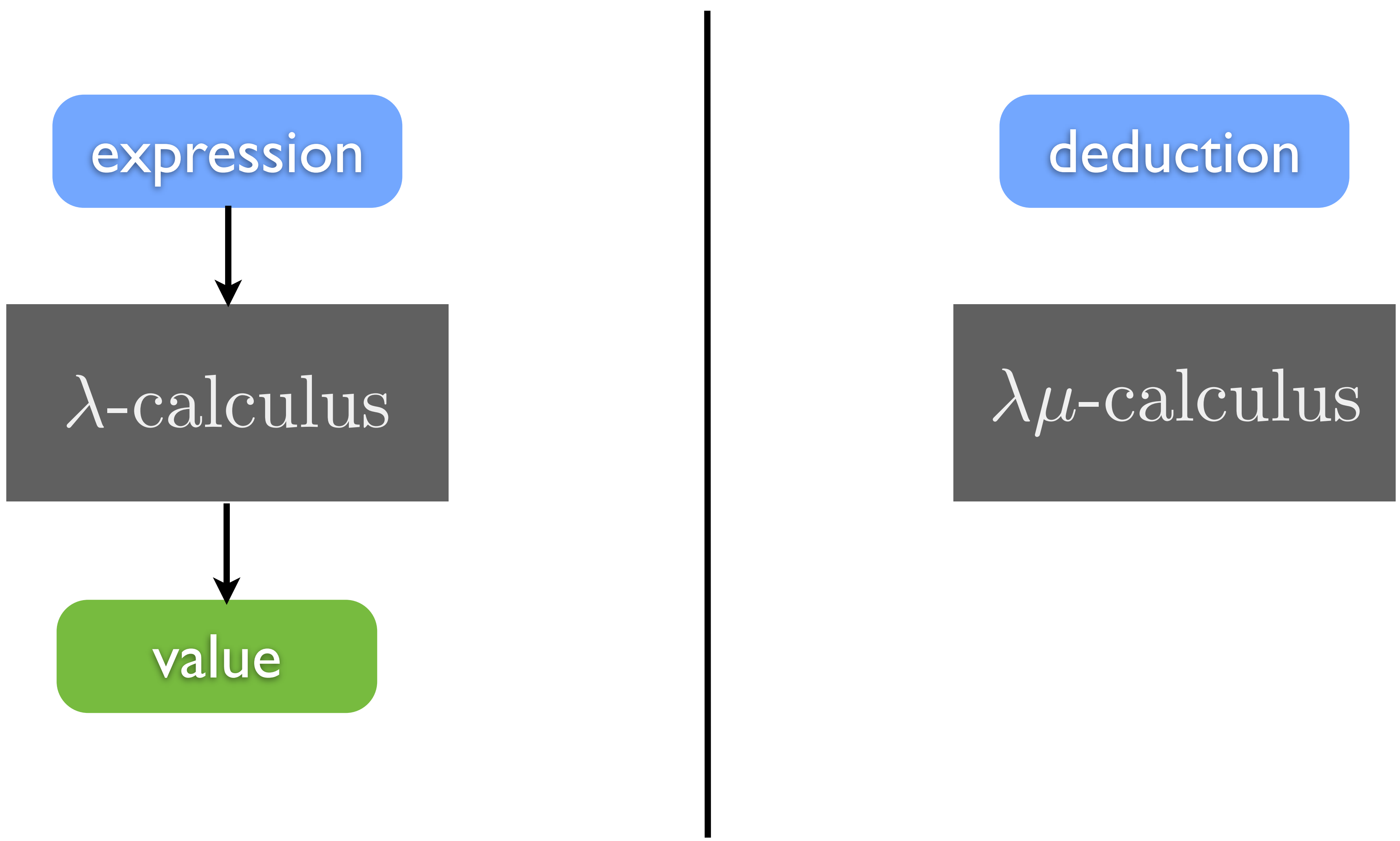

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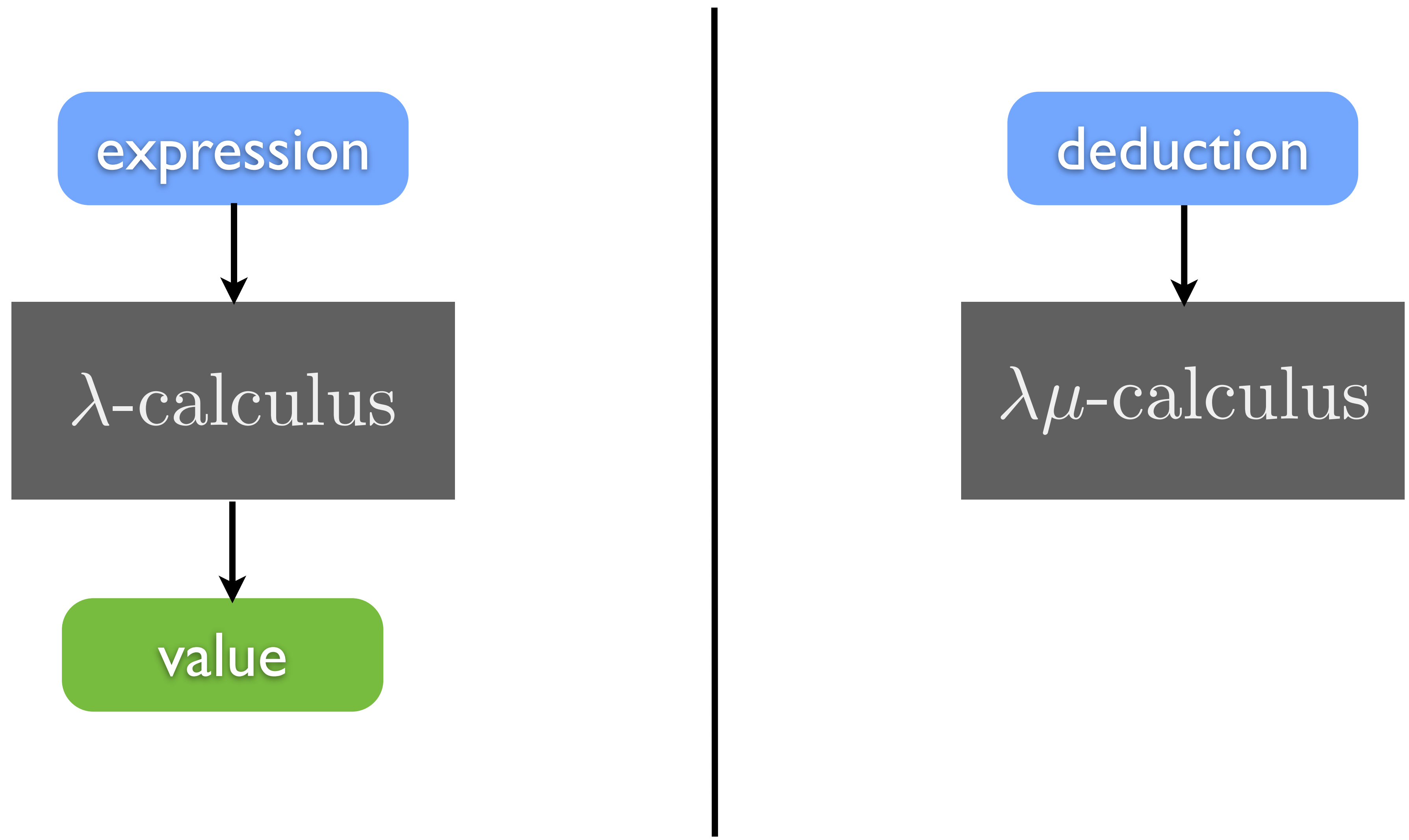

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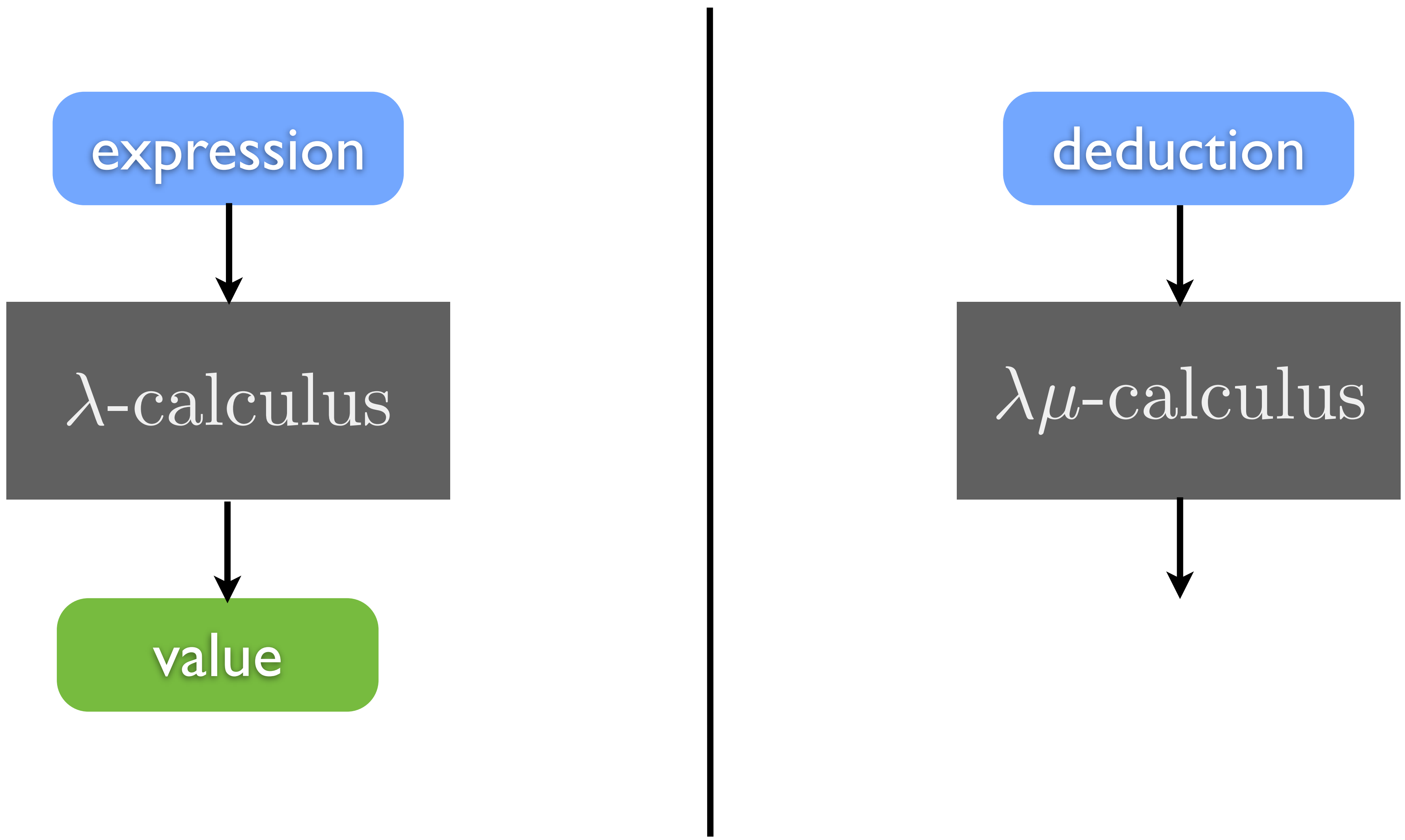

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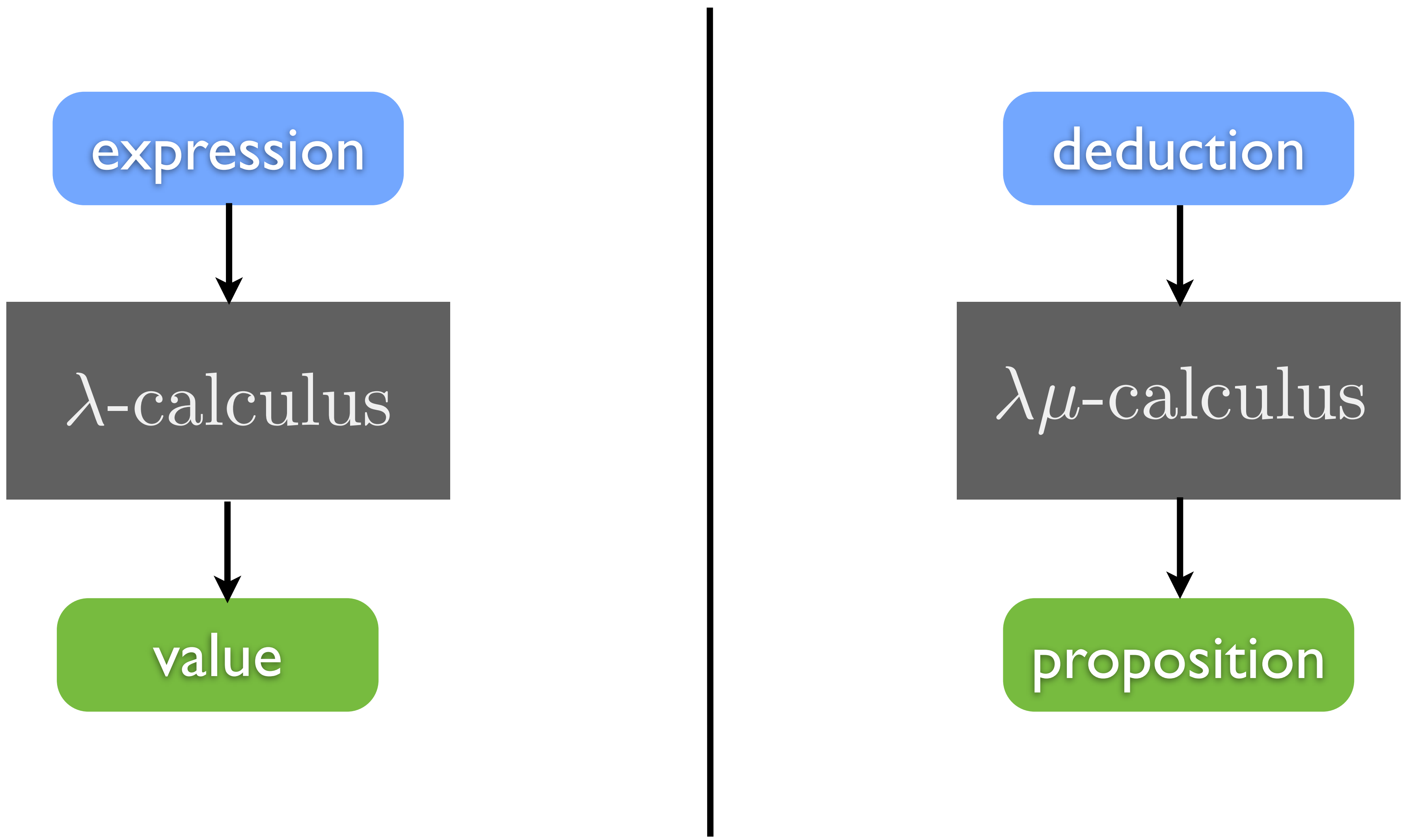

Proof Verification Using Denotational Proof Languages

Proof Verification Using Denotational Proof Languages

deduction

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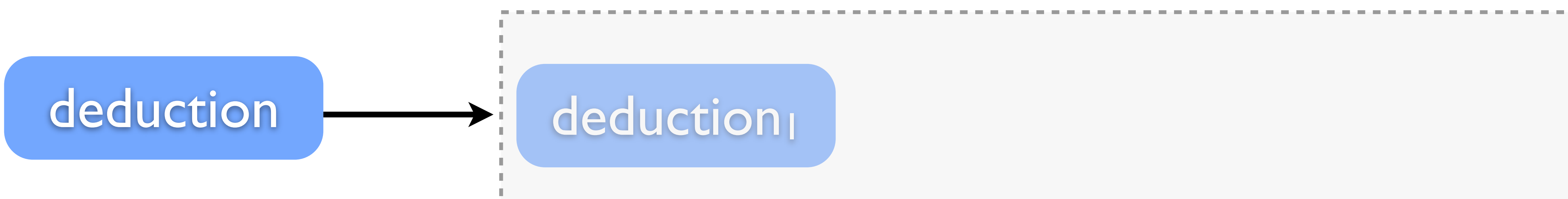

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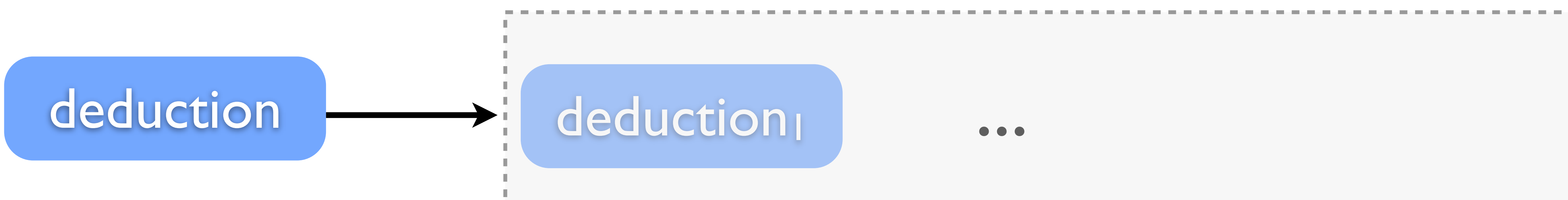

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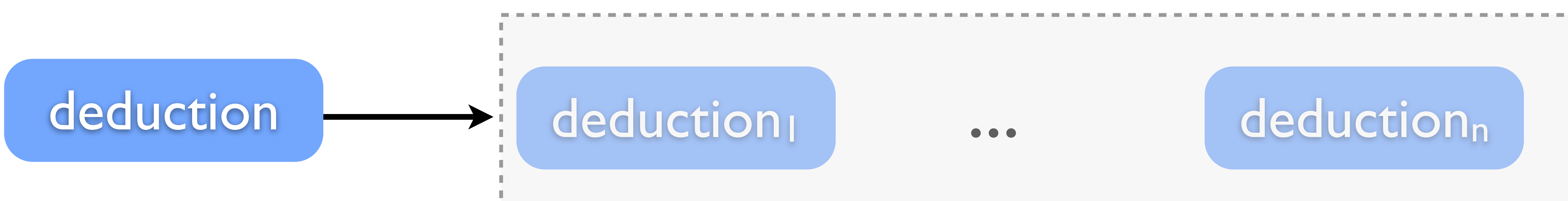

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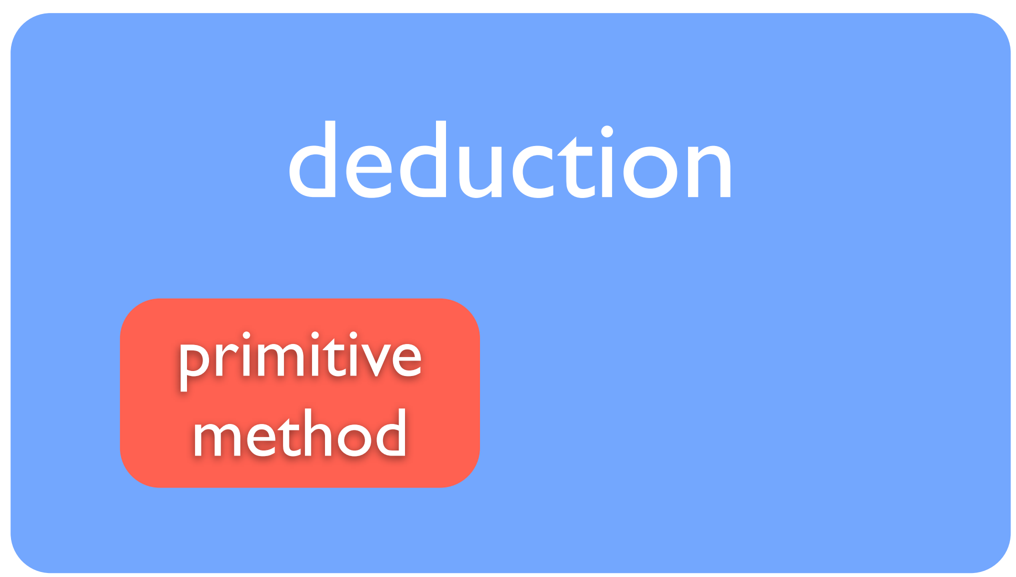

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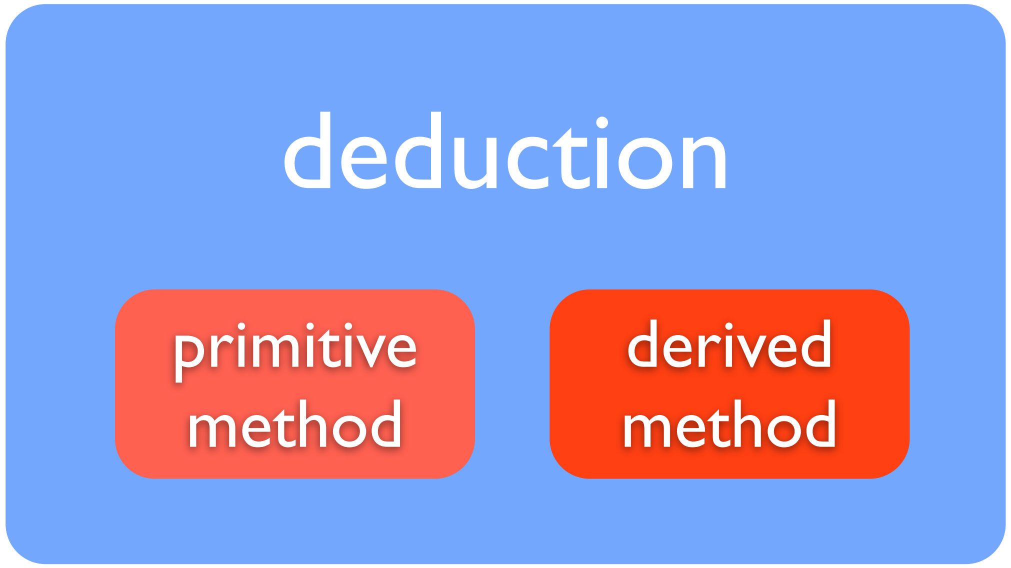

Important! Naïve, "black-box" use of an ATP won't work ...

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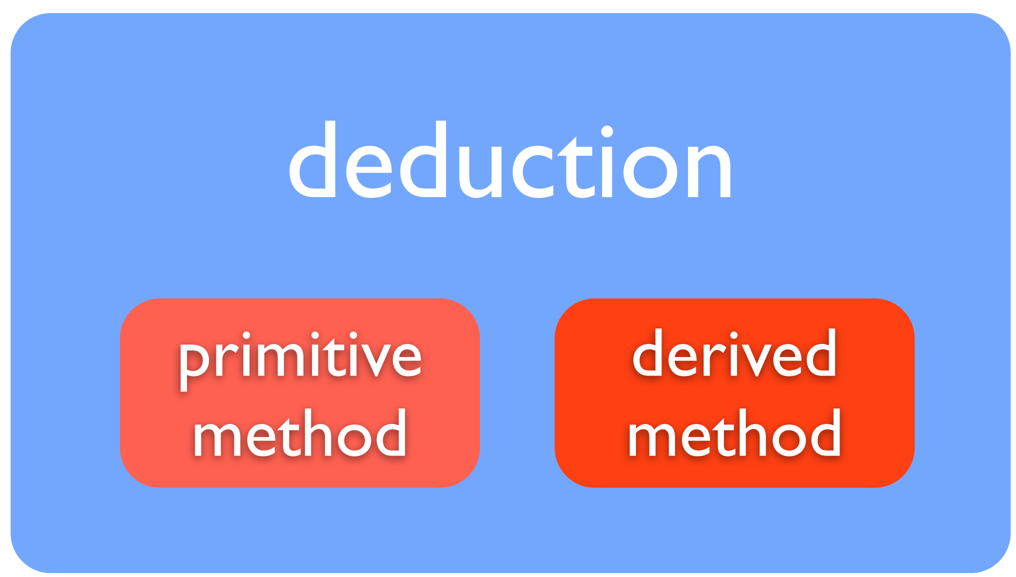

Semi-Automated Proof Discovery Using ATP

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a first-order language $\mathcal{L}$ whose sentences are $S(\mathcal{L})$

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$$\text{premises} \subset S(\mathcal{L})$$

$$\texttt{eval}(\langle \text{theorem}, \text{premises}, \texttt{nil} \rangle) = \begin{cases} \mathit{error} \text{ if premises} \not\subseteq \beta \\ \mathit{prover}(\mathit{theorem}, \mathit{premises}) \text{ if premises} \subseteq \beta \end{cases}$$

$$\texttt{eval}(\langle \text{theorem}, \text{premises}, [p_1, \ldots, p_n] \rangle) = (\wedge_i \texttt{eval}(p_i)) \Rightarrow \mathit{prover}(\mathit{theorem}, \mathit{premises} \cup \{\mathit{theorem}(p_1), \ldots, \mathit{theorem}(p_n)\}))$$

Semi-Automated Proof Discovery Using ATP

a first-order language $\mathcal{L}$ whose sentences are $S(\mathcal{L})$

let $\beta \subset S(\mathcal{L})$ be the entire set of axioms that one is allowed to use

$$\text{problem} := \langle \text{theorem}, \text{premises}, \texttt{list}\langle \text{problem} \rangle \rangle$$
$$\text{theorem} \in S(\mathcal{L})$$
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(This serves as well as a meta proof theory for semi-automated proofs that do not use natural deduction in Slate.)

Resolution Example from Russell and Norvig’s AIMA ...

The problem

ARTIFICIAL INTELLIGENCE: A MODERN APPROACH, 3/E

Either Jack or Curiosity killed the cat, who is named Tuna.

Did Curiosity kill the cat?

First, we express the original sentences, some background knowledge, and the negated goal G in first-order logic:

A. $\forall x \;\; [\forall y \;\; Animal(y) \;\Rightarrow\; Loves(x,y)] \;\Rightarrow\; [\exists y \;\; Loves(y,x)]$

B. $\forall x \;\; [\exists z \;\; Animal(z) \land Kills(x,z)] \;\Rightarrow\; [\forall y \;\; \neg Loves(y,x)]$

C. $\forall x \;\; Animal(x) \;\Rightarrow\; Loves(Jack,x)$

D. $Kills(Jack, Tuna) \lor Kills(Curiosity, Tuna)$

E. $Cat(Tuna)$

F. $\forall x \;\; Cat(x) \Rightarrow Animal(x)$

¬G. $\neg Kills(Curiosity, Tuna)$

Now we apply the conversion procedure to convert each sentence to CNF:

A1. $Animal(F(x)) \lor Loves(G(x),x)$

A2. $\neg Loves(x,F(x)) \lor Loves(G(x),x)$

B. $\neg Loves(y,x) \lor \neg Animal(z) \lor \neg Kills(x,z)$

C. $\neg Animal(x) \lor Loves(Jack,x)$

D. $Kills(Jack, Tuna) \lor Kills(Curiosity, Tuna)$

E. $Cat(Tuna)$

F. $\neg Cat(x) \lor Animal(x)$

¬G. $\boxed{\neg Kills(Curiosity, Tuna)}$

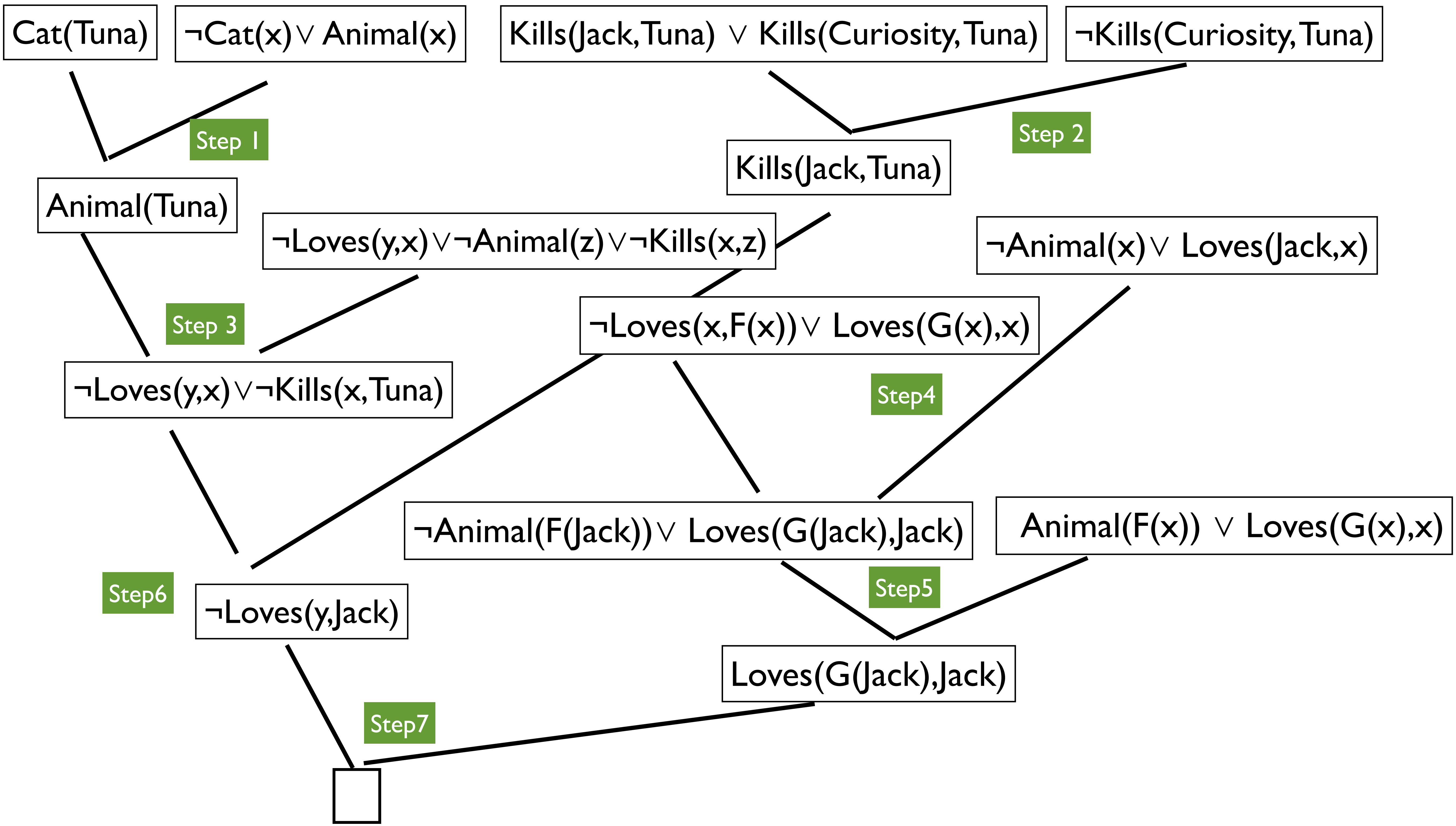
Negated goal

Signature

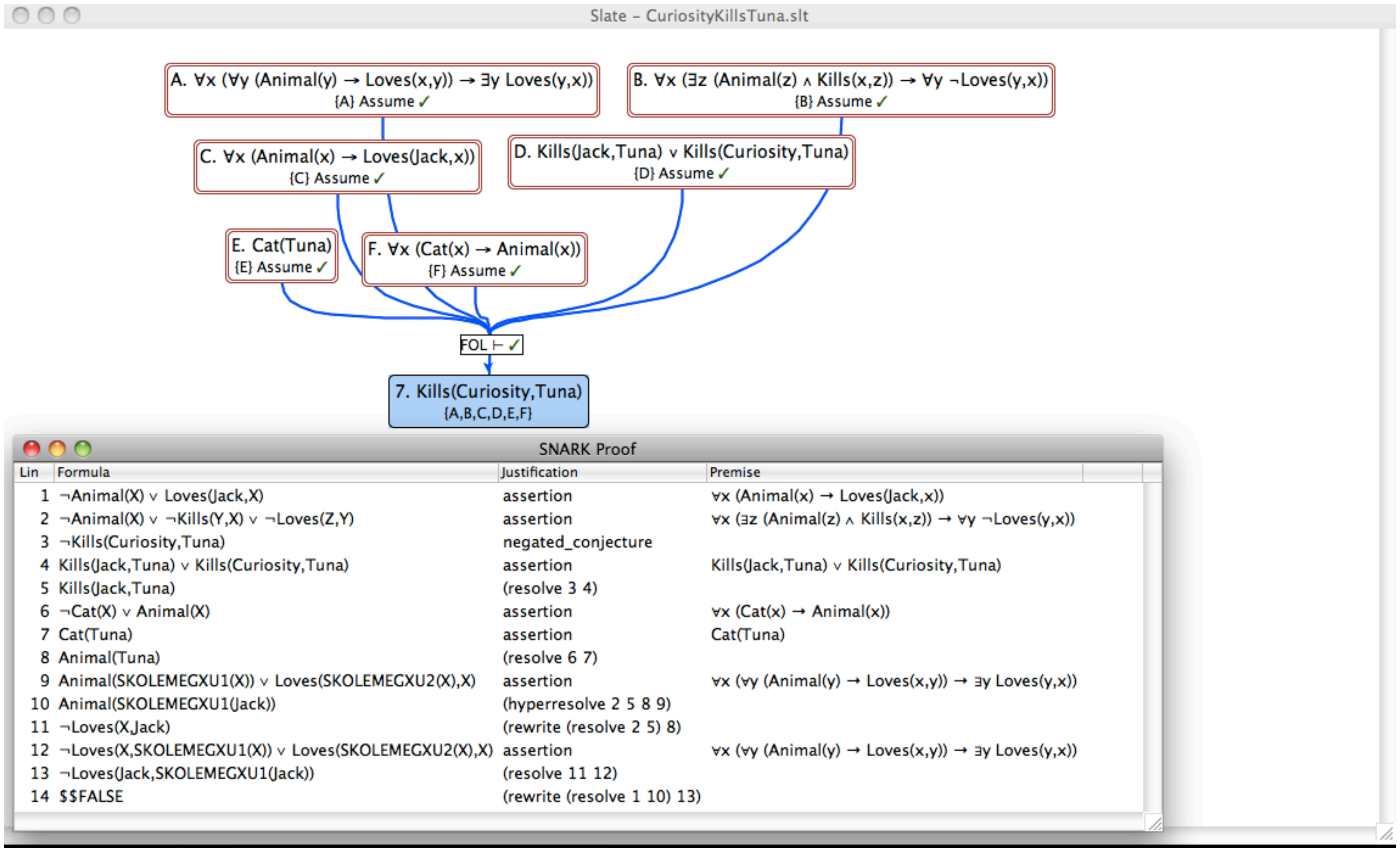
Predicate Symbol	Arity
Animal	1
Loves	2
Kills	2
Cat	1

Function Symbol	Arity
F	1
G	1
Jack	0
Curiosity	0
Tuna	0

A Resolution Proof



In Slate



Lin	Formula	Justification	Premise
1	¬Animal(X) ∨ Loves(Jack,X)	assertion	∀x (Animal(x) → Loves(Jack,x))
2	¬Animal(X) ∨ ¬Kills(Y,X) ∨ ¬Loves(Z,Y)	assertion	∀x (∃z (Animal(z) ∧ Kills(x,z)) → ∀y ¬Loves(y,x))
3	¬Kills(Curiosity,Tuna)	negated_conjecture	
4	Kills(Jack,Tuna) ∨ Kills(Curiosity,Tuna)	assertion	Kills(Jack,Tuna) ∨ Kills(Curiosity,Tuna)
5	Kills(Jack,Tuna)	(resolve 3 4)	
6	¬Cat(X) ∨ Animal(X)	assertion	∀x (Cat(x) → Animal(x))
7	Cat(Tuna)	assertion	Cat(Tuna)
8	Animal(Tuna)	(resolve 6 7)	
9	Animal(SKOLEMEGXU1(X)) ∨ Loves(SKOLEMEGXU2(X),X)	assertion	∀x (∀y (Animal(y) → Loves(x,y)) → ∃y Loves(y,x))
10	Animal(SKOLEMEGXU1(Jack))	(hyperresolve 2 5 8 9)	
11	¬Loves(X,Jack)	(rewrite (resolve 2 5) 8)	
12	¬Loves(X,SKOLEMEGXU1(X)) ∨ Loves(SKOLEMEGXU2(X),X)	assertion	∀x (∀y (Animal(y) → Loves(x,y)) → ∃y Loves(y,x))
13	¬Loves(Jack,SKOLEMEGXU1(Jack))	(resolve 11 12)	
14	$$FALSE	(rewrite (resolve 1 10) 13)	

Proof Verification & Discovery in Slate

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- Used to teach logic in RPI.

Examples ...

Special-Relativity Axiomatization

- Multi-sorted first-order logic:

Special-Relativity Axiomatization

- Multi-sorted first-order logic:

sorts

Special-Relativity Axiomatization

- Multi-sorted first-order logic:

$\boxed{B}$

sorts

Special-Relativity Axiomatization

- Multi-sorted first-order logic:

bodies (inertial and photons) $\boxed{B}$

sorts

Special-Relativity Axiomatization

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bodies (inertial and photons)

B	Q

sorts

quantities

Special-Relativity Axiomatization

- Multi-sorted first-order logic:

bodies (inertial and photons)	B	Q	sorts
			quantities

$$\mathrm{IB} : B \mapsto \mathrm{Bool}$$

$$\mathrm{Ph} : B \mapsto \mathrm{Bool}$$

$$+ : Q \times Q \mapsto Q$$

$$. : Q \times Q \mapsto Q$$

$$\mathrm{W} : B \times B \times Q^4 \mapsto \mathrm{Bool}$$

Special-Relativity Axiomatization

- Multi-sorted first-order logic:

bodies (inertial and photons)

B	Q

sorts

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signature

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AxSf	All observers observe themselves to be on the time axis.
AxSm	The speed of light for any observer is one, and all observers use the same units of measurement.

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Theorem Neat

(No Event at Two Places)

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$$\forall m,x,y\ ((IOb(m) \wedge Q(x) \wedge Q(y)) \rightarrow (x \neq y \rightarrow ev(m,x) \neq ev(m,y)))$$

Theorem Neat

(No Event at Two Places)

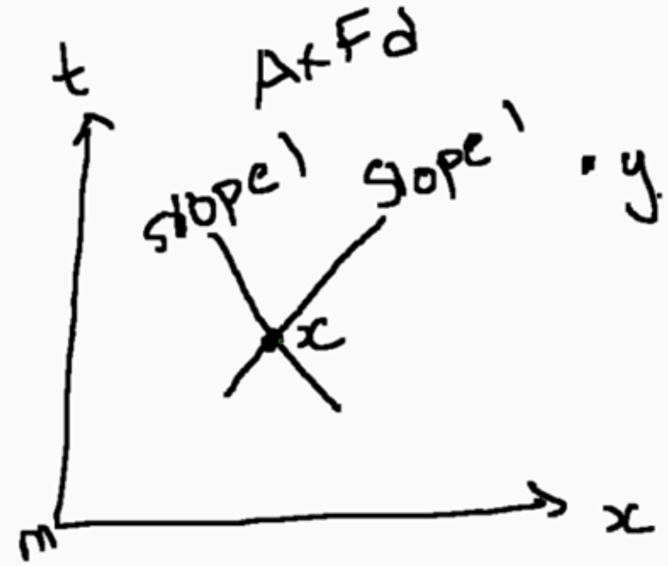
$$\forall m,x,y\ ((IOb(m) \wedge Q(x) \wedge Q(y)) \rightarrow (x \neq y \rightarrow ev(m,x) \neq ev(m,y)))$$

An inertial observer is just a Inertial body which coördinatizes some body.
IOb(m) == IB(m) & exists (k,x,y,z,t) W(m,k,x,y,z,t)

For all inertial observers *m* and quantities *x* and *y*, if and *x* and *y* are distinct, then the events that *m* observes at *x* is not the same as the events that *m* observes at *y*.

AxFd, AxPh

$c = \underline{1}$.



Semi-automated Proof in Slate

AxPh. $\forall m,x,y\ ((IOb(m) \wedge Q(x) \wedge Q(y)) \rightarrow (\exists p\ (Ph(p) \wedge W(m,p,x) \wedge W(m,p,y)) \leftrightarrow (speed(x,y) = cm)))$
{AxPh} Assume ✓

From AxFd. $\forall m,x,y\ ((IOb(m) \wedge Q(x) \wedge Q(y)) \rightarrow (x \neq y \rightarrow \exists z\ (Q(z) \wedge (speed(x,z) = cm) \wedge speed(z,y) \neq cm)))$
{From AxFd} Assume ✓

Sort Axiom. $\forall x\ (Ph(x) \rightarrow B(x))$
{Sort Axiom} Assume ✓

FOL ⊢ ✓

6. $\forall m,x,y\ ((IOb(m) \wedge Q(x) \wedge Q(y)) \rightarrow (x \neq y \rightarrow \exists b\ (B(b) \wedge ((W(m,b,x) \wedge \neg W(m,b,y)) \vee (\neg W(m,b,x) \wedge W(m,b,y))))))$
{AxPh,From AxFd,Sort Axiom}

Definition–Event–P. $\forall m,b,p\ ((IOb(m) \wedge B(b) \wedge Q(p)) \rightarrow (In(b,ev(m,p)) \leftrightarrow W(m,b,p)))$
{Definition–Event–P} Assume ✓

FOL ⊢ ✓

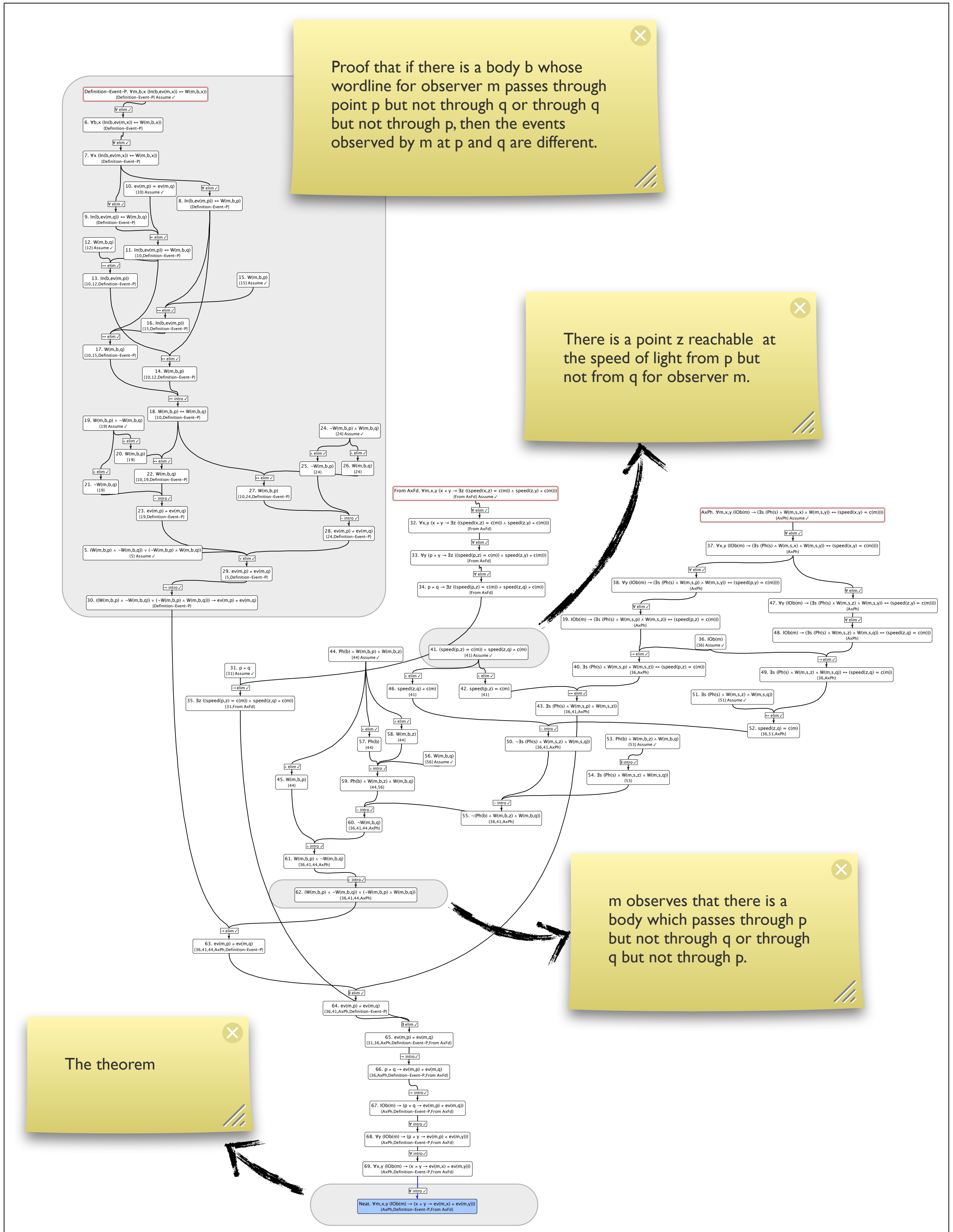
5. $\forall m,p,q\ ((IOb(m) \wedge Q(p) \wedge Q(q)) \rightarrow (p \neq q \rightarrow (\exists b\ (B(b) \wedge ((W(m,b,p) \wedge \neg W(m,b,q)) \vee (\neg W(m,b,p) \wedge W(m,b,q)))) \rightarrow ev(m,p) \neq ev(m,q))))$
{Definition–Event–P}

FOL ⊢ ✓

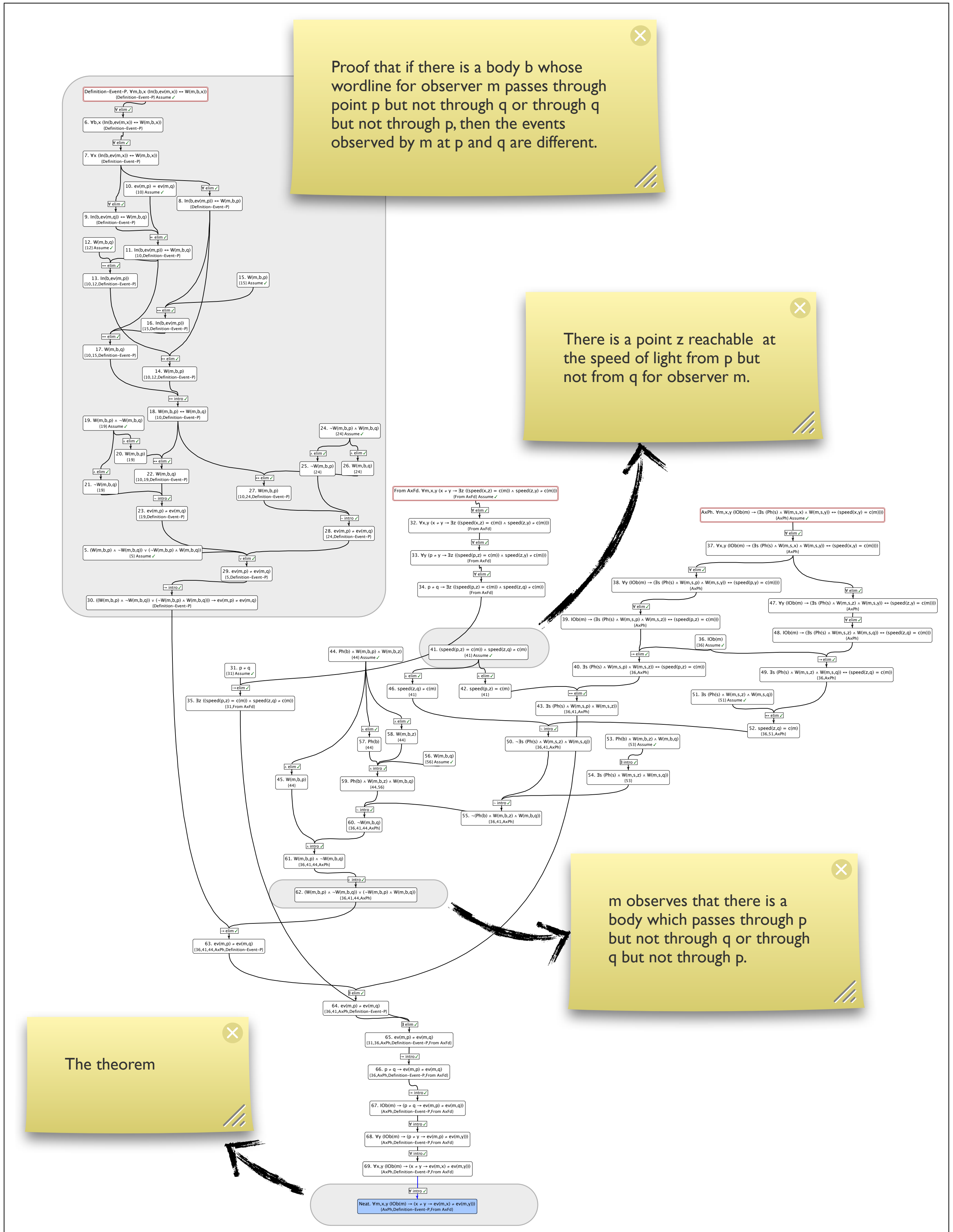
2. $\forall m,x,y\ ((IOb(m) \wedge Q(x) \wedge Q(y)) \rightarrow (x \neq y \rightarrow ev(m,x) \neq ev(m,y)))$
{AxPh,Definition–Event–P,From AxFd,Sort Axiom}

Manual Informal Proof

Manual Proof in Slate



Manual Proof in Slate



Theorem NTFLIO

(No Faster Than Light Travel)

$$\forall m,k,x,y\ ((wl(m,k,x) \land wl(m,k,y) \land x \neq y \land IOb(m) \land IOb(k)) \rightarrow (dist(x,y) < time(x,y)))$$

Theorem NTFLIO

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$$\forall m,k,x,y\ ((wl(m,k,x) \land wl(m,k,y) \land x \neq y \land IOb(m) \land IOb(k)) \rightarrow (dist(x,y) < time(x,y)))$$

For all *m* and *k*, if *m* observes *k* at *x* and *m* observes *k* at also *y*, and if *x* is not equal to *y* and if *m* and *k* are inertial observers, then the spatial distance between *x* and *y* is less than the temporal distance between *x* and *y* (giving us that the speed between *x* and *y* is less than 1, which is the speed of light normalized.)

Theorem NTFLIO
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in progress ...

For all *m* and *k*, if *m* observes *k* at *x* and *m* observes *k* at also *y*, and if *x* is not equal to *y* and if *m* and *k* are inertial observers, then the spatial distance between *x* and *y* is less than the temporal distance between *x* and *y* (giving us that the speed between *x* and *y* is less than 1, which is the speed of light normalized.)