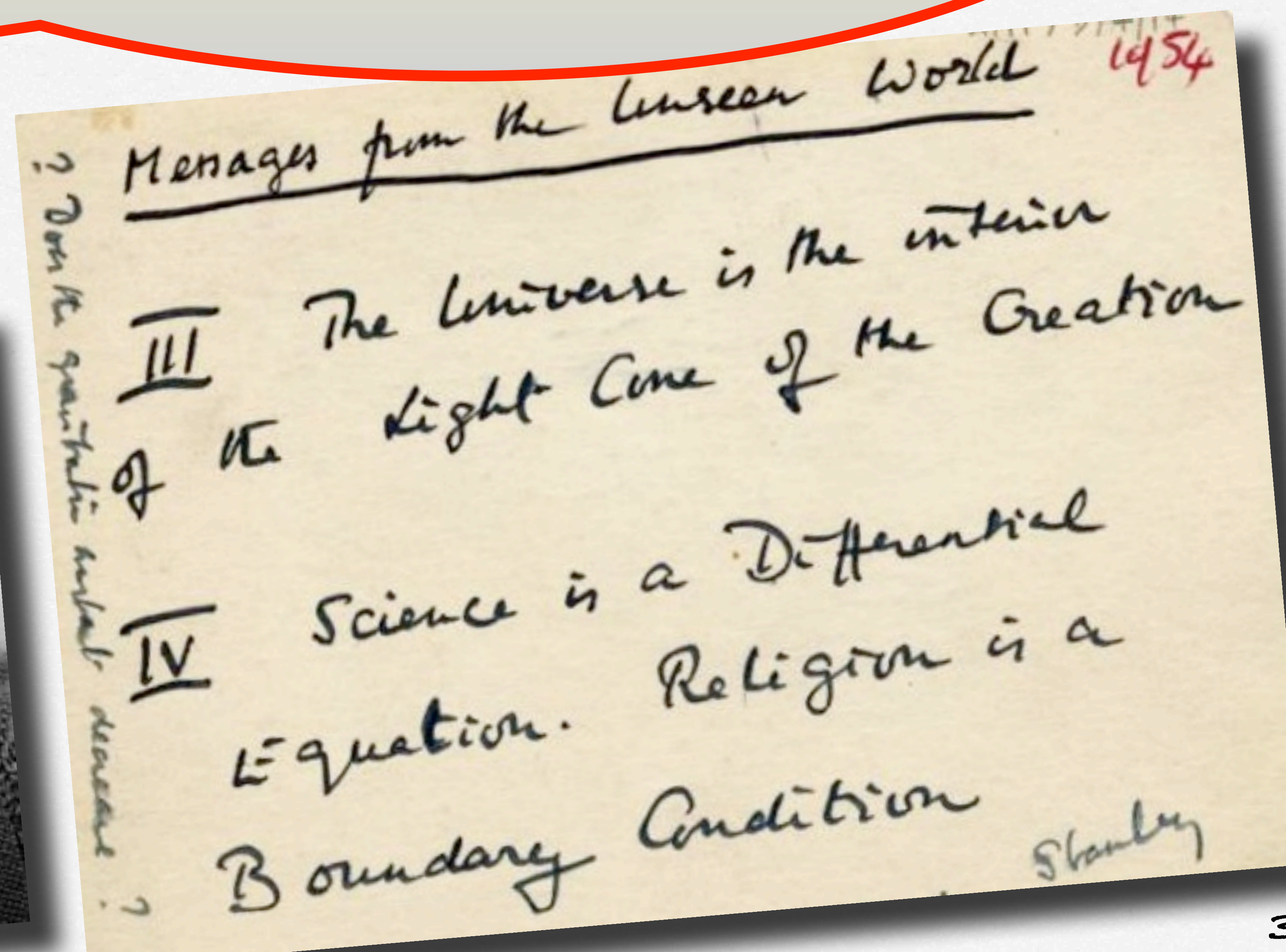
DESCRIPTION versus COMPUTATION ... and Levels of Abstraction

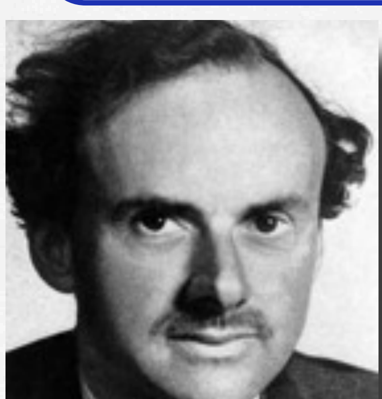
S. Barry Cooper - *First International Conference on Logic and Relativity: Honoring István Németi's 70th Birthday,* Sept. 8 - 12, 2012, Hungarian Academy of Sciences, Budapest

COMPUTATION - Disembodying Turing's Model

... at heart more of an applied than a pure mathematician

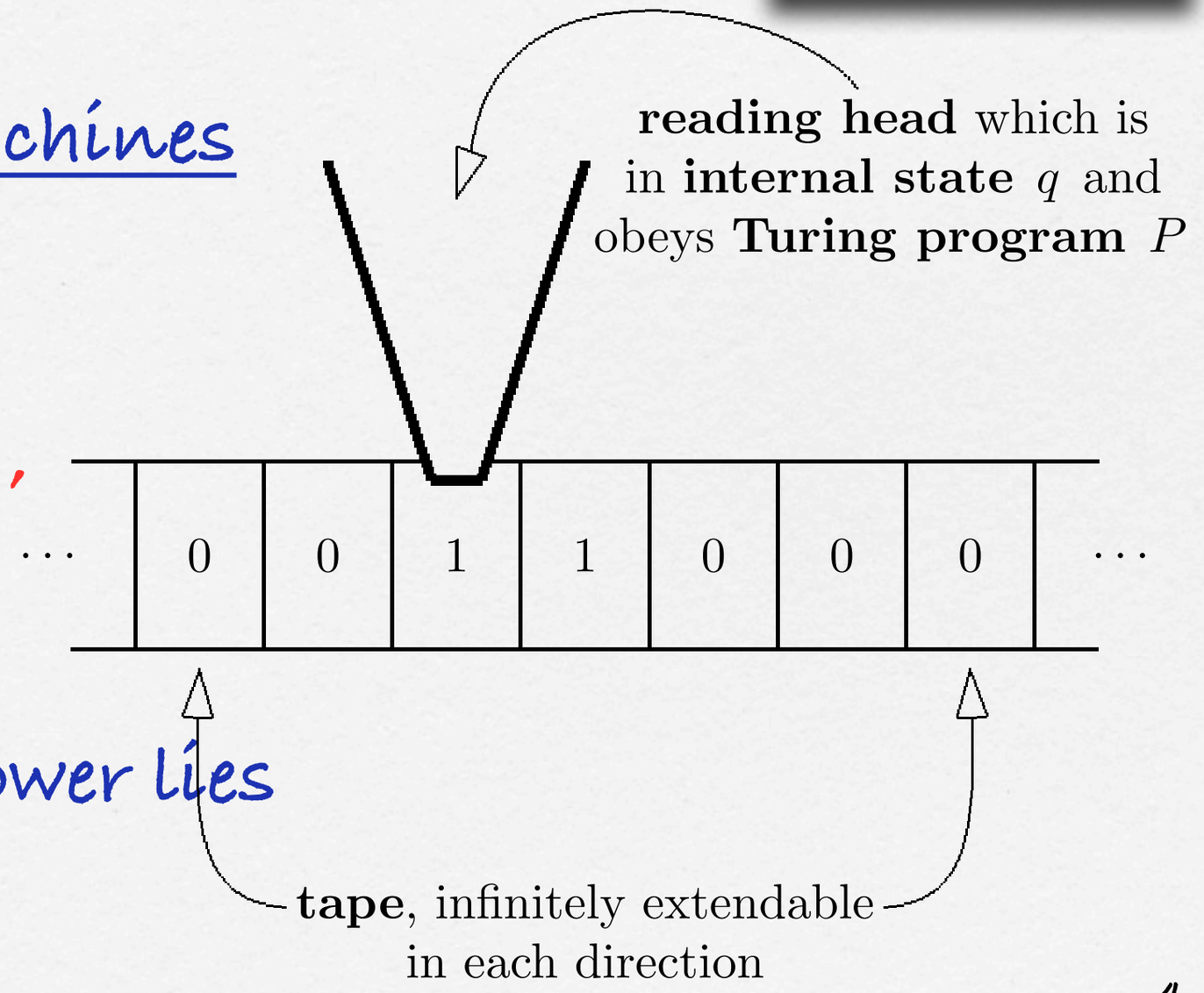


See A. Hodges in 'Collected Works'



A SELF-REFERENTIAL Computing Machine

- 1936 - Turing's machines
- Hardware trivial
- Computes 'anything' computable
- All the computing power lies in the program ...



Universality, and Programs as Data

Turing, 1936: Build a UNIVERSAL TURING MACHINE, which can simulate ANY other machine

Can computably code, and so list, the Turing programs, giving:

$$\varphi_e^{(k)} = \text{ the } k\text{-place partial function computed by } P_e.$$

$$\varphi_e = \varphi_e^{(1)} \textit{ the } \mathbf{e^{th}} \textbf{ partial computable (p.c.) function}$$

Universality, and Programs as Data

Turing, 1936: Build a UNIVERSAL TURING MACHINE, which can simulate ANY other machine

Depends on TYPE 0 description of e^{th} Turing machine

There exists a Turing machine U — the **Universal Turing Machine** — *which if given input (e, x) simulates the e^{th} Turing machine with input x.*

That is, $\varphi_U^{(2)}(e, x) = \varphi_e(x)$.

Universality, and Programs as Data

Turing, 1936: Build a UNIVERSAL TURING MACHINE, which can simulate ANY other machine

- An anticipation of the stored program computer
- A concept *immediately understood* by John von Neumann – as in his 1945 EDVAC report
- And '*program as data*' key to the first computer …

Universality, and Programs as Data

- First Draft of a Report on the EDVAC, 1945

- The General and Logical Theory of Automata, based on talk at the Hixon Symposium, Pasadena, September 1948

John von Neumann

A New Computing Paradigm ...

I am building a brain

The omnipotent computer –

Functionalism and AI – stress what a computer does as something realisable in different hardware – Hilary Putnam: "Minds and Machines", 1960

Virtual Machine (IBM, 1965) – software implementation of a programmable machine – JAVA, Unix

Hilary Putnam

A New Computing Paradigm ...

- Successful reduction of "natural" examples to the Turing model - e.g. quantum computation (David Deutsch)

I am sure
we will have [conscious computers], I expect they will be purely classical, and I expect that it will be a long time in the future. Significant advances in our philosophical understanding of what consciousness is, will be needed.

Question and Answers with David Deutsch, on New.Scientist.com News Service, December, 2006

A New Computing Paradigm ...

- Martin Davis versus selected hypercomputationalists ...

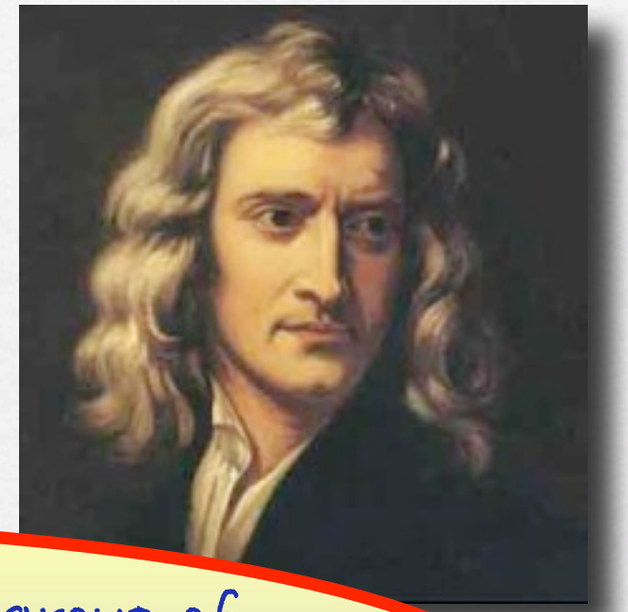
The great success of modern computers as all-purpose algorithm-executing engines embodying Turing's universal computer in physical form, makes it extremely plausible that the abstract theory of computability gives the correct answer to the question 'What is a computation?', and, by itself, makes the existence of any more general form of computation extremely doubtful.

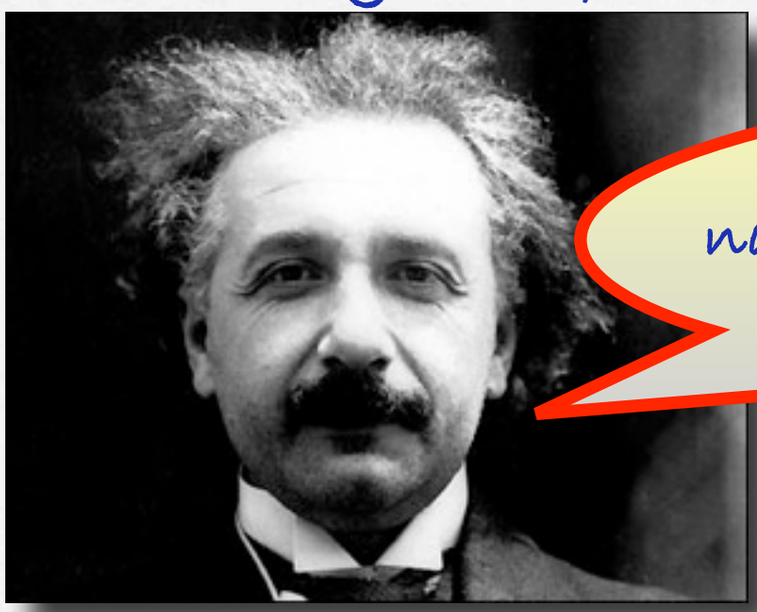
Martin Davis [2004], The myth of hypercomputation. In Alan Turing: Life and legacy of a great thinker (C. Teuscher, ed.), Springer-Verlag


June 2012:
The
Incomputable

Modelling how the World Computes?

- Newton onwards – mathematics rules science
  - we look for computable natural laws …
  - theories which computably predict …
  - try to capture truth via proofs …





When we say that we understand a group of natural phenomena, we mean that we have found a constructive theory which embraces them

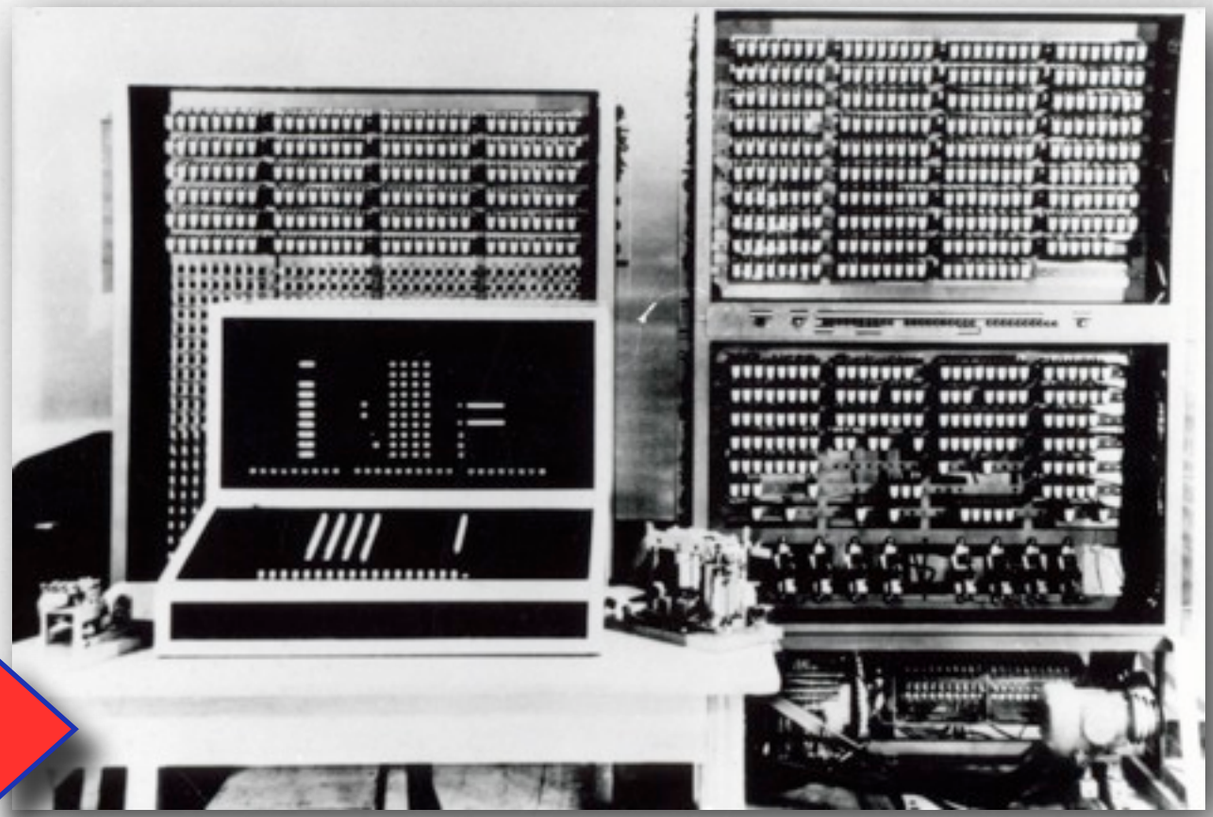
Albert Einstein: P.54, `Out of My Later Years', 1950

PHYSICALITY of Real World Computation ...

Programs as Data Embodied

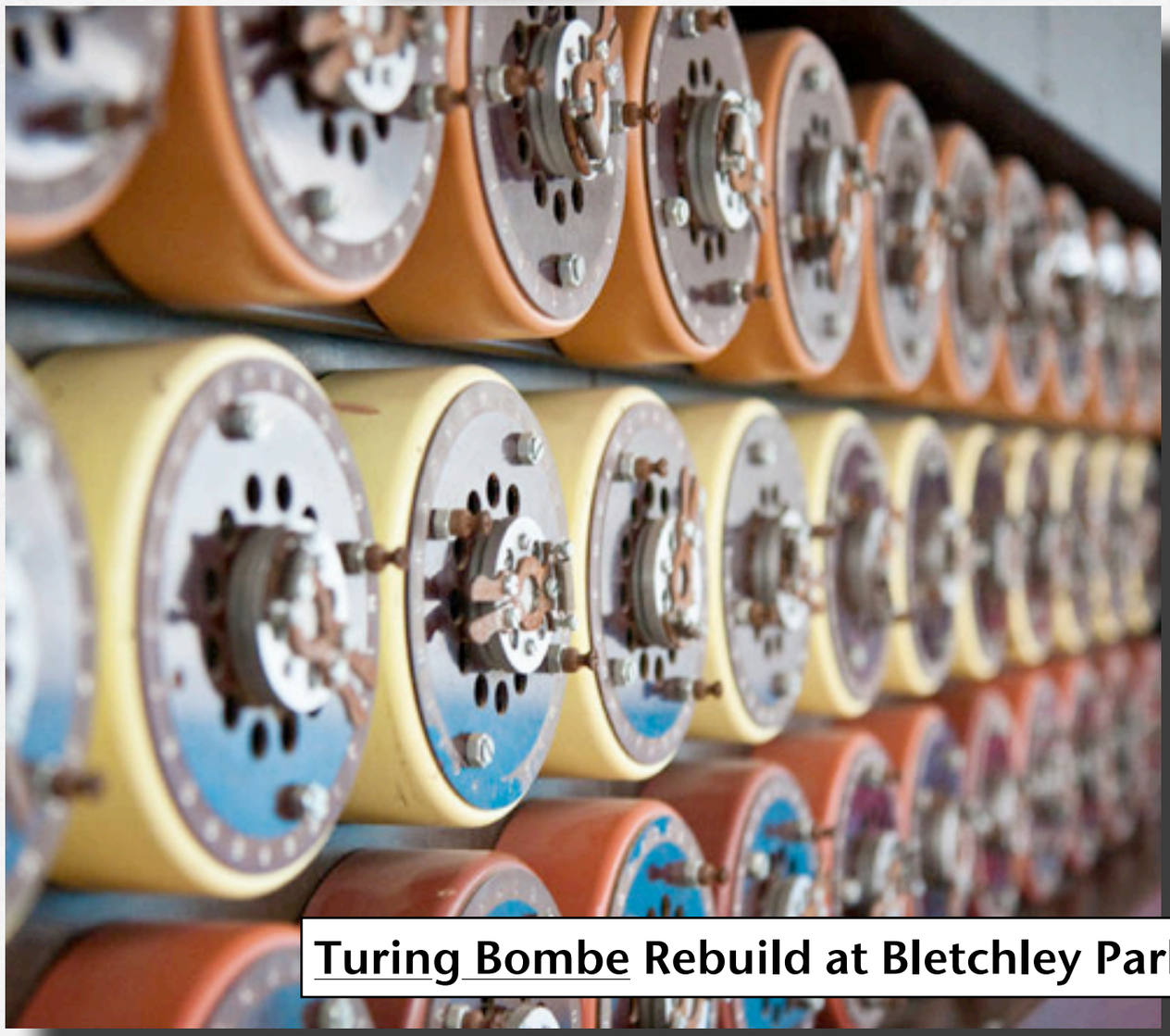
Konrad Zuse

- The first electronic digital computer - John Atanasoff (1937-42)?
- First stored program computer that worked - Manchester ‘Baby’ (1948)
- First commercial computer - EDSAC in Cambridge (Maurice Wilkes, 1949), or Eckert and Mauchly’s UNIVAC (1951)
- If ‘Program as data’ key to the first computer … out go Babbage; Zuse, 1930s; Colossus,1944; ENIAC,1946



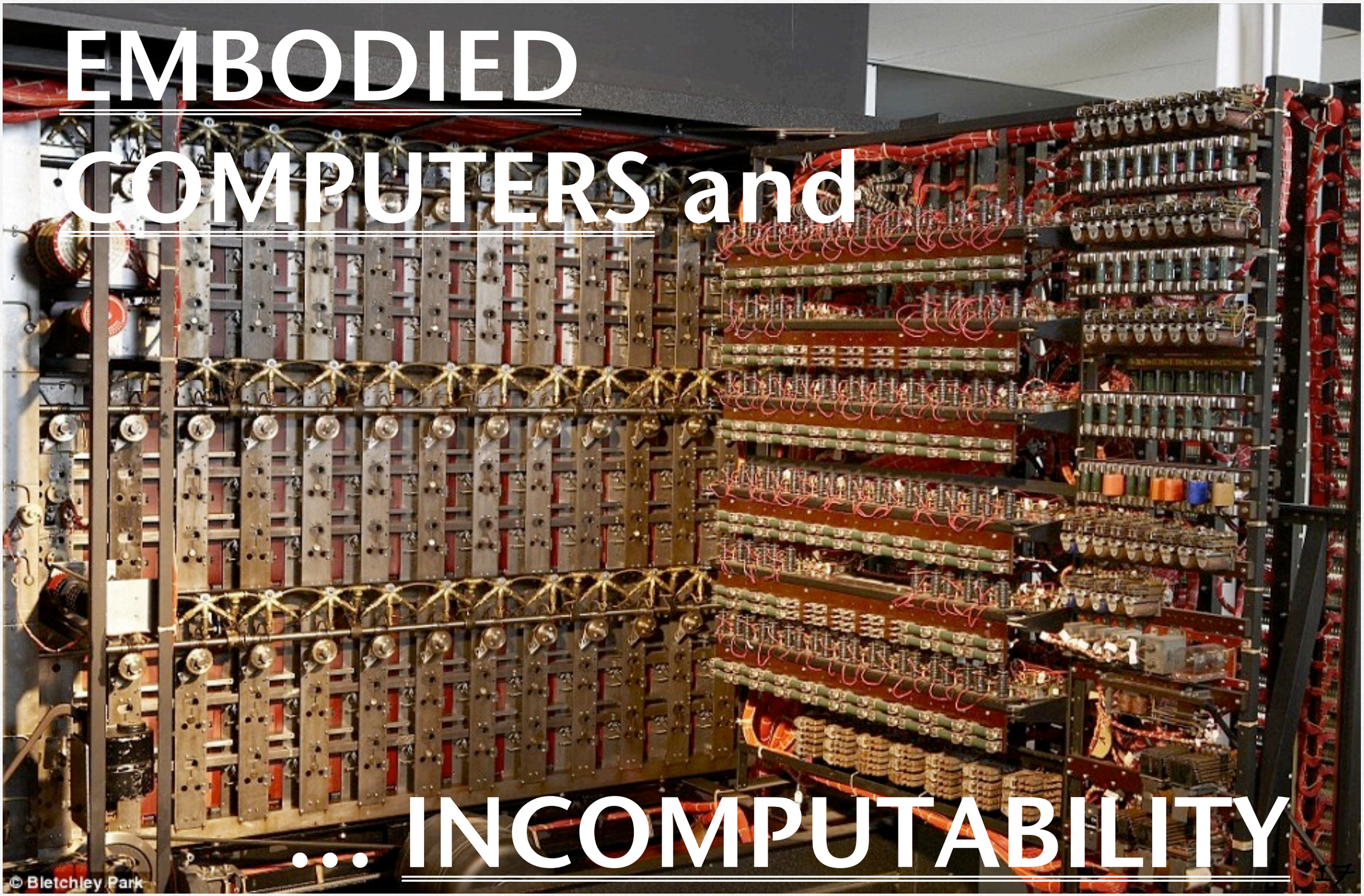
Programs as Data Embodied

Pilot ACE, May 10, 1950 - small version of plan contained in Turing's ACE Report of 1945



Turing Bombe Rebuild at Bletchley Park

EMBODIED COMPUTERS and ... INCOMPUTABILITY



The Discovery of 'Incomputability'

- Turing (1936): Take a Universal Turing machine **U**:

Unsolvability of the Halting Problem for U:
No computer can tell us, for each given input x, whether U will compute - where, remember - we allow an input to include a coded program!

'Church's Theorem':
No computer can tell us, for each given sentence, whether it is logically valid or not.

Real world randomness

- Quantum randomness is a familiar experimental and theoretical phenomenon

It passes all reasonable statistical properties of randomness

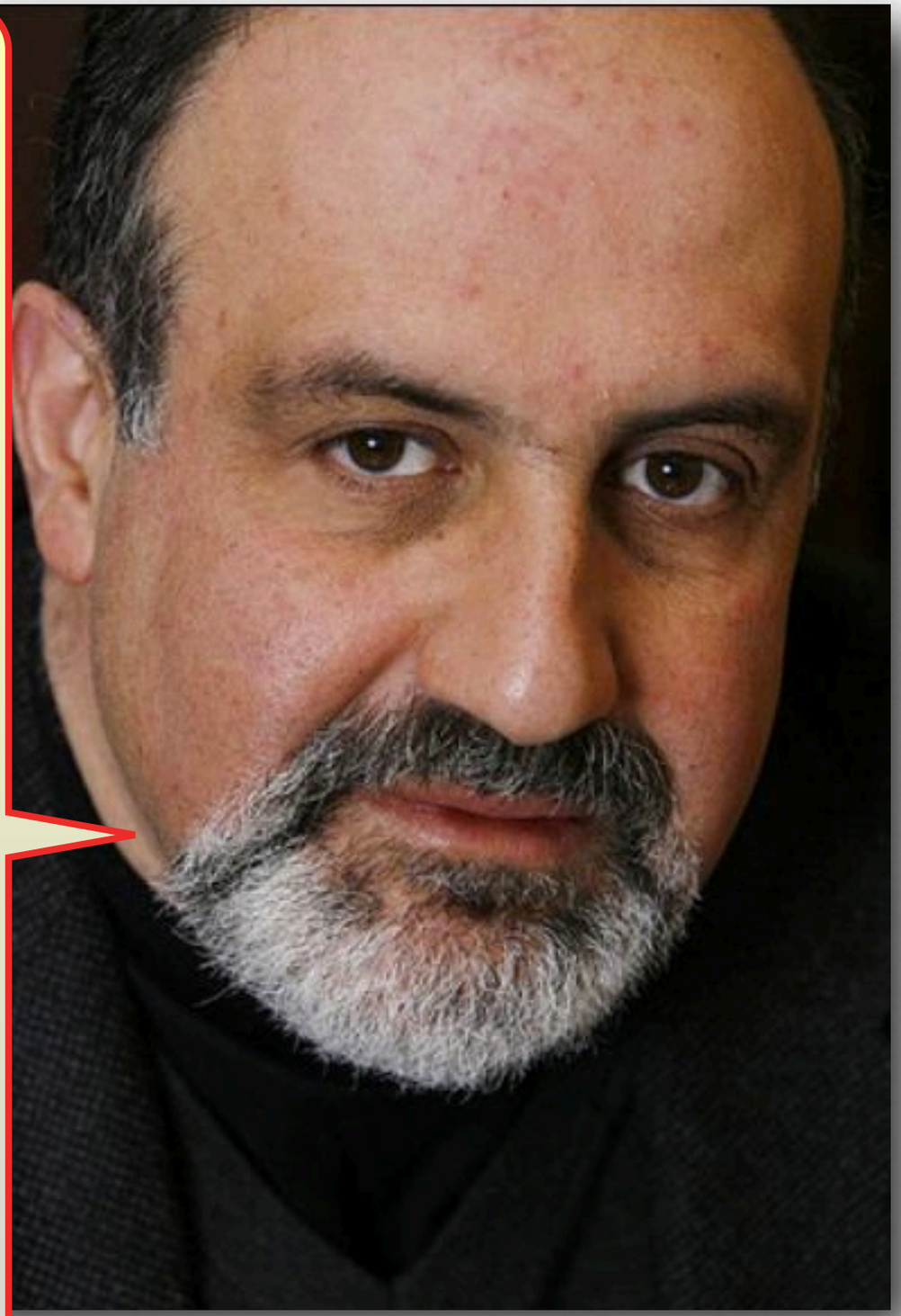
- Cris Calude/Karl Svozil: It is Turing incomputable

Open question: How random is quantum randomness ?

I have spent my entire life studying randomness, practicing randomness, hating randomness. The more that time passes, the worse things seem to me, the more scared I get, the more disgusted I am with Mother Nature.

Nassim Taleb

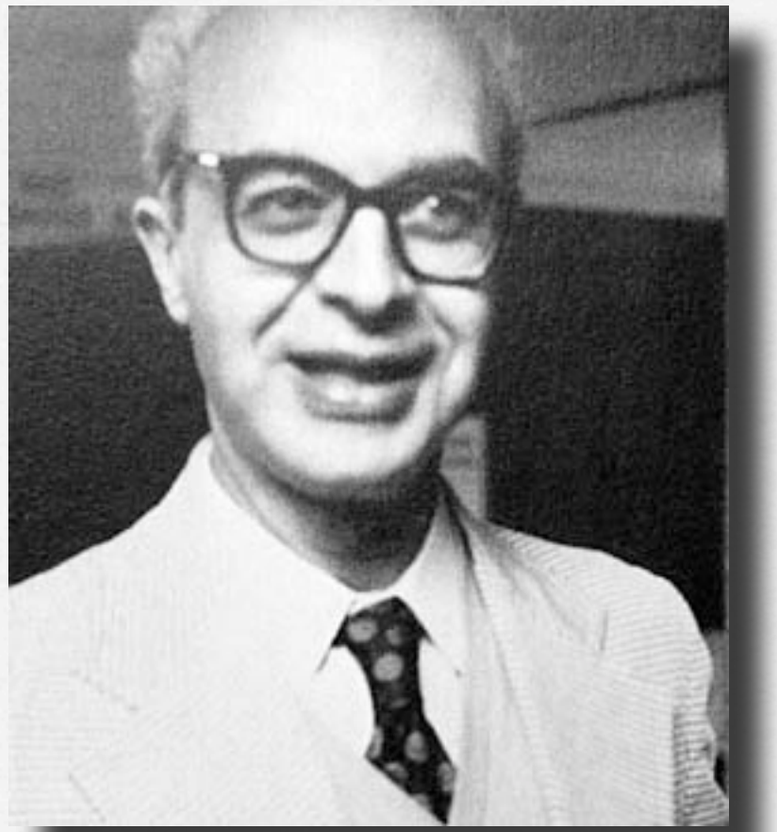
The more I think about my subject, the more I see evidence that the world we have in our minds is different from the one playing outside. Every morning the world appears to me more random than it did the day before, and humans seem to be even more fooled by it than they were the previous day. It is becoming unbearable. I find writing these lines painful; I find the world revolting.



Nassim Taleb

Co-operative phenomena

1970 - Georg Kreisel proposes a collision problem related to the 3-body problem, which might result in *"an analog computation of a non-recursive function"*



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Articles

UBIQUITY SYMPOSIUM 'WHAT IS COMPUTATION?' COMPUTATION IS PROCESS

November 2010 | BY DENNIS J. FRAILEY

Full text also available in the ACM Digital Library as PDF | HTML

Printer Friendly Version PDF

Various authors define forms of computation as specialized types of processes. As the scope of computation widens, the range of such specialties increases. Dennis J. Frailey posits that the essence of computation can be found in any form of process, hence the title and the thesis of this paper in the Ubiquity symposium discussion what is computation. --Editor

The concept of computation is arguably the most dramatic advance in mathematical thinking of the past century. Denning [2010], in his opening statement, describes how *computation* was originally defined in the 1930s and how that definition has progressed through the ensuing decades. Church, Gödel, and Turing defined it in terms of mathematical functions, which they divided into the decidable (can be evaluated by algorithms) and the un-

"What Is Computation?"

- **ACM *UBIQUITY* SYMPOSIUM: *Computation Is Process* by Dennis J Frailey:**

- The concept of computation is arguably the most dramatic advance in mathematical thinking of the past century

- Church, Gödel, and Turing defined it in terms of mathematical functions ... They were inclined to the view that only the algorithmic functions constituted computation

- I'll call this the "mathematician's bias" because I believe it limits our thinking and prevent us from fully appreciating the power of computation

A Gap in Classical Embodiment?

- A universal Turing machine can '<u>compute</u>' an '<u>incomputable</u>' real number ...
- ... which is not considered a computed outcome - as not available, or '<u>embodied</u>', in further computations
- The quantifier involved is an unembodied <u>chasm</u> ...
- ... but can we observe <u>definable</u> outcomes being better embodied in <u>more complex physical systems</u>?

PROBLEMS with PHYSICS ...

Definability in Physics, as ...

By 1973, physicists had in place what was to become a fantastically successful theory of fundamental particles and their interactions, a theory that was soon to acquire the name of the 'standard model'. Since that time, the overwhelming triumph of the standard model has been matched by a similarly overwhelming failure to find any way to make further progress on fundamental questions.

Introduction to Peter Woit: "Not Even Wrong - The Failure of String Theory and the Continuing Challenge to Unify the Laws of Physics", Jonathan Cape, 2006

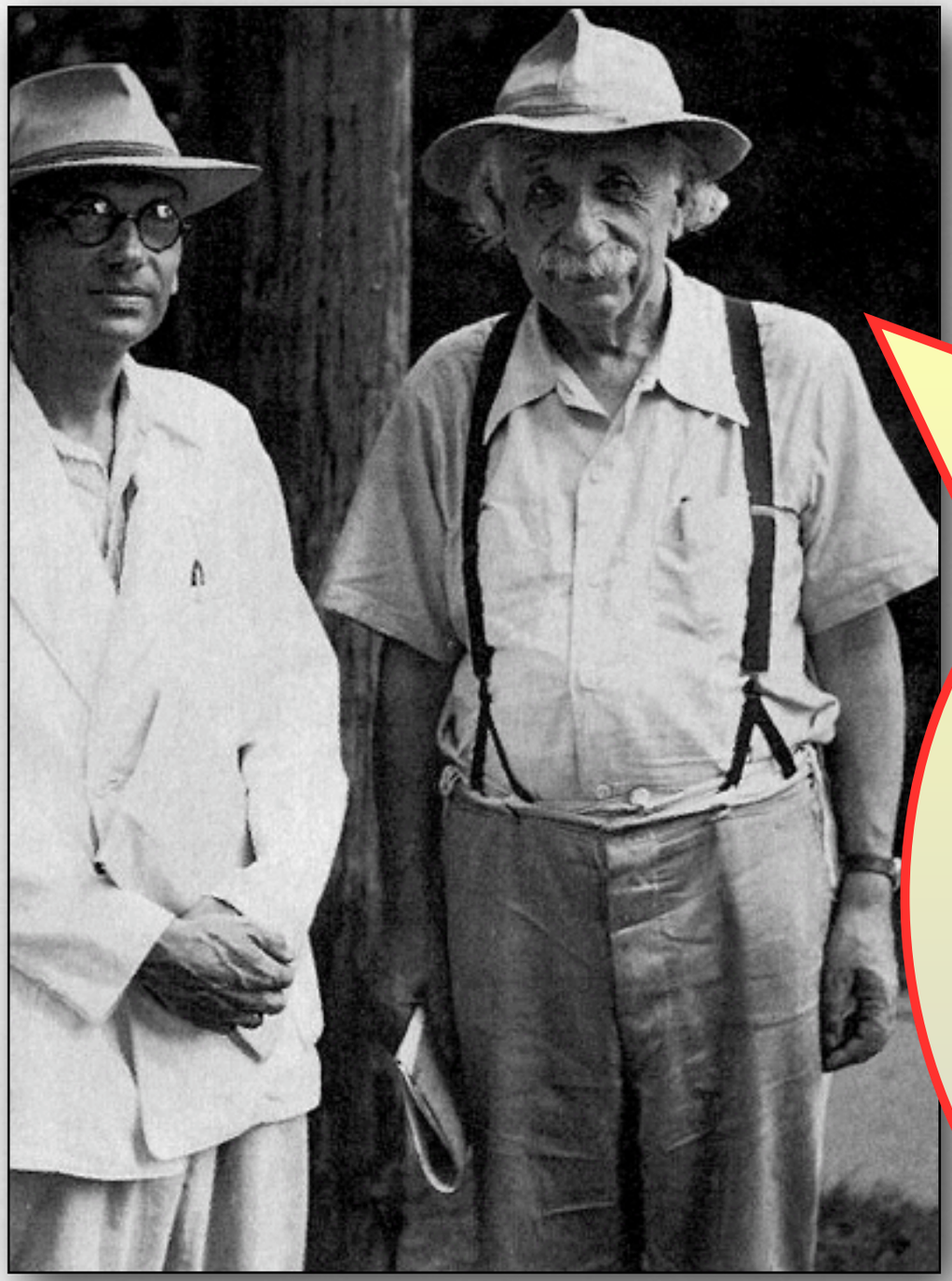
Causality Beyond Computation?

If the creation of the universe can be described as a quantum process, we would be left with one deep mystery of existence: What is it that determined the laws of physics?

A.H. Guth, *The Inflationary Universe - The Quest for a New Theory of Cosmic Origins*, Addison-Wesley, 1997

One way of thinking about what is unsatisfactory about the standard model is that it leaves seventeen non-trivial numbers still to be explained, ...

Peter Woit: *Not Even Wrong - The Failure of String Theory and the Continuing Challenge to Unify the Laws of Physics*, Jonathan Cape, 2006



From A. Einstein: "Autobiographical Notes", in "Albert Einstein: Philosopher-Scientist" (P. Schilpp, ed.), Open Court Publishing, 1969, p.63

... I would like to state a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e. intelligibility, of nature ... nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur (not constants, therefore, whose numerical value could be changed without destroying the theory) ...

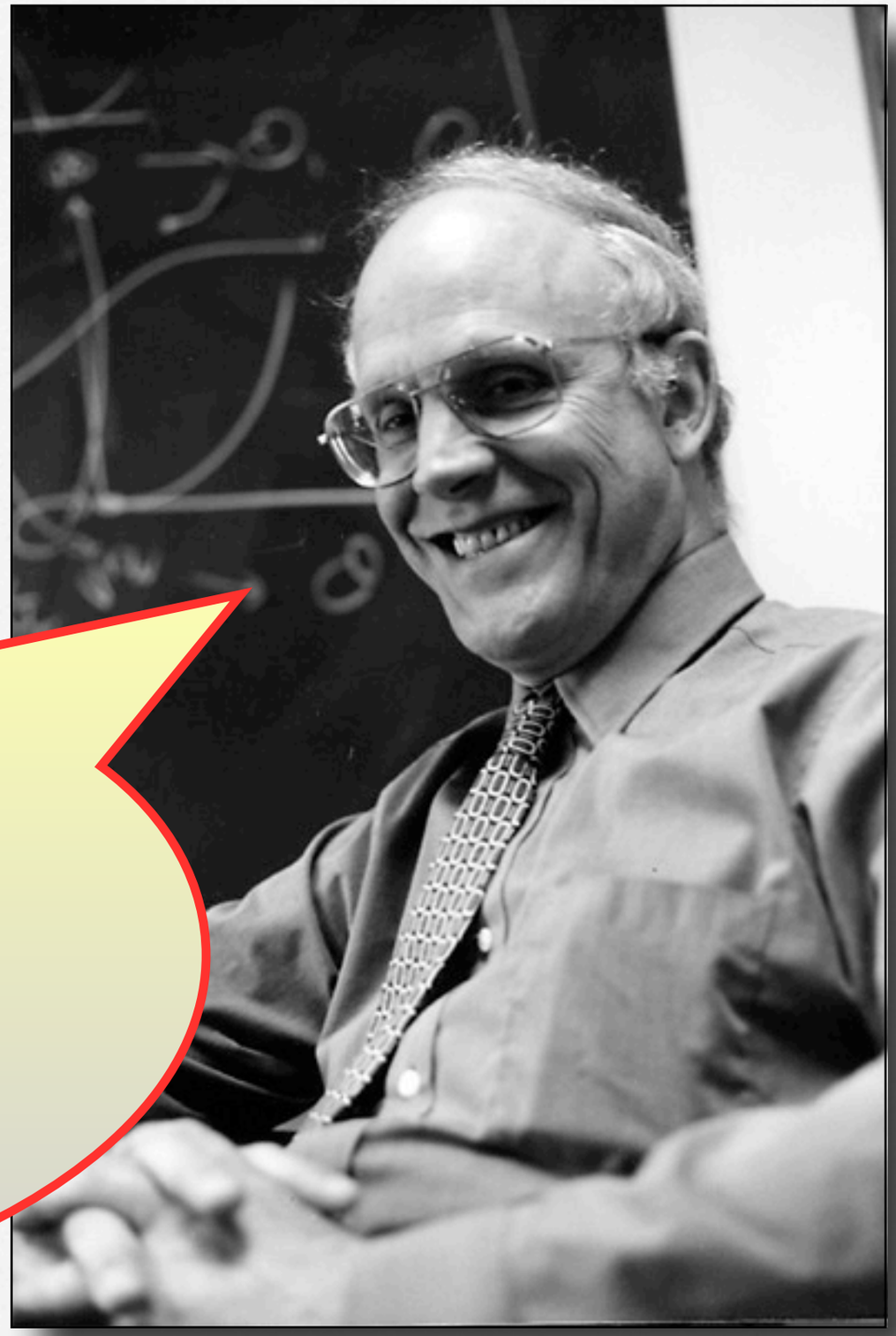
Strong Determinism

[According to Strong Determinism] ... all the complication, variety and apparent randomness that we see all about us, as well as the precise physical laws, are all exact and unambiguous consequences of one single coherent mathematical structure.

Roger Penrose: *Quantum physics and conscious thought*, in *Quantum Implications: Essays in honour of David Bohm* (B.J. Hiley and F.D. Peat, eds.), pp.106-107

David Gross, quoted in New Scientist, Dec. 10 2005, "Nobel Laureate Admits String Theory Is In Trouble":

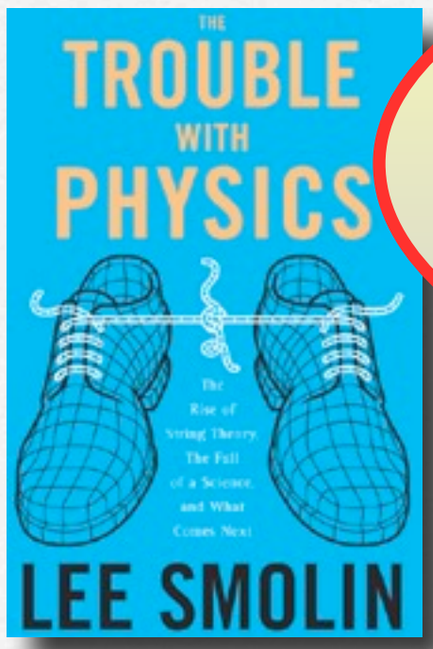
The state of physics today is like it was when we were mystified by radioactivity ... They were missing something absolutely fundamental. We are missing perhaps something as profound as they were back then.



… Causality and Computation

- In modelling the physical universe –

… causality itself is fundamental

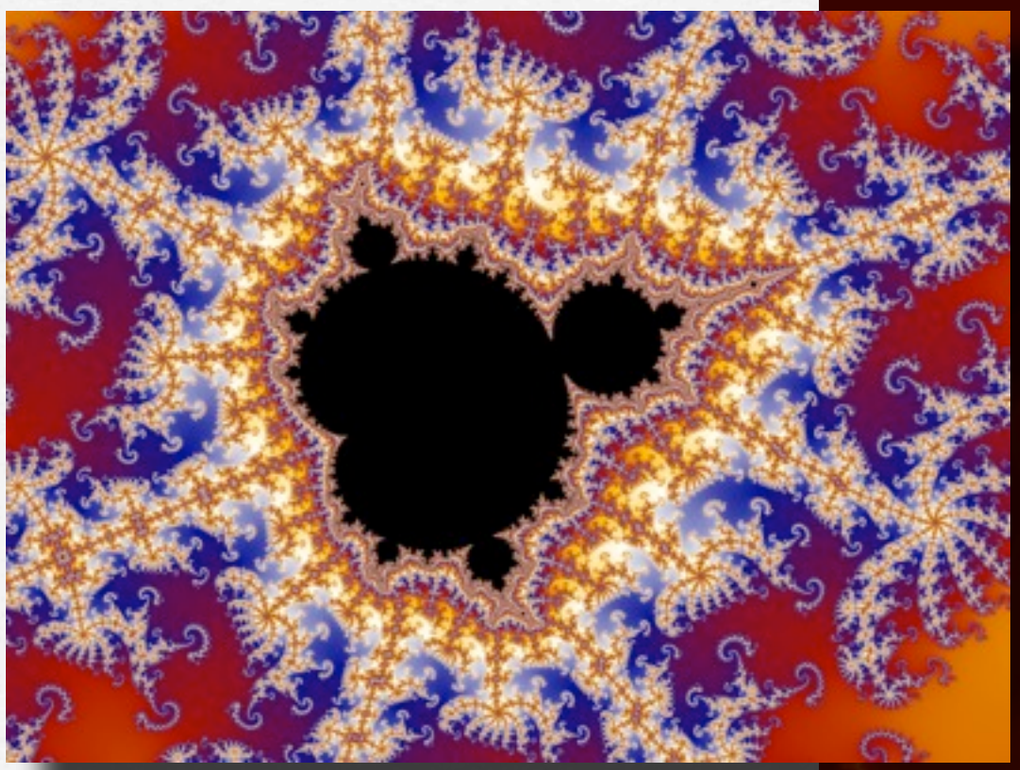


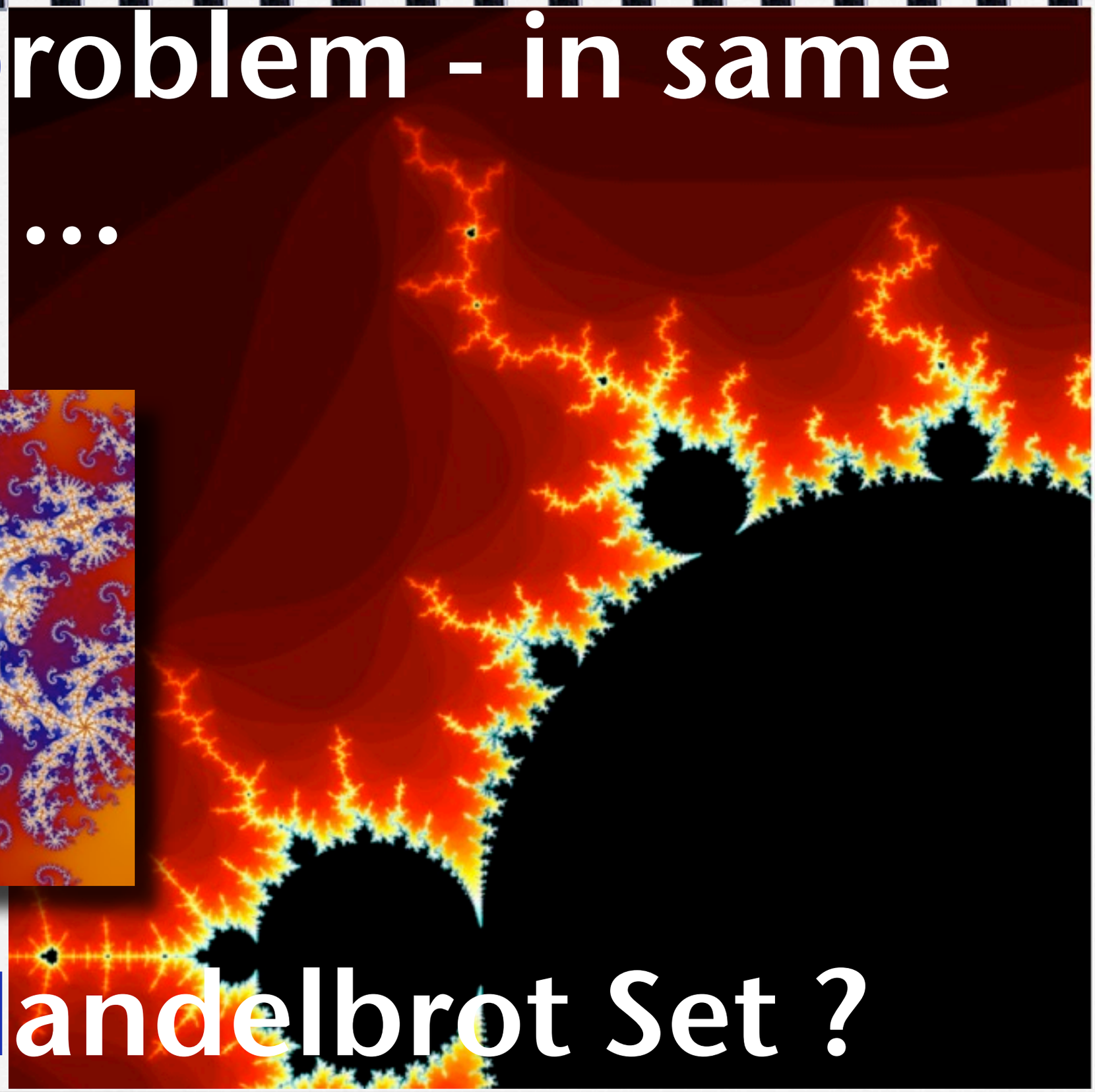
Lee Smolin, 'The Trouble With Physics', p.241

Complexity, Emergence, Approximation -

Grappling with HIGHER TYPE Computation

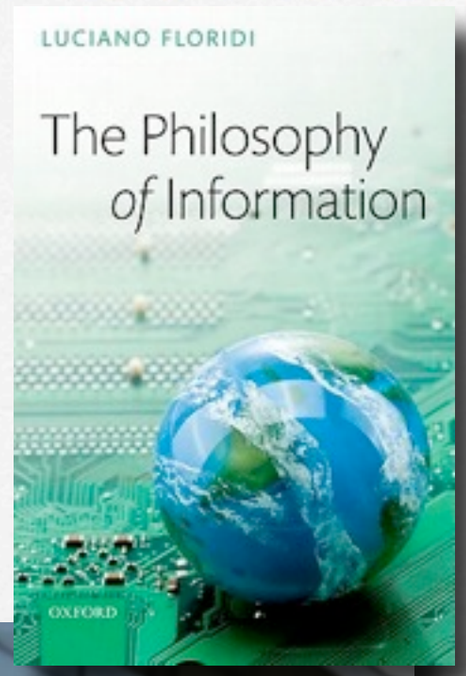
Halting problem - in same world as ...





... the Mandelbrot Set ?

LEVELS of ABSTRACTION



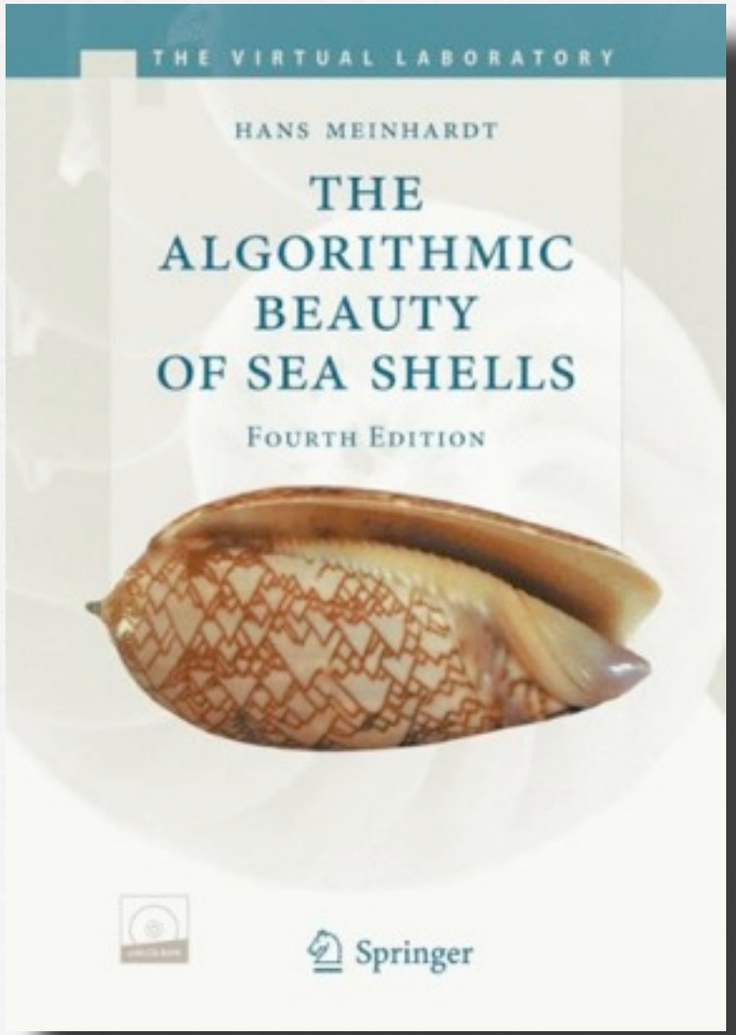
A *level of abstraction (LoA)* is a finite but non-empty set of observables. ...
The introduction of LoAs is often an important step prior to mathematical modelling of the phenomenon under consideration ... Use of LoAs is effective ... where a typed theory would be effective ... **[but] analysis ...may be conducted at different levels of epistemological levelism.**

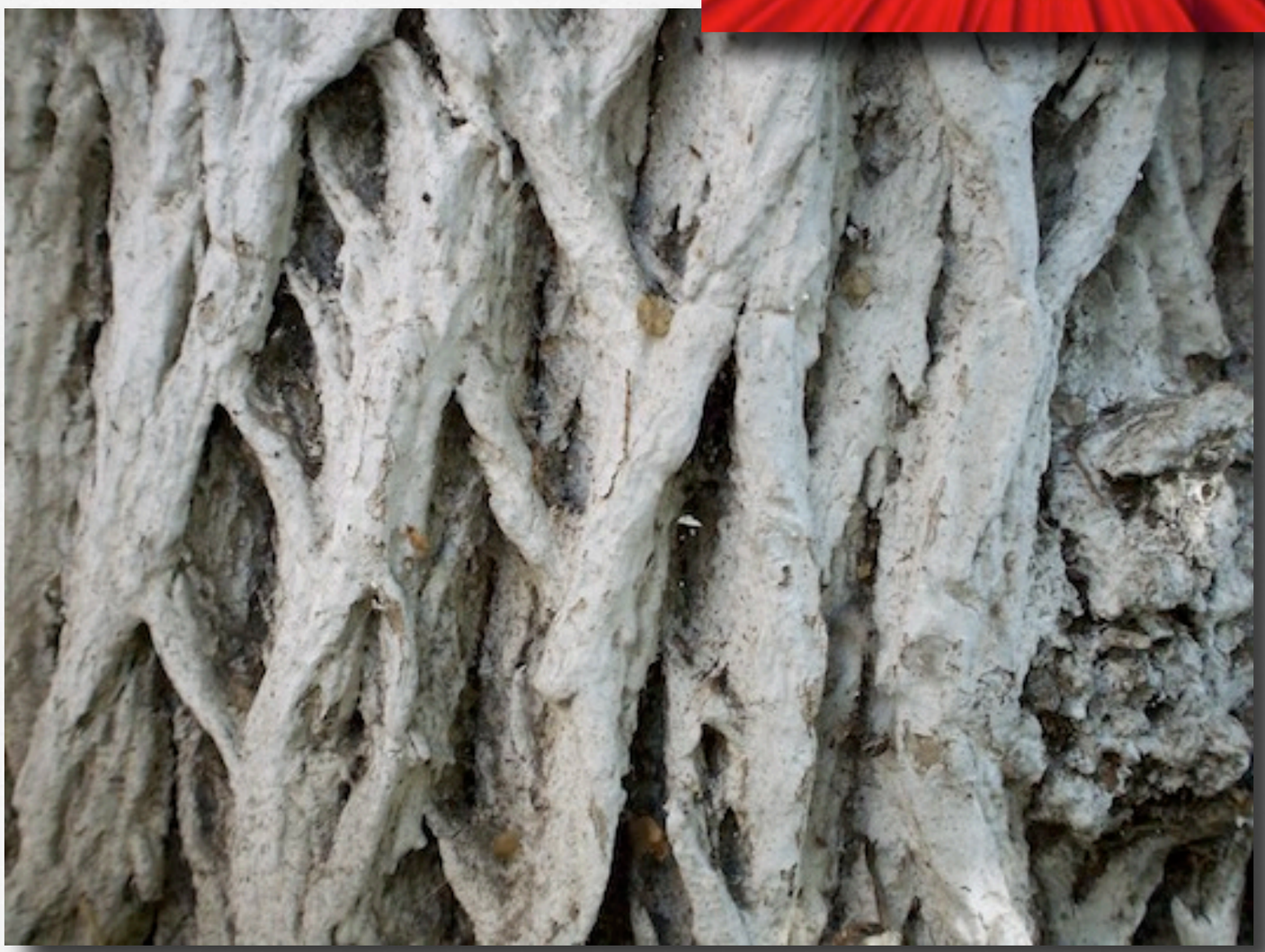
LEVELS of ABSTRACTION

By the theory of simple types I mean the doctrine which says that the objects of thought ... are divided into types, namely: individuals, properties of individuals, relations between individuals, properties of such relations, etc. ... , and that sentences of the form: " a has the property φ ", " b bears the relation R to c ", etc. are meaningless, if a, b, c, R, φ are not of types fitting together. Mixed types (such as classes containing individuals and classes as elements) and therefore also transfinite types (such as the class of all classes of finite types) are excluded. That the theory of simple types suffices for avoiding also the epistemological paradoxes is shown by a closer analysis of these.

(Kurt Godel: *Russell's mathematical logic*, 1944)

How Does Nature Compute?





Emergence of patterns in Nature

1950s - Alan Turing proposes a simple reaction-diffusion system describing chemical reactions and diffusion to account for morphogenesis

James D. Murray

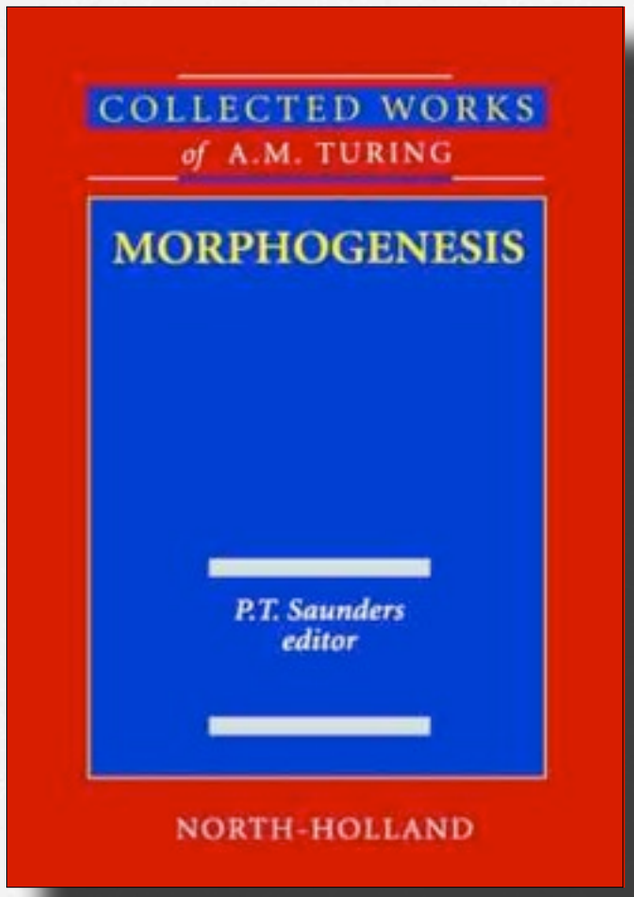
Philip Maini

From website of the Biological Modeling and Visualization research group, Department of Computer Science at the University of Calgary

See http://www.swintons.net/jonathan/turing.htm

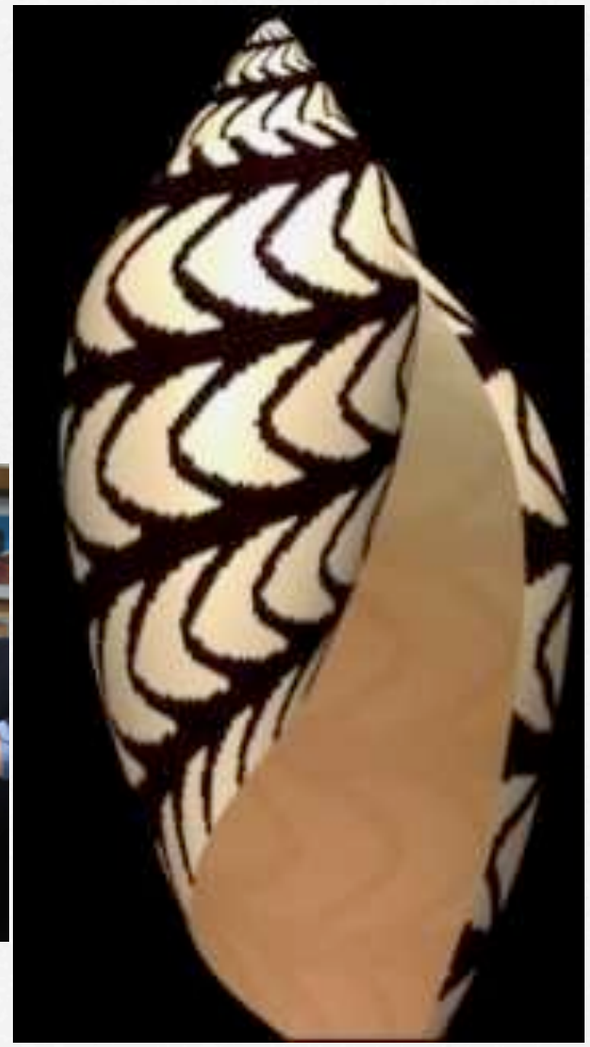
Emergence of patterns in Nature

1950s - Alan Turing proposes a simple reaction-diffusion system describing chemical reactions and diffusion to account for morphogenesis, i.e., the development of form and shape in biological systems.



From website of the Biological Modeling and Visualization research group, Department of Computer Science at the University of Calgary:

Przemyslaw Prusinkiewicz



See http://www.swintons.net/jonathan/turing.htm

Descriptions and Emergent Structure

- Notice – It is often possible to get descriptions of emergent properties in terms of the elementary actions

- E.g., this is what Turing did for the role of Fibonacci numbers in relation to the sunflower etc.

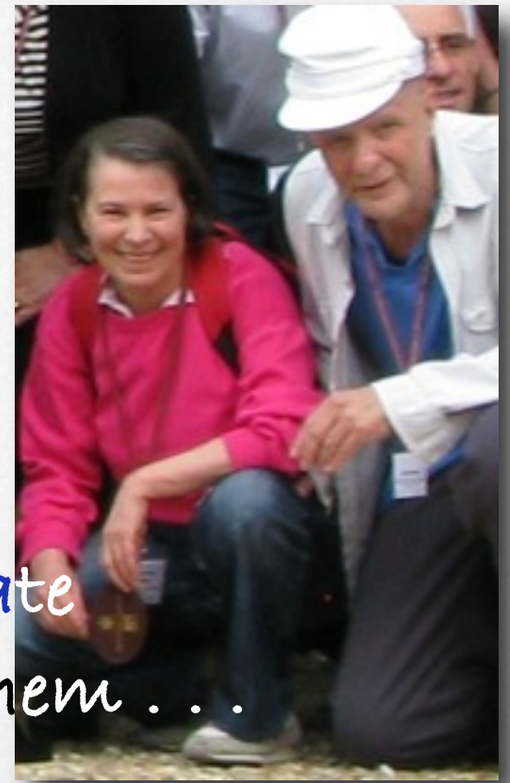
- In mathematics, it is well-known that complicated descriptions may entail incomputable phenomena

- A potential source of surprise in emergence …

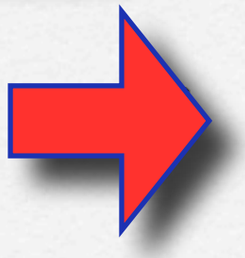
Definability in the Real World

Hans Reichenbach
(1891-1953)

Alfred Tarski



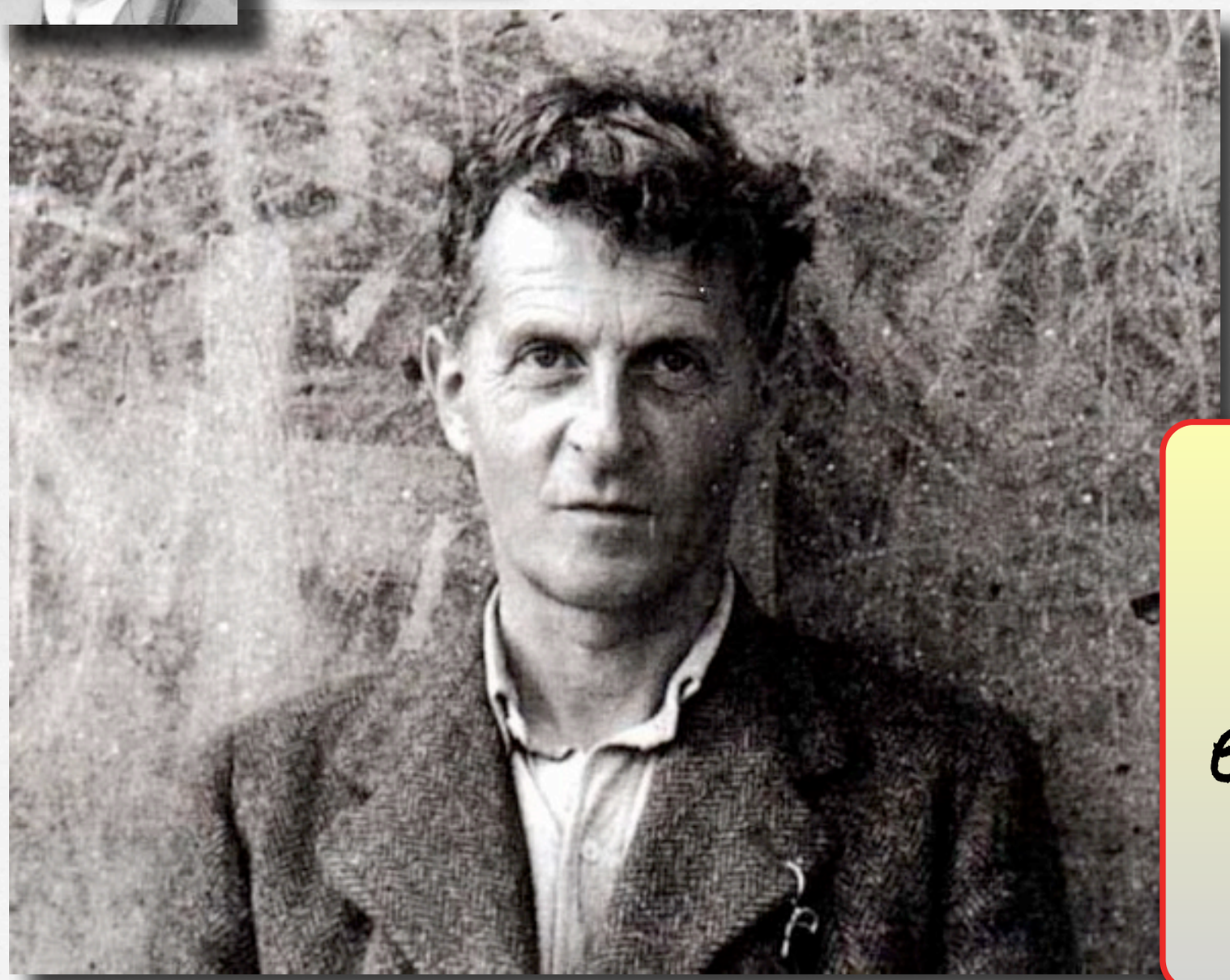
- Intuition: Natural phenomena not only generate descriptions, but arise and derive form from them . . .

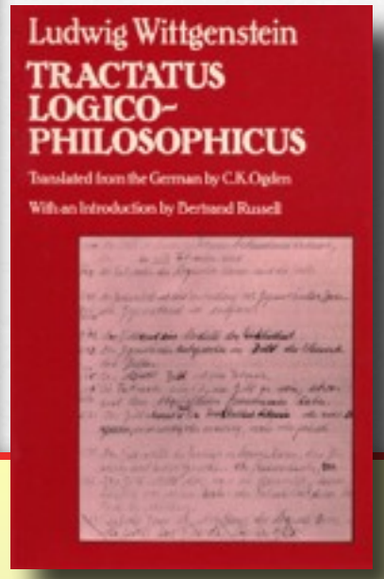


- . . . so - connecting with a useful abstraction - that of mathematical definability - or, more generally, invariance (under the automorphisms of the appropriate structure) . . .

- . . . giving precision to our experience of emergence as a potentially non-algorithmic determinant of phenomena

Definability in the Real World





"The world is everything that is the case"

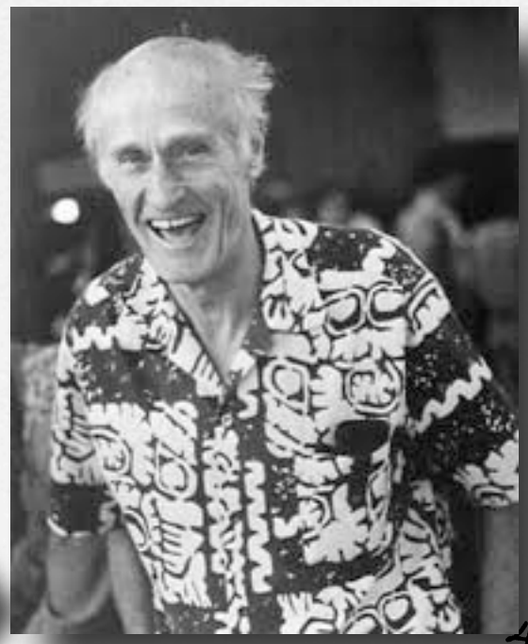
Definability as Higher Order Computation Embodied ...

- Aim to describe global relations in terms of local structure ...

- ... so capturing the emergence of large-scale formations ...

- Mathematically - formalise as definability - or as INVARIANCE under automorphisms - over basic computational structure

- Or ... as higher order computability



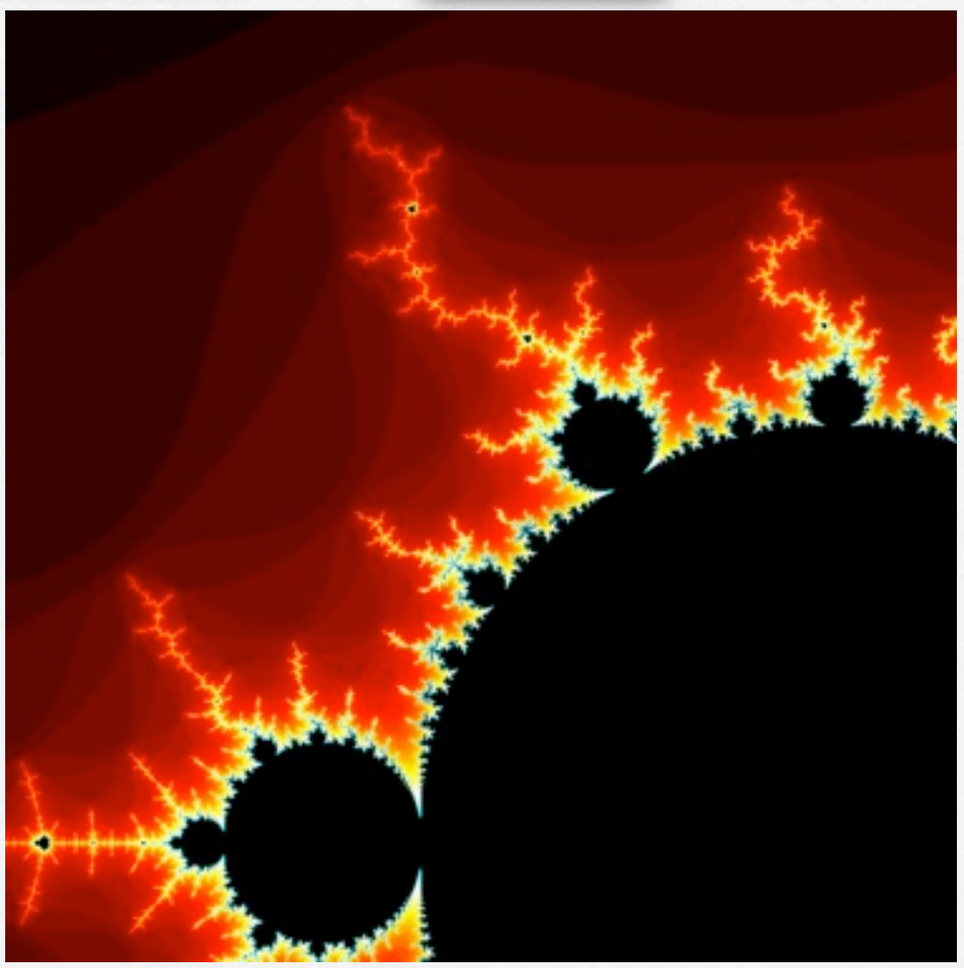
Simple Rules ...

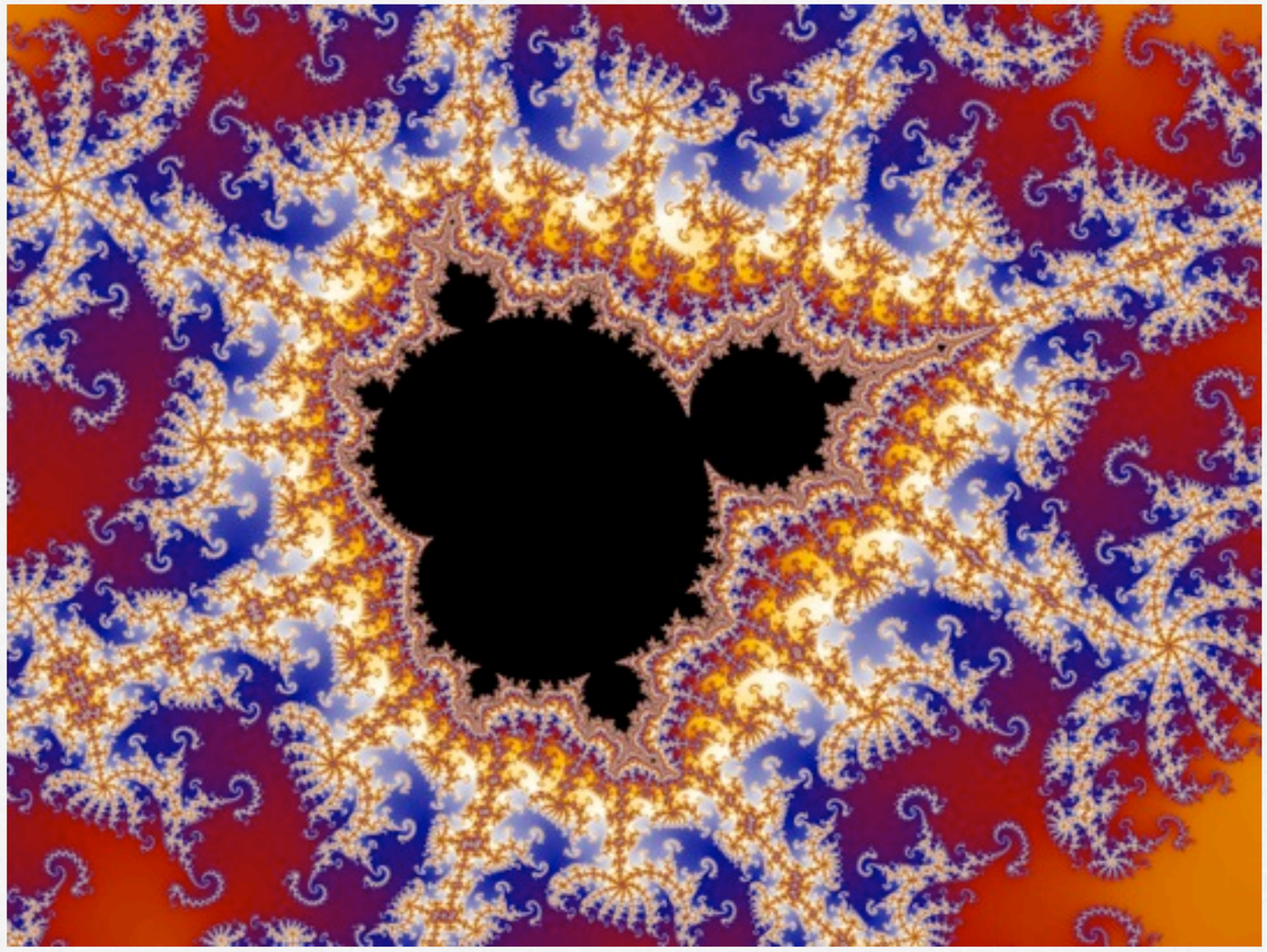
... Complex Outcomes ...

... Emergent Forms ...

... Computational Framework

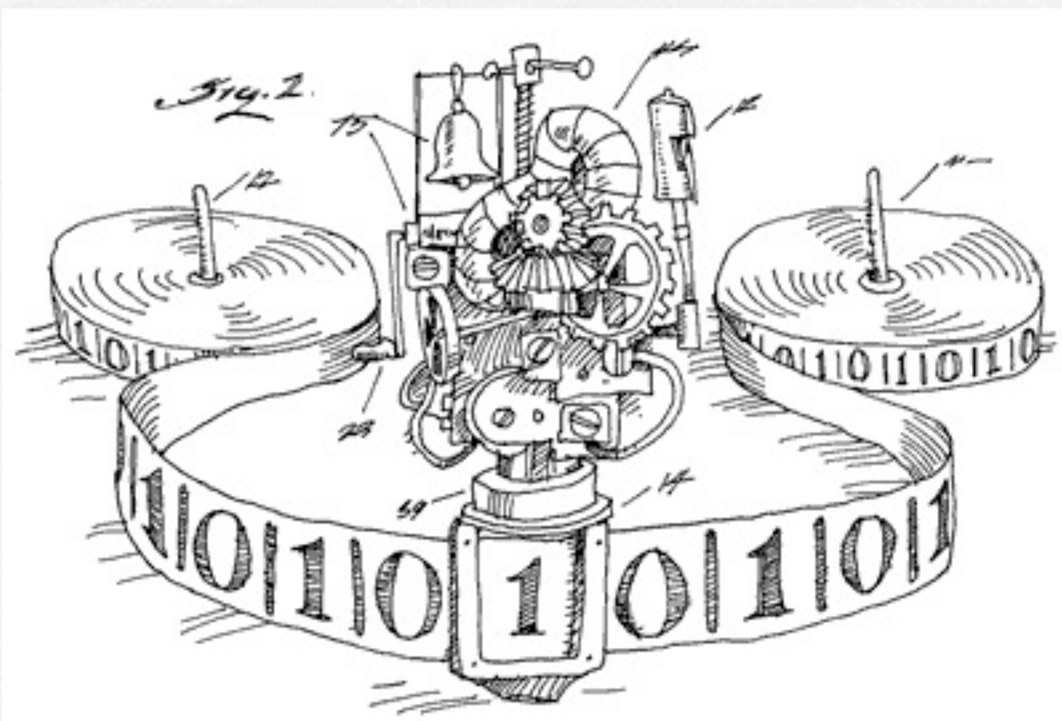
Morphogenesis - in same world as ...

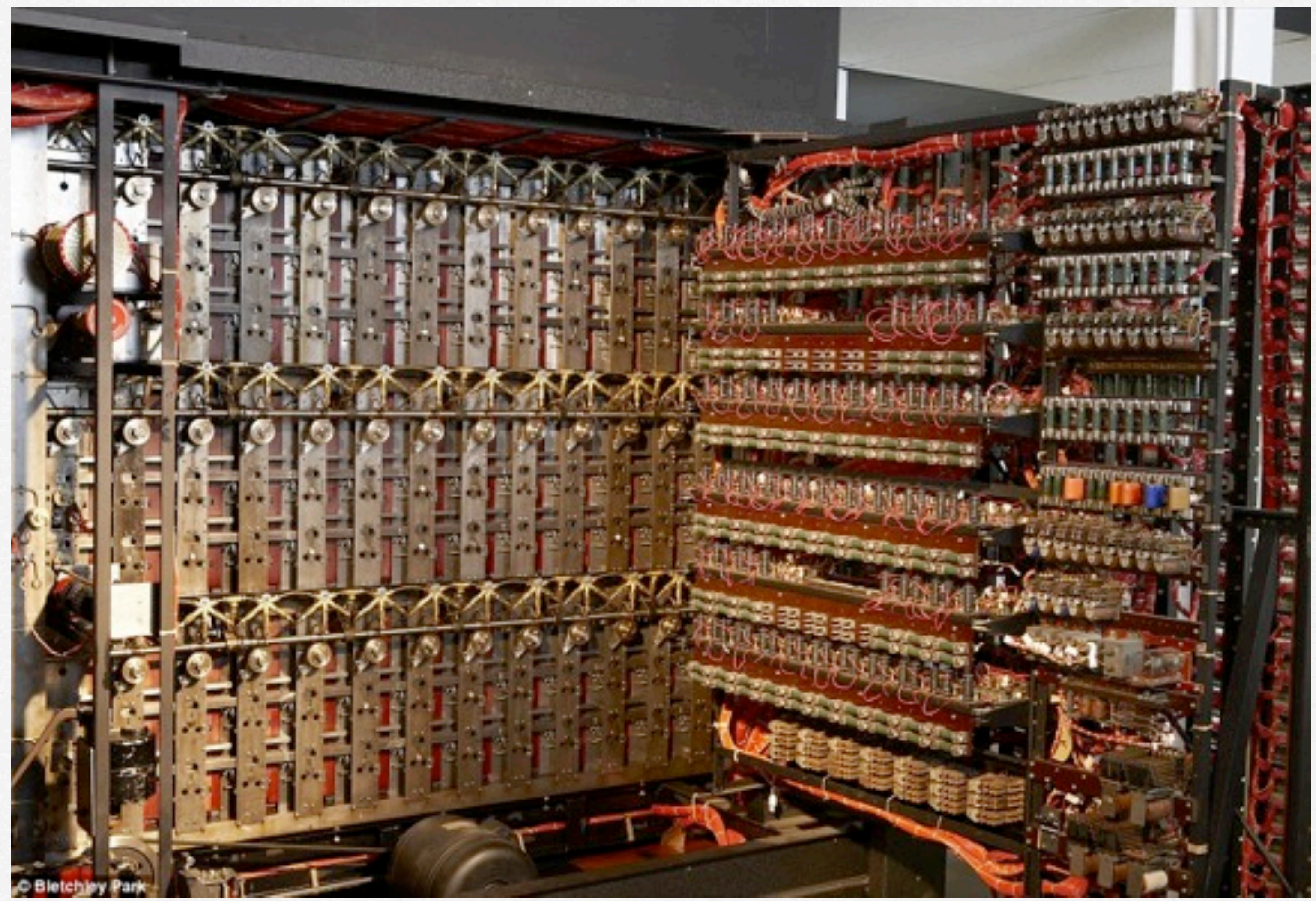




... the Mandelbrot set

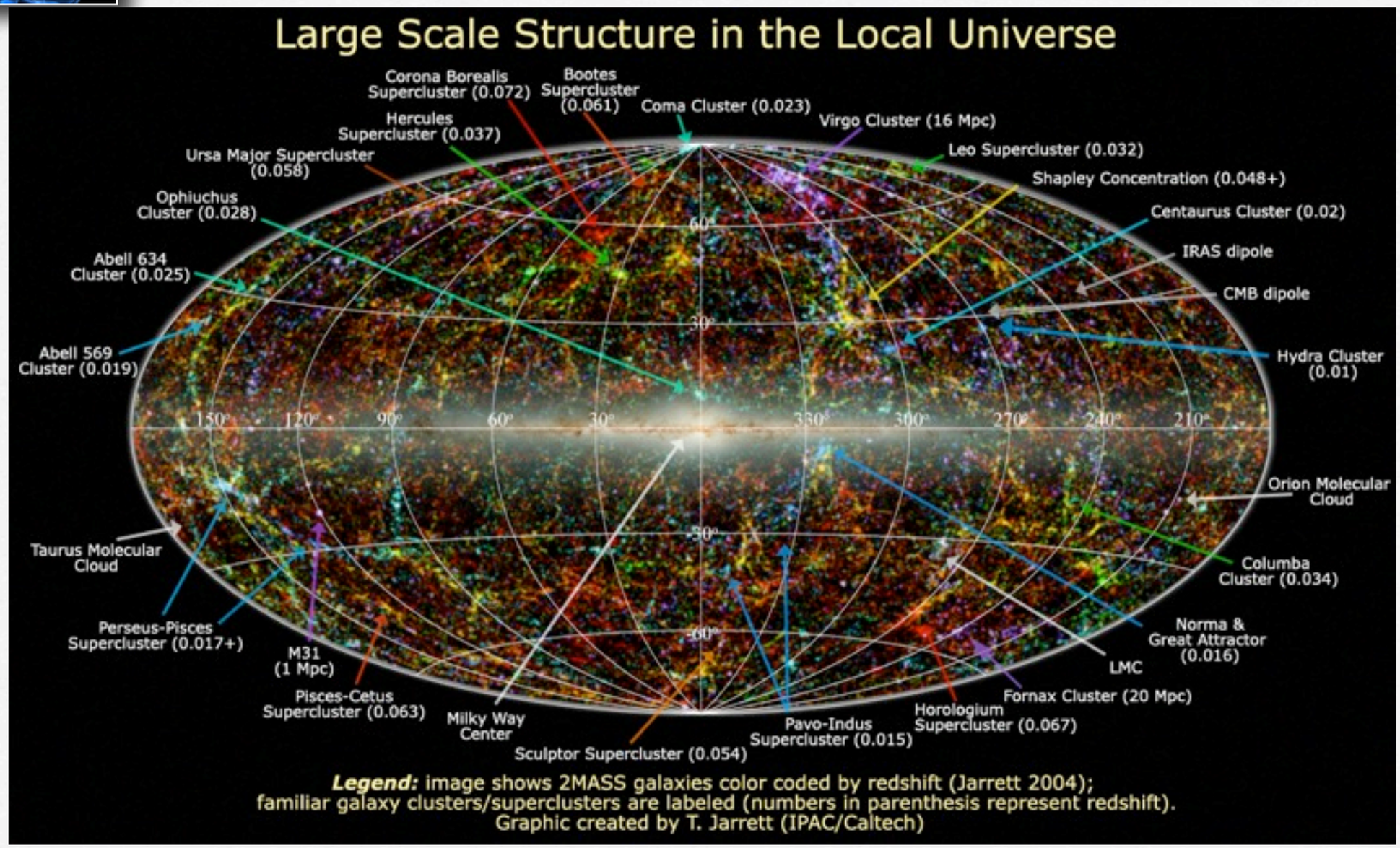
Morphogenesis - in same world as ...



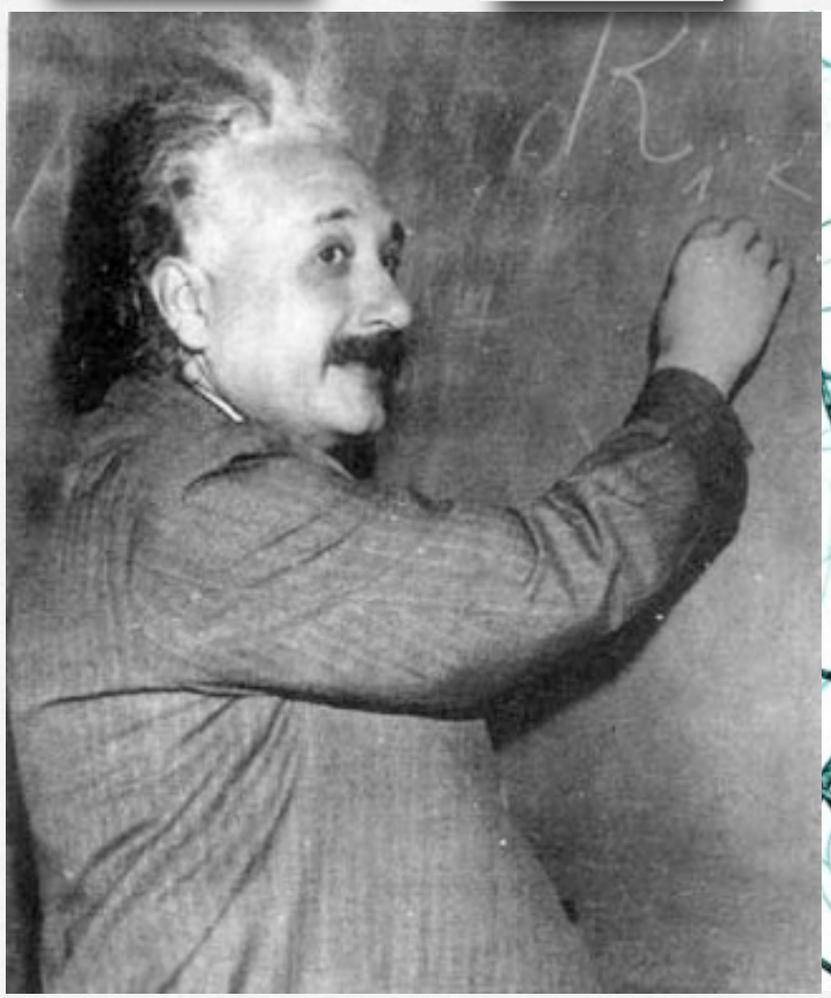


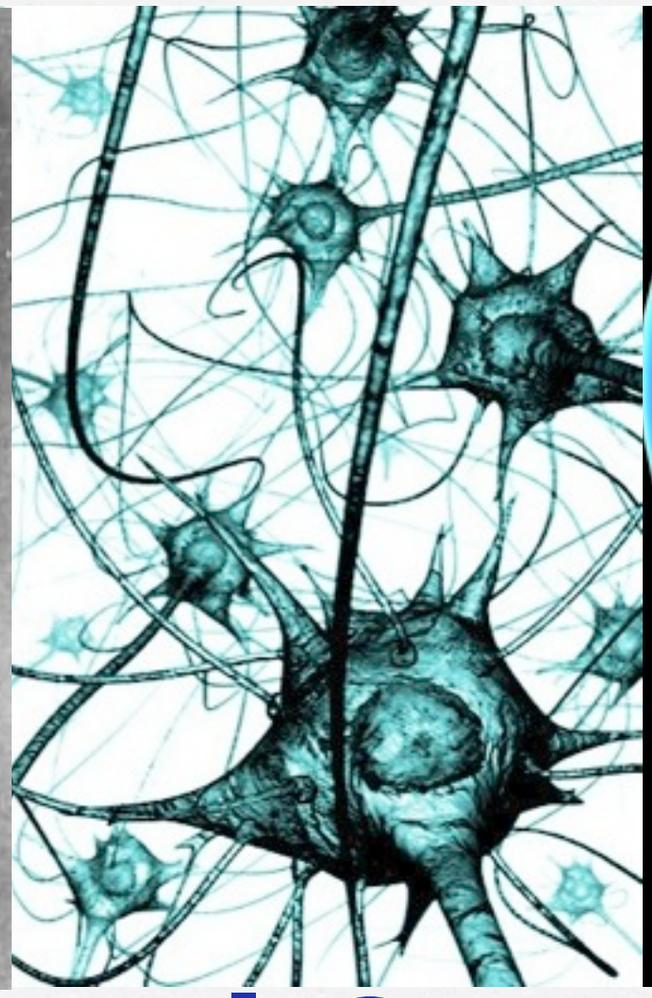
... the Halting Problem

Morphogenesis - in same world as ...



Morphogenesis - in same world as ...





... Mental Supervenience?

Human Brains & Higher Type Computation ...

... with mistakes

- **But what is intelligence? "... if a machine is expected to be infallible, it cannot also be intelligent. There are several theorems which say almost exactly that."**

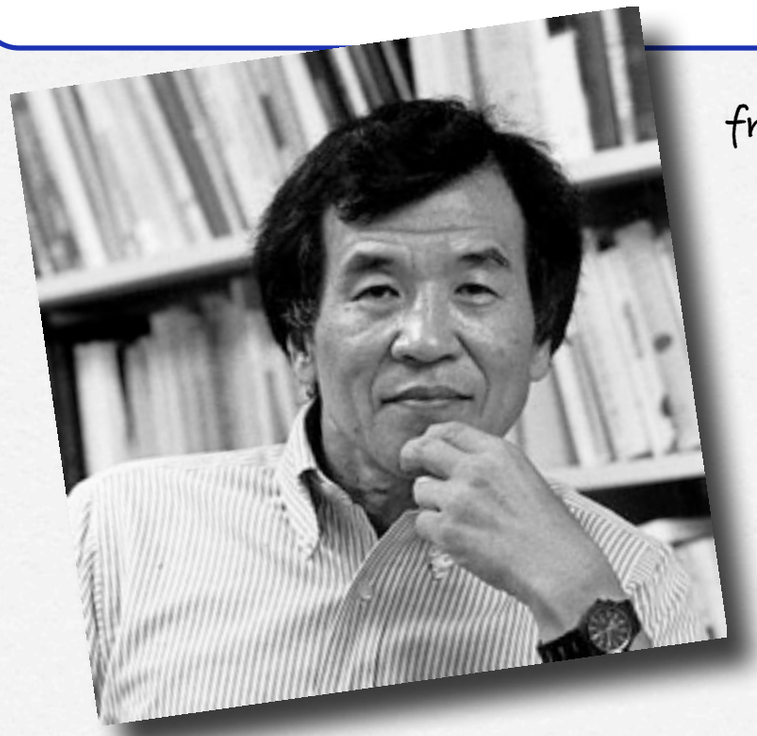
– A.M. Turing, talk to the London Mathematical Society, February 20, 1947, quoted in Andrew Hodges, p.361

- **"The results which have been described in this article are mainly of a negative character, setting certain bounds to what we can hope to achieve purely by reasoning. These, and some other results of mathematical logic may be regarded as going some way towards a demonstration, within mathematics itself, of the inadequacy of 'reason' unsupported by common sense."**

– final paragraph of Alan Turing, Solvable and Unsolvable Problems, Penguin Science News 31, 1954, p.23

Supervenience

Supervenience 'represents the idea that mentality is at bottom physically based, and that there is no free-floating mentality unanchored in the physical nature of objects and events in which it is manifested'



from Jaegwon Kim: "Mind in a Physical World", MIT Press, 1998, pp.14-15

"A set of properties A supervenes upon another set B just in case no two things can differ with respect to A-properties without also differing with respect to their B-properties."

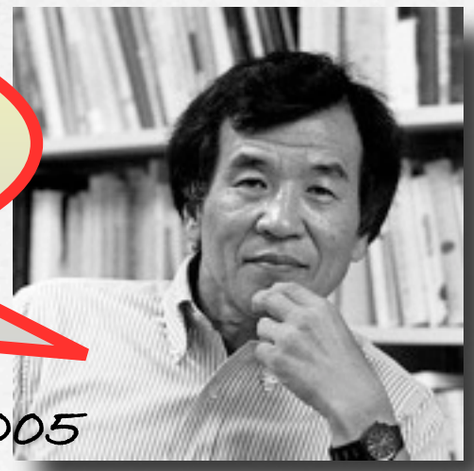
Stanford Encyclopedia of Philosophy

Supervenience

- How can mentality have a computational role in a world that is fundamentally physical?

- And what about 'overdetermination' - the problem of phenomena having both mental and physical causes?

... the problem of mental causation is solvable only if mentality is physically reducible; however, phenomenal consciousness resists physical reduction, putting its causal efficacy in peril.



-Jaegwon Kim: Physicalism, or Something Near Enough, Princeton, 2005

Connectionist Models?

There is a reasonable chance that connectionist models will lead to the development of new somewhat-general-purpose self-programming, massively parallel analog computers, and a new theory of analog parallel computation: they may possibly even challenge the strong construal of Church's Thesis as the claim that the class of well-defined computations is exhausted by those of Turing machines.

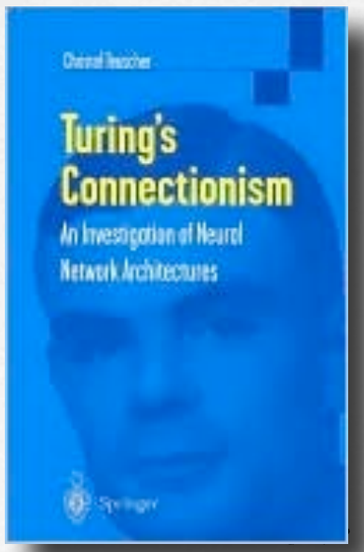
Paul Smolensky [1988] (recipient 2005 David E. Rumelhart Prize),
On the proper treatment of connectionism, in Behavioral and Brain Sciences, 11, pp. 1-74

Connectionist Models?

- These have come a long way since Turing's [1948] discussion of 'unorganised machines', and McCulloch and Pitts [1943] early paper on neural nets

- But for Steven Pinker "... neural networks alone cannot do the job".

And focussing on our elusive higher functionality, he points to a "kind of mental fecundity called recursion" . . .



Connectionist Models?

We humans can take an entire proposition and give it a role in some larger proposition. Then we can take the larger proposition and embed it in a still-larger one. Not only did the baby eat the slug, but the father saw the baby eat the slug, and I wonder whether the father saw the baby eat the slug, the father knows that I wonder whether he saw the baby eat the slug, and I can guess that the father knows that I wonder whether he saw the baby eat the slug, and so on.

Steven Pinker,

How the Mind Works, W. W. Norton, New York, 1997

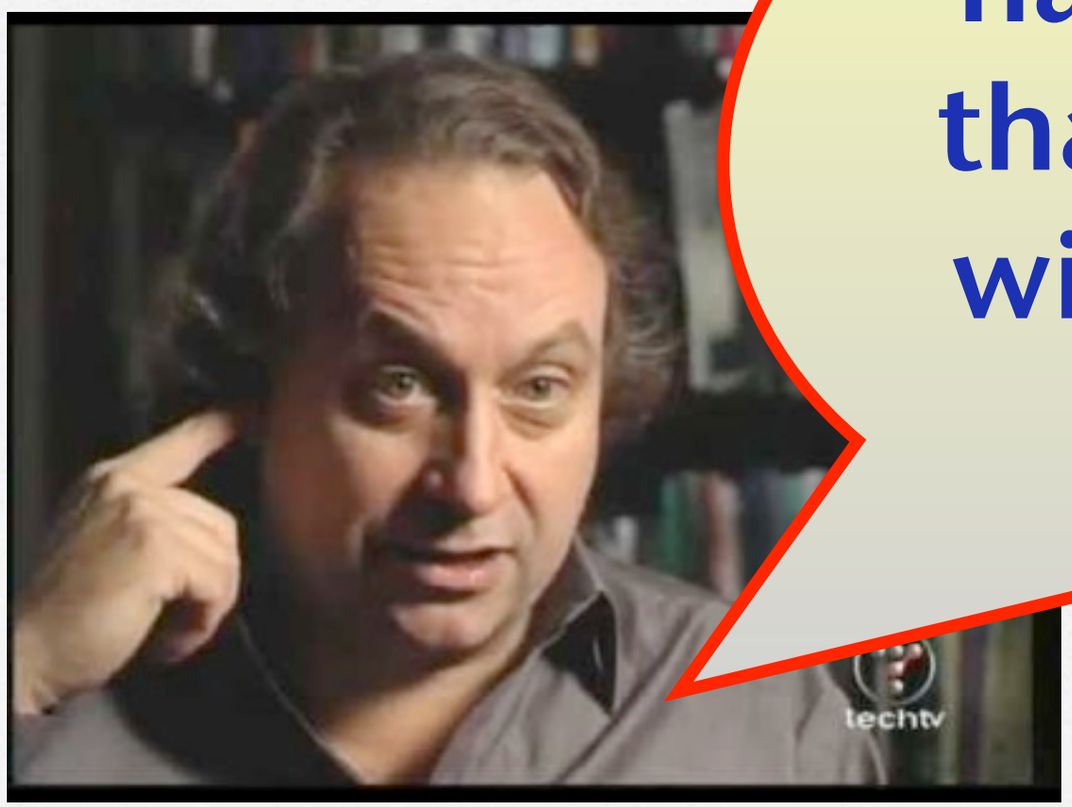
Capturing and Recycling Emergence

“As the brain forms images of an object - such as a face, a melody, a toothache, the memory of an event - and as the images of the object affect the state of the organism, yet another level of brain structure creates a swift nonverbal account of the events that are taking place in the varied brain regions activated as a consequence of the object-organism interaction. The mapping of the object-related consequences occurs in first-order neural maps representing the proto-self and object; the account of the causal relationship between object and organism can only be captured in second-order neural maps. … one might say that the swift, second-order nonverbal account narrates a story:

that of the organism caught in the act of representing its own changing state as it goes about representing something else.”

– Antonio Damasio [1999], The Feeling Of What Happens, p.170

Intelligent Machines?



... neither AI nor Alife has produced artifacts that could be confused with a living organism for more than an instant

□ Rodney Brooks in Nature, 2001

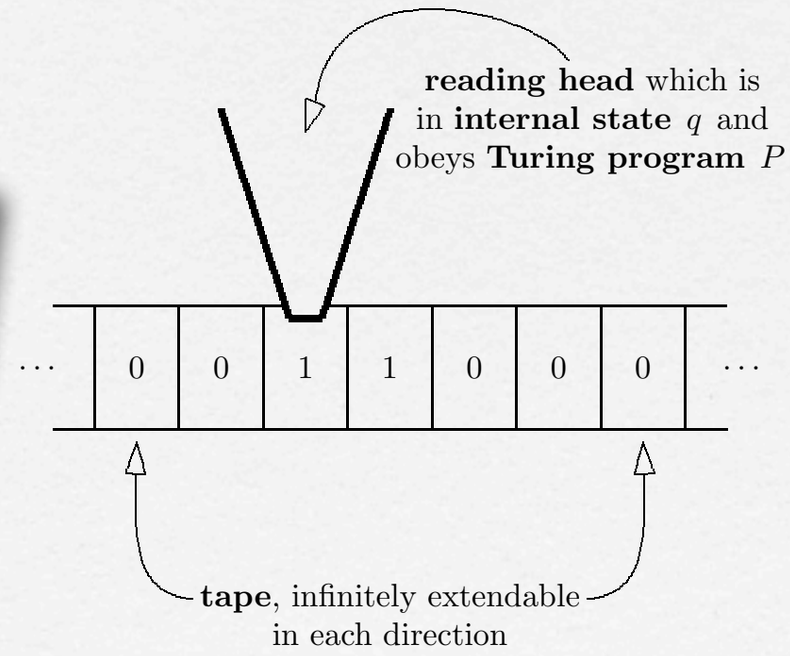
Computational Host for Information

Computation also about Information

- Turing, 1939 - Oracle Turing Machines ...

- Provide a model of how we compute using data given to us from unknown sources

- A model within which Newtonian computability etc comfortably fit ...



The Mathematics of Relative Computability

- 1939 - Turing's *oracle Turing machines*

- Provide a model of *computable content* of *structures*, *based on p.c. functionals over the reals*

- 1944 - Post defines the *degrees of unsolvability* as a *classification of reals* in terms of their relative computability

- Giving *a landscape with a rich structure*

Phyllis, Emil and Gertrude Post

Hartley Rogers' Programme

Fundamental problem: Characterise the Turing definable/invariant relations

- Intuition: These are key to pinning down how higher order relations on the real world can appear to be computed

- So: The richness of Turing structure discovered so far becomes the raw material for a multitude of definable - and hence visibly computable - relations

Physical Computation, Turing Landscape and Emergence ...

- Can describe global relations in terms of local structure ...

- ... so capturing the computation of large-scale formations

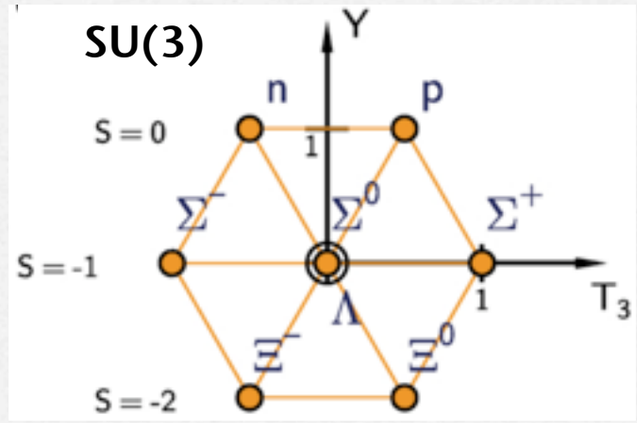
Mathematically - formalise as definability over structure based on oracle computations

More generally - as Invariance under automorphisms

Examples of Failure of Definability?

Murray Gell-Mann

- Symmetries play a huge role in science ..



- ... expressing appropriate automorphisms
- ... or particular lapses in definability

... so giving a clear route: from fundamental mathematical structures, and their automorphisms and breakdowns in definability - to far-reaching macro-symmetries in nature

Examples of Failure of Definability?

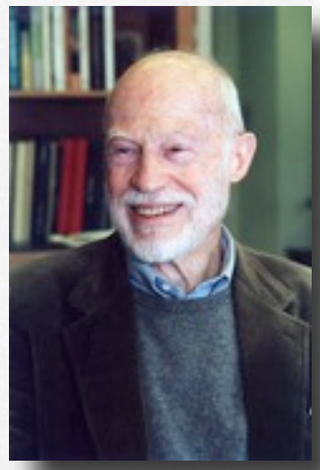
Processes for change of wave equation describing quantum state of a physical system:

- Deterministic continuous evolution via Schrödinger's equation - involves superpositions of basis states

- Probabilistic non-local discontinuous change due to measurement - observe a jump to a single basis state

Putative mathematical explanation in terms of assertion of previously imperfect definability ... providing ...

... an alternative to Many Worlds

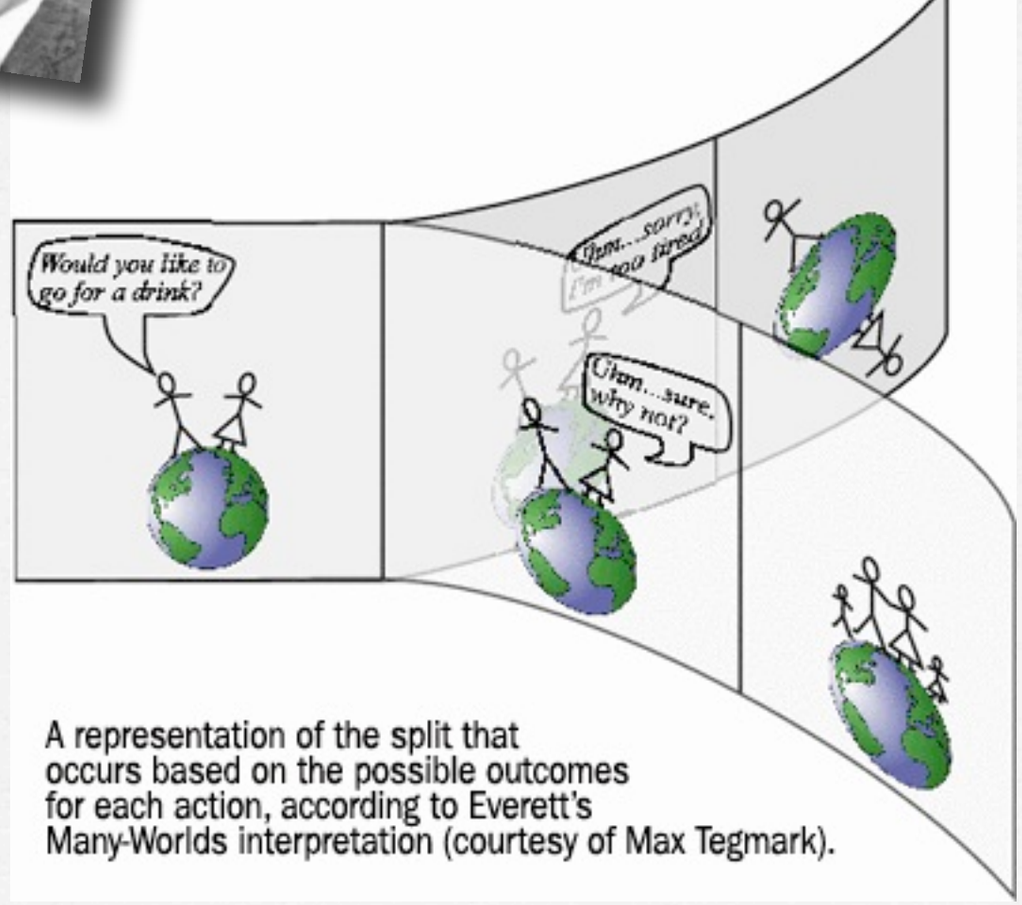


Bryce DeWitt

John Wheeler

Hugh Everett III

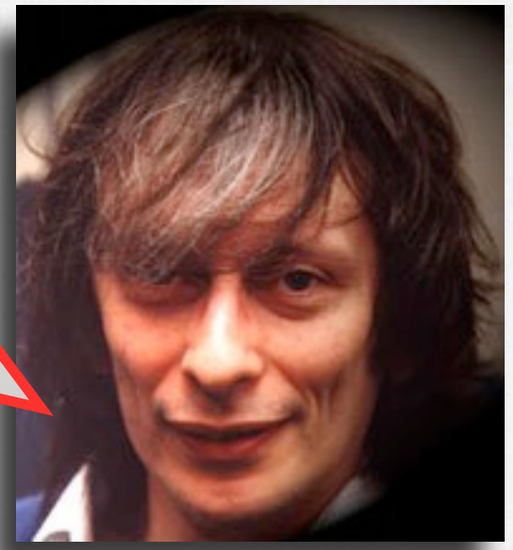
(Nov.11, 1930- July 19, 1982)



A representation of the split that occurs based on the possible outcomes for each action, according to Everett's Many-Worlds interpretation (courtesy of Max Tegmark).

and the Multiverse +Anthropic Principle

... understanding the multiverse is a precondition for understanding reality as best we can. Nor is this said in a spirit of grim determination to seek the truth no matter how unpalatable it may be ... It is, on the contrary, because the resulting world-view is so much more integrated, and makes more sense in so many ways, than any previous world-view, and certainly more than the cynical pragmatism which too often nowadays serves as surrogate for a world-view amongst scientists.



David Deutsch, The Fabric of Reality, Allen Lane, 1997, p.48

By Max Tegmark

Parallel Universes

Not just a staple of science fiction, other universes are a direct implication of cosmological observations

Is there a copy of you

reading this article? A person who is not you but who lives on a planet called Earth, with misty mountains, fertile fields and sprawling cities, in a solar system with eight other planets? The life of this person has been identical to yours in every respect. But perhaps he or she now decides to put down this article without finishing it, while you read on.

The idea of such an alter ego seems strange and implausible, but it looks as if we will just have to live with it, because it is supported by astronomical observations. The simplest and most popular cosmological model today predicts that you have a twin in a galaxy about 10 to the 10^{28} meters from here. This distance is so large that it is beyond astronomical, but that does not make your doppelgänger any less real. The estimate is derived from elementary probability and does not even assume speculative modern physics, merely that space is infinite (or at least sufficiently large) in size and almost uniformly filled with matter, as observations indicate. In infinite space, even the most unlikely events must take place somewhere. There are infinitely many other inhabited planets, including not just one but infinitely many that have people with the same appearance, name and memories as you, who play out every possible permutation of your life choices.

You will probably never see your other selves. The farthest you can observe is the distance that light has been able to travel during the 14 billion years since the big bang expansion began. The most distant visible objects are now about 4×10^{26} meters away—a distance that defines our observable universe, also called our Hubble volume, our horizon volume or simply our universe. Likewise, the universes of your other selves are spheres of the same size centered on their planets. They are the most straightforward example of parallel universes. Each universe is merely a small part of a larger "multiverse."

By this very definition of "universe," one might expect the notion of a multiverse to be forever in the domain of metaphysics. Yet the borderline between physics and metaphysics is defined by whether a theory is experimentally testable, not by whether it is weird or involves unobservable entities. The frontiers of physics have gradually expanded to incorporate ever more abstract (and once metaphysical) concepts such as a round Earth, invisible electromagnetic fields, time slowdown at high speeds, quantum superpositions, curved space, and black holes. Over the past several years the concept of a multiverse has joined this list. It is grounded in well-tested theories such as relativity and quantum mechanics, and it fulfills both of the basic criteria

ALFRED T. KAMAJIAN

and the Multiverse + Anthropic Principle

The issue of what is to be regarded as an ensemble of 'all possible' universes is unclear, it can be manipulated to produce any result you want ... The argument that this infinite ensemble actually exists can be claimed to have a certain explanatory economy (Tegmark 1993), although others would claim that Occam's razor has been completely abandoned in favour of a profligate excess of existential multiplicity, extravagantly hypothesized in order to explain the one universe that we do know exists.

George Ellis, The Unique Nature of Cosmology, in Revisiting the Foundations of Relativistic Physics (eds. Abhay Ashtekar et al), Kluwer, 1996, p.198

Reality of Higher Type Computation

- Described *in terms of reals* … with natural laws based on *algorithmic relations between reals*
- Emergence described as *definability/higher type computation*
- . . . with failures of definable information content modelling *mental phenomena*, *quantum ambiguity*
- . . . which generate *new levels of computable structure*
- . . . and a *fragmented scientific enterprise*

Thank you!