Tutorial on Axiomatization of Relativity Theory (part 2)

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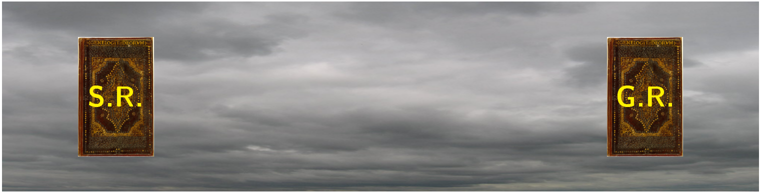
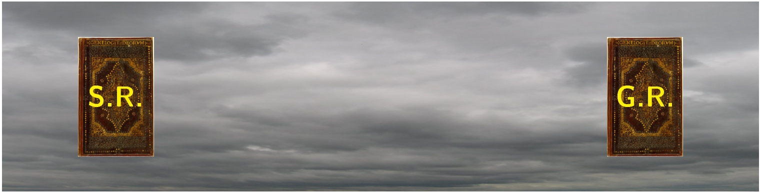
Theory in between



SpecRel

Theory in between



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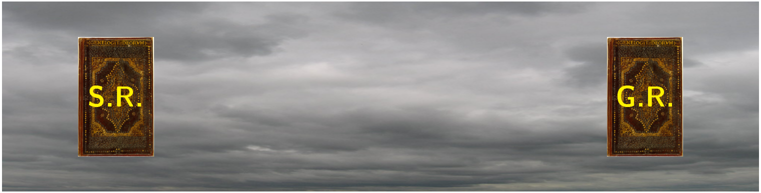
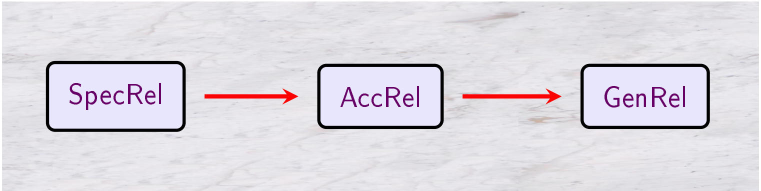
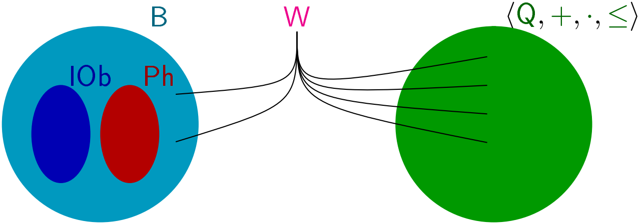




AccRel: a theory of accelerated observers

The language of AccRel is the same.



B $\rightsquigarrow$ Bodies (things that move)

IOb $\rightsquigarrow$ Inertial Observers   Ph $\rightsquigarrow$ Photons (light signals)

Q $\rightsquigarrow$ Quantities $+$, $\cdot$ and $\leq$ $\rightsquigarrow$ field operations and ordering

W $\rightsquigarrow$ Worldview (a 6-ary relation of type BBQQQQ)

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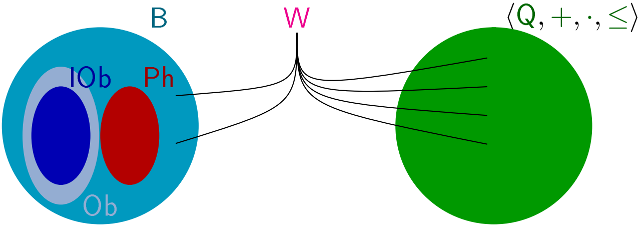

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IOb ⇝ Inertial Observers   Ph ⇝ Photons (light signals)

Q ⇝ Quantities $+$, $\cdot$ and $\leq$ ⇝ field operations and ordering

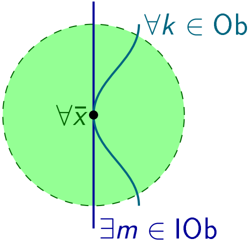
W ⇝ Worldview (a 6-ary relation of type BBQQQQ)

Observers: $\mathsf{Ob}(k) \stackrel{def}{\Longleftrightarrow} \exists xyzt\ b\ \mathsf{W}(k, b, x, y, z, t)$

Axioms of AccRel

AxCmv :

At each moment of its life, every observer coordinatizes the nearby world for a short while in the same way as an inertial observer does.



$$\forall k \in Ob\ \forall \bar{x} \in wline_k(k)\ \exists m \in IOb \quad d_{\bar{x}} w_{mk} = Id, \text{ where}$$

$$d_{\bar{x}} w_{mk} = L \overset{def}{\Longleftrightarrow} \forall \varepsilon > 0\ \exists \delta > 0\ \forall \bar{y}\ \ |\bar{y} - \bar{x}| \leq \delta \rightarrow |w_{mk}(\bar{y}) - L(\bar{y})| \leq \varepsilon |\bar{y} - \bar{x}|.$$

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AxEv^- :

Any observer encounters the events in which he was observed.

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The worldview transformations have linear approximations at each point of their domain (i.e., they are differentiable).

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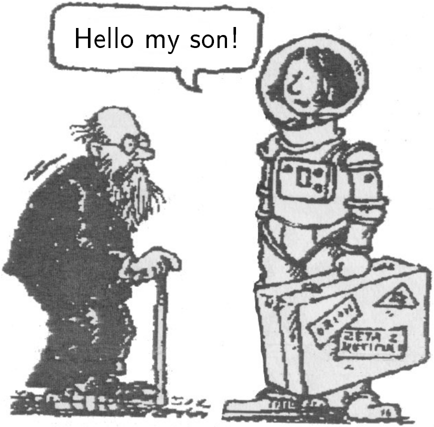
CONT :

Every definable, bounded and nonempty subset of $\mathbb{Q}$ has a supremum.

$$\text{AccRel} = \text{SpecRel} + \text{AxCmv} + \text{AxEv}^- + \text{AxSelf}^- + \text{AxDiff} + \text{CONT}$$

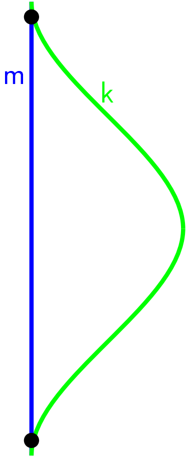
AccRel:		Theorems:
SpecRel		
AxCmv		?
AxEv^-		??
AxSelf^-	⟶	???
AxDiff		Etc.
CONT		

Twin paradox $\rightsquigarrow$ TwP



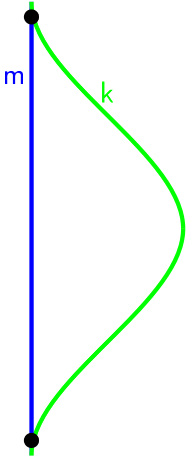
Theorem:

AccRel $\vdash$ TwP



Theorem:

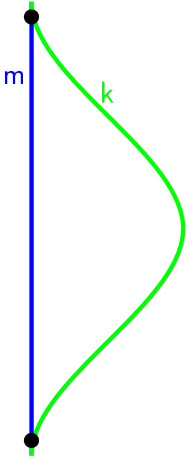
$$\mathsf{AccRel} - \mathsf{AxDiff} \vdash \mathsf{TwP}$$



Theorem:

$$\mathsf{AccRel} - \mathsf{AxDiff} \vdash \mathsf{TwP}$$

$$\mathsf{AccRel} - \mathsf{CONT} \nvdash \mathsf{TwP}$$



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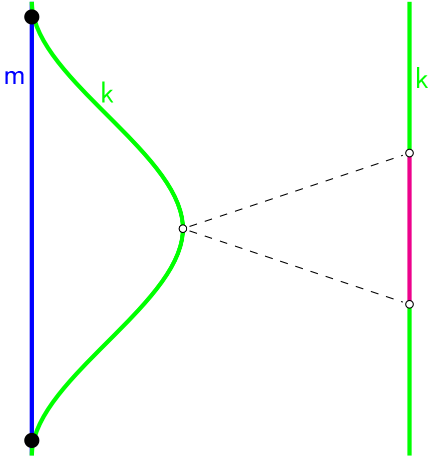
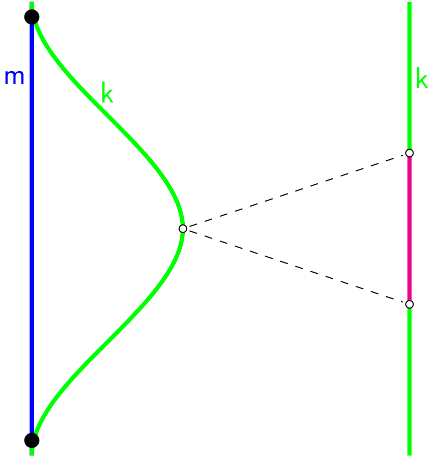


Theorem:

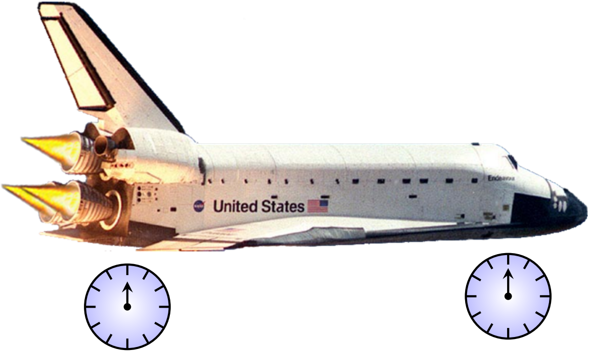
$$\mathrm{AccRel} - \mathrm{AxDiff} \vdash \mathrm{TwP}$$

$$Th(\mathbb{R}) + \mathrm{AccRel} - \mathrm{CONT} \nvdash \mathrm{TwP}$$



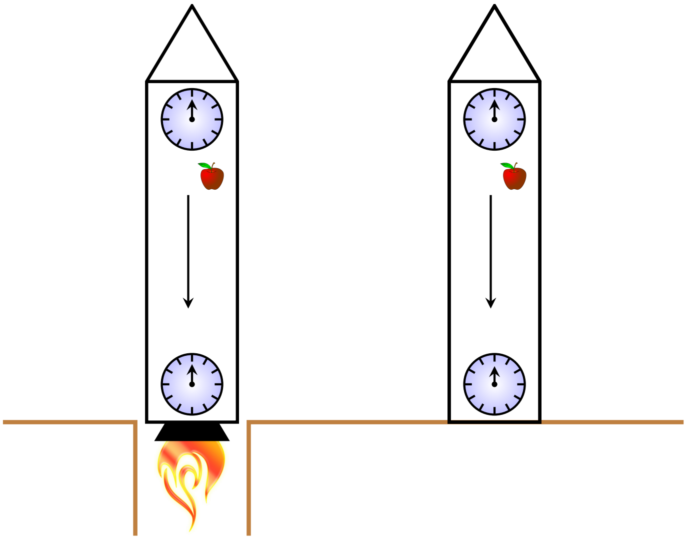
Theorem:

AccRel $\vdash$ *„Acceleration slows time down.”*

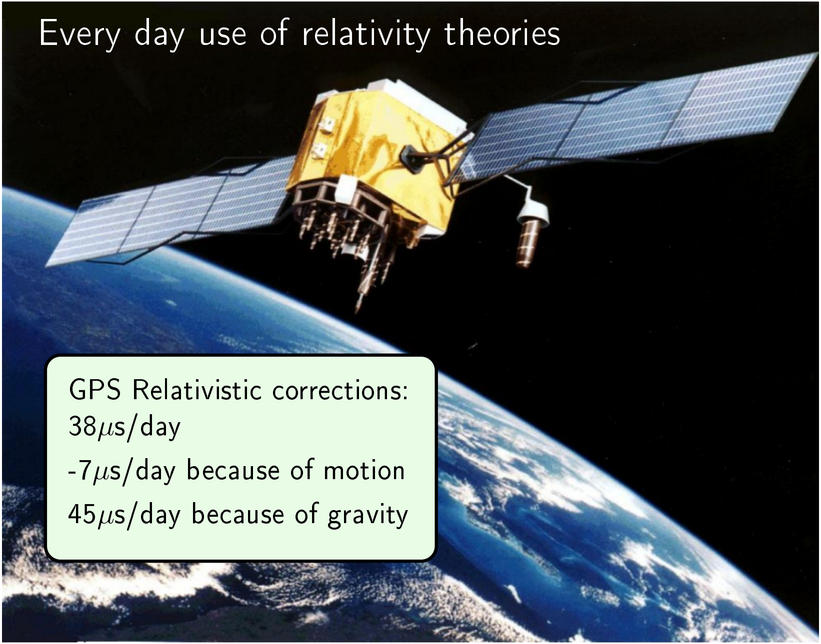


GRAVITY CAUSES SLOW TIME

via Einstein's Principle of Equivalence



Every day use of relativity theories



GPS Relativistic corrections:

$38\mu s$/day

-$7\mu s$/day because of motion

$45\mu s$/day because of gravity

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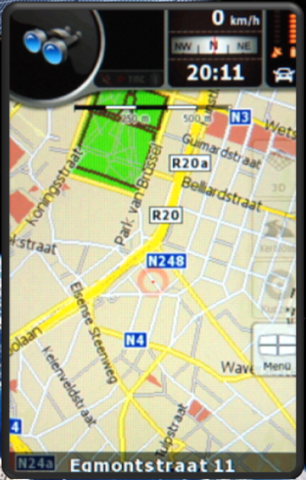

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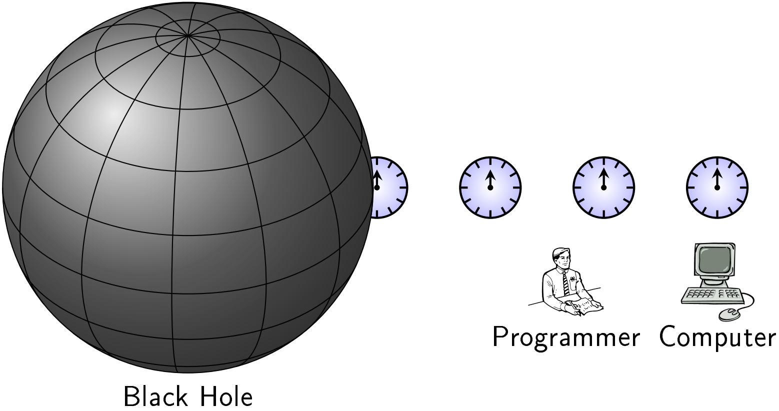
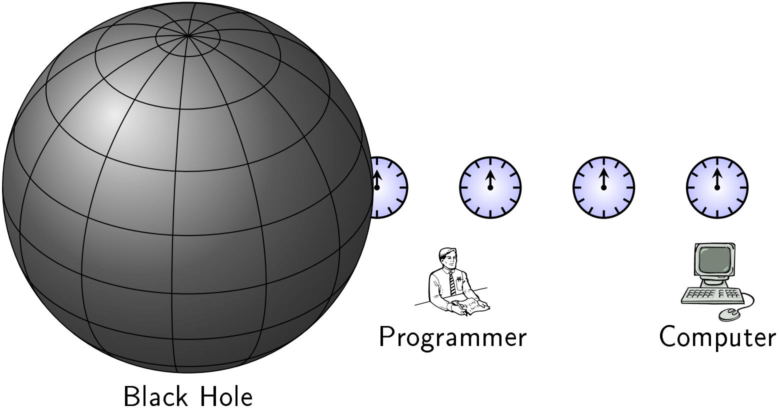
AccRel:		Theorems:
SpecRel		
AxCmv		Twin Paradox
AxEv^-		Gravit. time dilation
AxSelf^-	⟶	GenRel
AxDiff		Etc.
CONT		

Relativistic computation

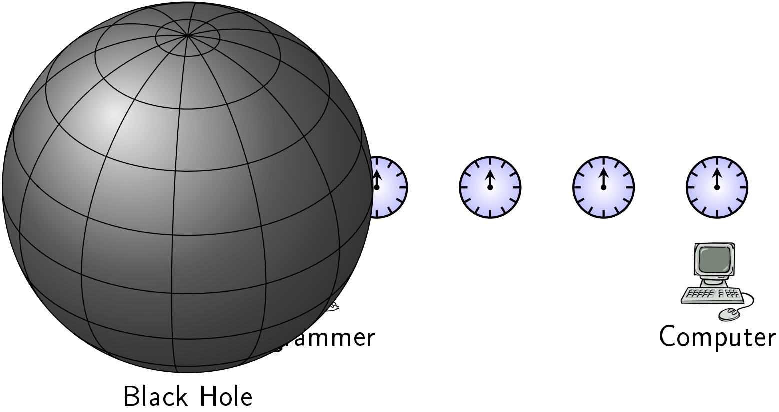
Basic idea of relativistic (hyper)computation



Basic idea of relativistic (hyper)computation

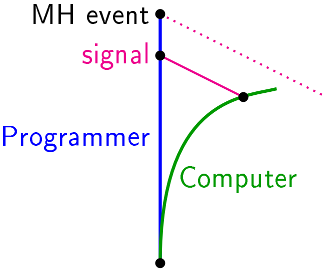


Basic idea of relativistic (hyper)computation



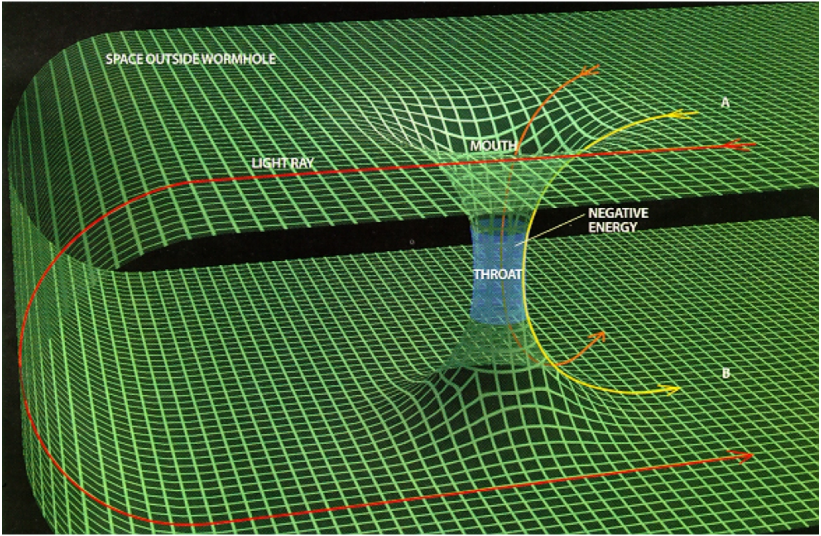
HypComp :

There is an event (MH event) such that the Computer has infinite time to compute sending signals reaching the Programmer before this event.

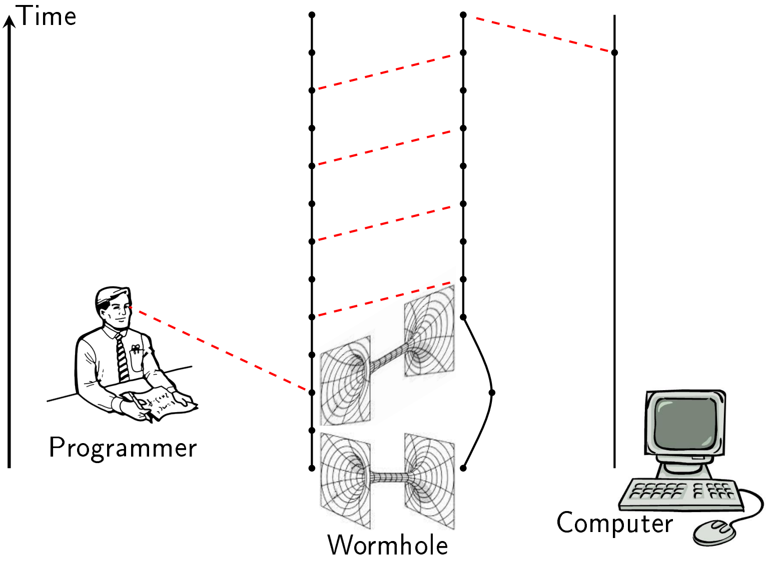


$$\exists p c \tau \Big[\mathsf{Ob}(p) \land \mathsf{Ob}(c) \land \forall m x \Big(\mathsf{IOb}(m) \land x \geq 0 \to x \in Dom\, \mathsf{lc}_m(c) \land$$
$$\forall t \Big[t > 0 \to \exists t' s_t \big[0 < t' < \tau \land s_t \in \mathsf{ev}_m\big(\mathsf{lc}_m(c)(t)\big) \cap \mathsf{ev}_m\big(\mathsf{lc}_m(p)(t')\big) \big] \Big] \Big) \Big].$$

Wormholes connecting different parts of the same universe



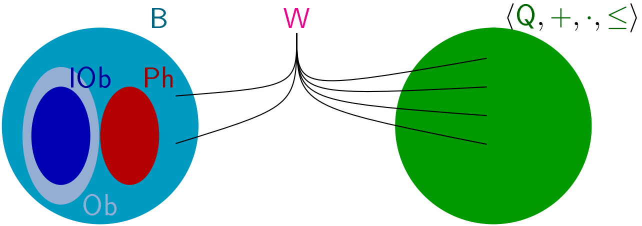
Hypercomputation via wormholes:



The story of relativistic hypercomputation continues in Péter Németi's talk.

GenRel: an axiomatic theory of general relativity

The language of GenRel is the same.



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Axioms of GenRel

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„Let all observers be equal at the level of axioms." (Einstein)

$$\left.\begin{array}{c}\text{AxPh}\\ \text{AxEv}\\ \text{AxSelf}\\ \text{AxSym}\end{array}\right\} \xrightarrow[\text{AxDiff}]{\text{AxCmv}} \left\{\begin{array}{c}\text{AxPh}^-\\ \text{AxEv}^-\\ \text{AxSelf}^-\\ \text{AxSym}^-\end{array}\right.$$

E.g., $\text{AxPh}, \text{AxCmv} \vdash \text{AxPh}^-$.

Axioms of GenRel

AxPh$^-$:

The instantaneous velocity of photons is 1 in the moment when they are sent out according to the observer who sent them out, and any observer can send out a photon in any direction with this instantaneous velocity.

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AxSym$^-$:

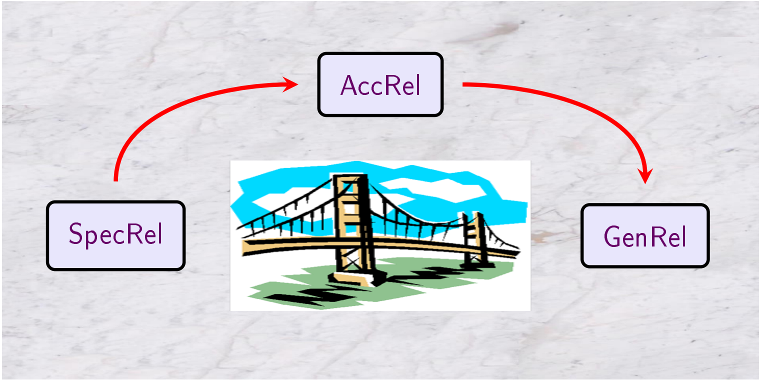
Any two observers meeting see each others' clocks behaving in the same way at the event of meeting.

$$\mathsf{GenRel} = \mathsf{AxFd} + \mathsf{AxPh}^- + \mathsf{AxEv}^- + \mathsf{AxSelf}^- + \mathsf{AxSym}^- + \mathsf{AxDiff} + \mathsf{CONT}$$

GenRel:		Theorems:
AxPh$^-$		
AxEv$^-$		?
AxOField		??
AxSelf$^-$	→	???
AxSym$^-$		
AxDiff		Etc.
CONT		

Theorem:

$$\mathsf{SpecRel} \dashv \mathsf{AccRel} \models \mathsf{GenRel}.$$



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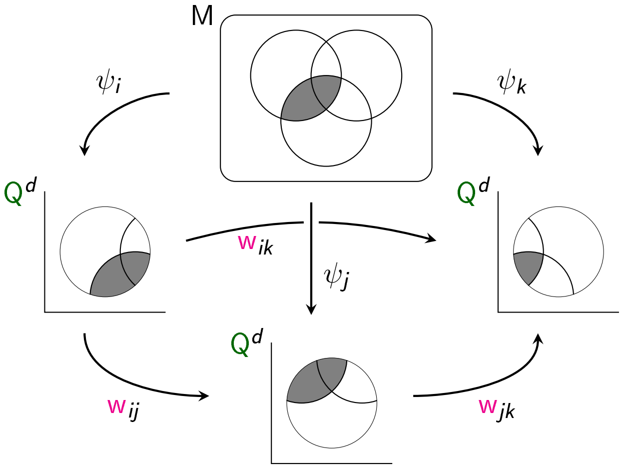
Theorem:

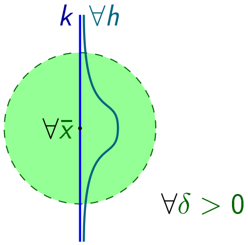
$\mathsf{GenRel} \vdash \forall m, k \in \mathsf{Ob}\ \ \forall \bar{x} \in wline_m(k) \cap wline_m(m) \rightarrow$ "w_{mk} *is differentiable at* $\bar{x}$ *and* $d_{\bar{x}}\mathsf{w}_{mk}$ *is a Lorentz transformation.*"

Theorem: (Completeness)

GenRel *is complete with respect to the „standard models of GR", i.e., the differentiable Lorentzian manifolds over real closed fields.*



Geodesics: The worldline of an observer is called *timelike geodesic* if it „locally maximizes measured time."



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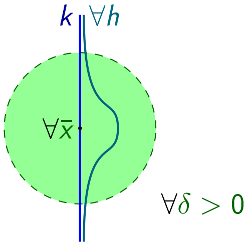

COMPR :

For any parametrically definable timelike curve in any observers worldview, there is another observer whose worldline is the range of this curve.

$$\mathsf{GenRel}^{+} = \mathsf{GenRel} + \mathsf{COMPR}$$

In GenRel$^+$ the notion of geodesics coincides with its standard notion. Via geodesics, we can define the other notions of general relativity, such as Riemann curvature tensor.

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Einstein's field equations: $\boxed{R_{ij} - \frac{1}{2} R g_{ij} = T_{ij}}$.

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Definition or axiom?

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Einstein's field equations: $$\boxed{R_{ij} - \frac{1}{2} R g_{ij} = T_{ij}}.$$

Definition or axiom? No real difference.

GenRel$^+$:

AxPh$^-$

AxEv$^-$

AxOField

AxSelf$^-$

AxSym$^-$

AxDiff$^-$

CONT

COMPR

$\longrightarrow$

Theorems:

Local Lorenz transf.

Completeness

Geodesics eq.

Happy End?

Happy ~~End?~~ Beginning!