An Infinitesimally Superluminal Neutrino is Left-Handed, Conserves Lepton Number and Solves the Autobahn Paradox (Illustrative Discussion)

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Consider a gedanken experiment in which a massive left-handed neutrino, traveling on an autobahn at a speed of $v = 0.999\,c$ is overtaken by a tuned-up CAGIVA V–RAPTOR 1000 traveling at a speed of $v = 0.999999\,c$. The biker, looking back, would see a right-handed neutrino. This 'autobahn paradox' implies that a massive subluminal (tardyonic) neutrino necessarily has to be a Majorana particle, i.e, equal to its own antiparticle. In turn, this would require us to assign the same lepton number to charged leptons and antileptons, essentially voiding the concept of lepton number. By contrast, an infinitesimally superluminal (tachyonic) neutrino is not equal to its own antiparticle and allows us to assign proper lepton number, just as if the neutrino were a Weyl particle. Furthermore, if Lorentz symmetry holds, then an infinitesimally tachyonic neutrino remains superluminal upon Lorentz transformation, which implies that it is impossible to overtake it in a gedanken experiment. Consistently, right-handed neutrino and left-handed antineutrino states have recently been shown to acquire negative norm under the assumption of an ever-so-slightly tachyonic neutrino, and it would thus not be necessary to invoke a seesaw mechanism. An infinitesimally superluminal neutrino does not necessarily violate causality, as has been discussed in the literature. This paper is devoted to an illustrative discussion on a scenario which could unfold if the observation of neutrinoless double beta decay is not confirmed. In this case, an infinitesimally superluminal neutrino could appear to solve at least as many problems as it raises.

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I. INTRODUCTION

From the point of view of fundamental symmetries, the introduction of a Majorana neutrino is not as innocent as it seems. The Majorana mass term violates lepton number, which a global symmetry that tracks the difference between particles and antiparticles. While there is nothing sacred about global symmetries, 'lepton number aficionados' may find the lack of a proper distinction of particles and antiparticles disturbing. A Majorana neutrino, if it exists, would be the only spin-$\frac{1}{2}$ particle in an (extended) standard model which is equal to its own antiparticle (the original standard model called for a Weyl neutrino). Furthermore, a Majorana neutrino, would be the only particle in an extended standard model described by an equation which, on the level of first quantization, does not allow plane-wave solutions of the form $u \exp(-\mathrm{i}k \cdot x)$, where u is a spinor (or polarization vector in the case of spin-1 particles) and $k \cdot x = E\,t - \vec{k} \cdot \vec{r}$ is the scalar product of energy-momentum and space-time coordinate (see also Appendix A).

These difficulties appear because the neutrino has been observed to have mass. In the 'good old times', neutrinos were supposed to be massless, and thus Weyl fermions which can come in only one helicity state. They would transform according to the fundamental $(\frac{1}{2}, 0)$ representation of the Lorentz group, and could thus be incorporated easily into the standard model. The discovery of neutrino oscillations implies that the neutrinos need to have some kind of mass. If we assume that neutrinos are massive fermions, then, due to the 'autobahn paradox' they have to be Majorana fermions. As already mentioned, a tuned-up CAGIVA overtaking a left-handed neutrino on an autobahn without speed limits will see a right-handed neutrino; this is possible only if the neutrino is its own antiparticle and thus a Majorana fermion. However, massive Majorana fermions necessarily come in two helicities, nullifying the 'Weyl' argument for the suppression of right-handed neutrino states (an excellent overview is presented in Ref. [1]). Also, because a Majorana neutrino is its own antiparticle, we cannot assign different lepton number to neutrinos and antineutrinos. In order to still suppress the right-handed neutrino state, one has to invoke the seesaw mechanism [2–4] which assign a very heavy mass to the right-handed neutrino and left-handed antineutrino states, but it has a hierarchy problem: namely, the mass generation for the neutrinos happens via a nonrenormalizable dimension-5 interaction, and the left-handed neutrino masses (of the order of v^2/Λ where v is the vacuum expectation value of the Higgs) are extremely sensitive to the fine-tuning of the GUT scale Λ (see Ref. [5] for a brief overview).

Experienced drivers know that there usually is a realm of tolerance just over the speed limit (if there is one) where one does not run into trouble. Neutrinos have been traveling the universe for billions of years. The cosmic speed limit is the speed of light, and just over the speed limit, there is a realm of tolerance set forth by the uncertainty principle (see Sec. 2.13 of Ref. [6] and Ref. [7]). If we assume that the neutrino is a ever-so-slightly tachyonic, superluminal fermion, but within the bounds set by the uncertainty principle, then all problems mentioned above simply evaporate, more or less.

Thus, one would assume that the neutrino is much less superluminal than recent false and meanwhile retracted claims of experimentalists would suggest, but still, *infinitesimally* superluminal. How much? This would be a question for experimentalists to answer. The reason behind the theoretical benefits are as follows. First, let us recall that a tachyonic particle, which travels a little faster than the speed of light, still is compatible with special relativity [8, 9]. Relativity theory is based on two assumptions: (i) The principle of relativity states that the laws of physics are the same for all observers in uniform motion relative to one another. (ii) The speed of light in a vacuum is the same for all observers, regardless of their relative motion or of the motion of the source of the light. Superluminal propagation is not excluded by these axioms, and in fact is compatible with the Lorentz transformation [8, 9].

Due to the Lorentz transformation, superluminal particles always remain superluminal under a transformation between subluminal reference frames, whereas due to special relativity, it is impossible to accelerate even the most tuned-up fictitious motorbike *through* the light barrier. Thus, the above mentioned gedanken experiment becomes void. Next, a superluminal neutrino is described by the tachyonic Dirac equation, which is $\mathcal{CP}$ invariant, but not $\mathcal{T}$ invariant [10], and this implies that the neutrino is not equal to its own antiparticle, saving the lepton number as a valid concept. Last, but certainly not least, the right-handed neutrino and the left-handed antineutrino states automatically acquire negative norm in the quantized tachyonic theory [11, 12], thus voiding the necessity to introduce the seesaw mechanism.

The assumption of an infinitesimally tachyonic neutrino could potentially solves a number of theoretical issues with neutrino physics. If the observation of neutrinoless double beta decay is not confirmed [13–19], then it may be worthwhile to investigate the tachyonic hypothesis further. Here, we start with a somewhat didactic recollection of some basic facts in Sec. II, continue with a generalized Dirac equation for tardyonic (subluminal) particles in Sec. III, treat virtual tachyonic particles in the framework of a generalized Dirac equation with two mass terms (Sec. IV). Via a detour over massless particles (Sec. V), we show that real as opposed to virtual spin-1/2 particles are always left-handed (see Sec. VI). Conclusions are given in Sec. VII. Finally, Appendix A is devoted to the Majorana equation. Units with $\hbar = c = \epsilon_0 = 1$ are used throughout this article.

II. IMPLICATIONS

Regarding the philosophical consequences and implications of infinitesimally superluminal neutrinos, let us recall the following facts:

- Dirac [20, 21] saw that if one linearizes the Klein–Gordon equation, one has to introduce a 4×4 matrix structure. The resultant equation has negative-energy solutions. These are interpreted as antiparticles.

- According to the Feynman prescription the propagator of the quantized Dirac field has an advanced, strictly speaking *acausal* part which describes antiparticles moving backward in time, and it also has a causal, retarded part, describing particles moving forward in time. However, the only physically relevant amplitudes predicted by theory are elements of the scattering matrix, which is called S-matrix. Antiparticles moving backward in time are reinterpreted as particles moving in time, with all kinetic variables (energy, momentum) changing sign [22, 23].

- This is a manifestation of the principle of reinterpretation. The transition amplitude predicted by S matrix theory connects two space-time points, and the time coordinate of one of the events happens to be earlier than the other. Particles, seen by an observer, always move forward in time because it is always possible to identify the 'earlier' event whose time coordinate is less than that of the other event. If the particles described by the theory are subluminal (move slower than light), then this reinterpretation is undisputed within the community and forms one the core foundations of experimentally verified quantum field theory. This concept has been generalized to superluminal particles [9].

- The theory of relativity does not forbid faster-than-light propagation [8, 9]. Predictions of relativity theory regarding the relativity of simultaneity, time dilation and length contraction would not change if superluminal particles did exist. Via a geometric construction (Minkowski diagram), one can show that a superluminal velocity remains superluminal if one changes Lorentz frames. In particular, the Einstein velocity addition theorem remains valid in the superluminal world. Superluminal velocities always remain superluminal upon Lorentz transformation [8, 9].

- Nimtz and coworkers [24–26] claim to have demonstrated in their (disputed) experiments that electromagnetic signal propagation with up to four times of the speed of light is possible if one is willing to accept exponential damping of the signal (tunneling effect), i.e., over short distances.

- Sudarshan and Feinberg, and coworkers (Refs. [8, 9, 27–30]) have suggested to extend the reinterpretation principle to superluminal particles, and have suggested equations which describe superluminal particles respecting Lorentz invariance. These are called tachyons. Based on the reinterpretation principle, it is possible to resolve the causality paradox regarding the possible change in the time ordering of space-like events upon Lorentz transformation, or, to ascertain that if one accepts reinterpretation for the advanced, acausal part of the Feynman (subluminal) propagator, one should do the same for the superluminal propagator. We recall that the Feynman prescription is key in replacing the 'holes' in hole theory by actual particles, the antiparticles, at the cost of introducing a manifestly advanced, 'acausal' part of the propagator. The quantum field theory of superluminal particles is plagued with a series of problems, and it was soon realized that not all of the so-called Osterwalder–Schrader axioms [31] can be retained if one tries to incorporate tachyonic particles into field theory. In a series of recent papers [10–12], we have used the concept of a Lorentz non-invariant vacuum state, breaking one of the Osterwalder–Schrader axioms, but we have retained absolute conformity with Lorentz covariance and the special theory of relativity. Also, we have offered an alternative picture ('re-reinterpretation') in Sec. 4 of Ref. [11]) where we work with a Lorentz-invariant vacuum while transforming only the space-time argument of the creation and annihilation operators, but not, as it would seem necessary otherwise, some of the annihilation operators of the tachyonic field into creation operators and vice versa.

One might then ask: What are the relevant physical degrees of freedom in a tachyonic theory of spin-1/2 particles? The answer is as follows.

- One can describe quantum particles using 'good' quantum numbers. For free spin-1/2 particles, one can use the four-momentum wave number $\vec{k}$. For the second quantum number, one can use the spin projection $\lambda = \pm 1/2$ (for a particle at rest), or the helicity $\sigma = \pm$, which describes an orientation of the spin either in the flight direction of the particles as a left-handed screw, or a right-handed screw. The helicity operator commutes with the Hamiltonian and is a good quantum number.
- Beware: Helicity is not equal to chirality. Both quantities are equal in the massless limit, but are not equal for massive particles. Chirality is an eigenvalue of the fifth-current γ^5 matrix, but helicity describes the left-handed or right-handed orientation of the spin. Chirality is not a good quantum number for massive particles. Neutrinos have been observed in specific helicity, not chirality, states.

III. SUBLUMINAL SPIN-1/2 PARTICLES COME IN TWO HELICITIES

A generalized Dirac equation with two tardyonic (subluminal) mass terms is as follows:

$$\left(\mathrm{i}\gamma^\mu \partial_\mu - m_1 - \mathrm{i}\,\gamma^5 m_2\right) \psi^{(t)}(x) = 0\,, \tag{1}$$

where all the matrices are used in the standard (Dirac) representation. The Lagrangian density is

$$\mathcal{L}^{(t)} = \overline{\psi}^{(t)}(x) \left(\mathrm{i}\gamma^\mu \partial_\mu - m_1 - \mathrm{i}\,\gamma^5 m_2\right) \psi^{(t)}(x)\,. \tag{2}$$

The Hamiltonian is Hermitian,

$$H^{(t)} = \vec{\alpha} \cdot \vec{p} + \beta\, m_1 + \mathrm{i}\,\beta\,\gamma^5\, m_2\,, \qquad H^{(t)} = H^{(t)+}\,. \tag{3}$$

The fundamental positive-energy eigenspinors in the plane-wave case ($\mathrm{i}\,\partial\!\!\!/ \to k\!\!\!/$) are given by $U_\pm^{(t)}(\vec{k})\, \mathrm{e}^{-\mathrm{i}k\cdot x}$ with

$$U_+^{(t)}(\vec{k}) = \begin{pmatrix} \dfrac{m_1 - \mathrm{i}\, m_2 + E^{(t)} - |\vec{k}|}{\sqrt{(E^{(t)} - |\vec{k}|)^2 + m_1^2 + m_2^2}}\, \dfrac{a_+(\vec{k})}{\sqrt{2}} \\ \dfrac{m_1 - \mathrm{i}\, m_2 - E^{(t)} + |\vec{k}|}{\sqrt{(E^{(t)} - |\vec{k}|)^2 + m_1^2 + m_2^2}}\, \dfrac{a_+(\vec{k})}{\sqrt{2}} \end{pmatrix}, \quad U_-^{(t)}(\vec{k}) = \begin{pmatrix} \dfrac{m_1 + \mathrm{i}\, m_2 + E^{(t)} - |\vec{k}|}{\sqrt{(E^{(t)} - |\vec{k}|)^2 + m_1^2 + m_2^2}}\, \dfrac{a_-(\vec{k})}{\sqrt{2}} \\ \dfrac{-m_1 - \mathrm{i}\, m_2 + E^{(t)} - |\vec{k}|}{\sqrt{(E^{(t)} - |\vec{k}|)^2 + m_1^2 + m_2^2}}\, \dfrac{a_-(\vec{k})}{\sqrt{2}} \end{pmatrix}. \tag{4a}$$

where the $a_\pm(\vec{k})$ are the fundamental helicity spinors. The negative-energy eigenspinors are given by $V_\pm^{(t)}(\vec{k})\, \mathrm{e}^{\mathrm{i}k\cdot x}$ with

$$V_+^{(t)}(\vec{k}) = \begin{pmatrix} \dfrac{-m_1 + \mathrm{i}\, m_2 + E^{(t)} - |\vec{k}|}{\sqrt{(E^{(t)} - |\vec{k}|)^2 + m_1^2 + m_2^2}}\, \dfrac{a_+(\vec{k})}{\sqrt{2}} \\ \dfrac{-m_1 + \mathrm{i}\, m_2 - E^{(t)} + |\vec{k}|}{\sqrt{(E^{(t)} - |\vec{k}|)^2 + m_1^2 + m_2^2}}\, \dfrac{a_+(\vec{k})}{\sqrt{2}} \end{pmatrix}, \quad V_-^{(t)}(\vec{k}) = \begin{pmatrix} \dfrac{-m_1 - \mathrm{i}\, m_2 + E^{(t)} - |\vec{k}|}{\sqrt{(E^{(t)} - |\vec{k}|)^2 + m_1^2 + m_2^2}}\, \dfrac{a_-(\vec{k})}{\sqrt{2}} \\ \dfrac{m_1 + \mathrm{i}\, m_2 + E^{(t)} - |\vec{k}|}{\sqrt{(E^{(t)} - |\vec{k}|)^2 + m_1^2 + m_2^2}}\, \dfrac{a_-(\vec{k})}{\sqrt{2}} \end{pmatrix}. \tag{4b}$$

In spherical coordinates, we have

$$a_+(\vec{k}) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) \, \mathrm{e}^{\mathrm{i}\,\varphi} \end{pmatrix}, \quad a_-(\vec{k}) = \begin{pmatrix} -\sin\left(\frac{\theta}{2}\right) \, \mathrm{e}^{-\mathrm{i}\,\varphi} \\ \cos\left(\frac{\theta}{2}\right) \end{pmatrix}. \tag{5}$$

These spinors are normalized according to the 'normal' scalar product $u^+ \, u = 1$. Changing the normalization to

$$\mathcal{U}^{(t)}_\sigma(\vec{k}) = \left(\frac{E^{(t)}}{m_1} \right)^{1/2} U^{(t)}_\sigma(\vec{k}) \,, \qquad \mathcal{V}^{(t)}_\sigma(\vec{k}) = \left(\frac{E^{(t)}}{m_1} \right)^{1/2} V^{(t)}_\sigma(\vec{k}) \,, \qquad E^{(t)} = \sqrt{\vec{k}^2 + m_1^2 + m_2^2} \,, \tag{6}$$

a little algebra reveals that the eigenspinors fulfill the following sum rules,

$$\sum_\sigma \mathcal{U}^{(t)}_\sigma(\vec{k}) \otimes \overline{\mathcal{U}}^{(t)}_\sigma(\vec{k}) = \frac{\not{k} + m_1 - \mathrm{i}\,\gamma^5 \, m_2}{2m_1} \,, \quad \sum_\sigma \mathcal{V}^{(t)}_\sigma(\vec{k}) \otimes \overline{\mathcal{V}}^{(t)}_\sigma(\vec{k}) = \frac{\not{k} - m_1 + \mathrm{i}\,\gamma^5 \, m_2}{2m_1} \,. \tag{7}$$

IV. VIRTUAL SUPERLUMINAL SPIN-1/2 PARTICLES ALSO COME IN TWO HELICITIES

A suitable tachyonic generalized Dirac equation with two mass terms is

$$\left(\mathrm{i}\gamma^\mu \partial_\mu - \mathrm{i}m_1 - \gamma^5 m_2 \right) \psi'(x) = 0 \,. \tag{8}$$

The Lagrangian density is

$$\mathcal{L}' = \overline{\psi}'(x) \, \gamma^5 \, \left(\mathrm{i}\gamma^\mu \partial_\mu - \mathrm{i}m_1 - \gamma^5 m_2 \right) \psi'(x) \,. \tag{9}$$

The Hamiltonian is pseudo-Hermitian [32–40], or 'γ^5 Hermitian' [41],

$$H' = \vec{\alpha} \cdot \vec{p} + \mathrm{i}\,\beta \, m_1 + \beta \, \gamma^5 \, m_2 \,, \qquad H' = \gamma^5 \, H'^+ \, \gamma^5 \,. \tag{10}$$

The fundamental plane-wave solutions are $U'_\pm(\vec{k}) \, \mathrm{e}^{-\mathrm{i}k\cdot x}$ with

$$U'_+(\vec{k}) = \begin{pmatrix} \dfrac{\mathrm{i}\,m_1 + m_2 - E' + |\vec{k}|}{\sqrt{(E' - |\vec{k}|)^2 + m_1^2 + m_2^2}} \, \dfrac{a_+(\vec{k})}{\sqrt{2}} \\ \dfrac{\mathrm{i}\,m_1 + m_2 + E' - |\vec{k}|}{\sqrt{(E' - |\vec{k}|)^2 + m_1^2 + m_2^2}} \, \dfrac{a_+(\vec{k})}{\sqrt{2}} \end{pmatrix}, \quad U'_-(\vec{k}) = \begin{pmatrix} \dfrac{\mathrm{i}m_1 + m_2 + E' - |\vec{k}|}{\sqrt{(E' - |\vec{k}|)^2 + m_1^2 + m_2^2}} \, \dfrac{a_-(\vec{k})}{\sqrt{2}} \\ \dfrac{-\mathrm{i}m_1 - m_2 + E' - |\vec{k}|}{\sqrt{(E' - |\vec{k}|)^2 + m_1^2 + m_2^2}} \, \dfrac{a_-(\vec{k})}{\sqrt{2}} \end{pmatrix}. \tag{11a}$$

where again the $a_\pm(\vec{k})$ are the helicity spinors. The negative-energy eigenspinors read as $V'_\pm(\vec{k}) \, \mathrm{e}^{\mathrm{i}k\cdot x}$, where

$$V'_+(\vec{k}) = \begin{pmatrix} \dfrac{\mathrm{i}m_1 - m_2 - E' + |\vec{k}|}{\sqrt{(E' - |\vec{k}|)^2 + m_1^2 + m_2^2}} \, \dfrac{a_+(\vec{k})}{\sqrt{2}} \\ \dfrac{\mathrm{i}m_1 - m_2 - E' - |\vec{k}|}{\sqrt{(E' - |\vec{k}|)^2 + m_1^2 + m_2^2}} \, \dfrac{a_+(\vec{k})}{\sqrt{2}} \end{pmatrix}, \quad V'_-(\vec{k}) = \begin{pmatrix} \dfrac{-\mathrm{i}m_1 - m_2 + E' - |\vec{k}|}{\sqrt{(E' - |\vec{k}|)^2 + m_1^2 + m_2^2}} \, \dfrac{a_-(\vec{k})}{\sqrt{2}} \\ \dfrac{\mathrm{i}m_1 + m_2 + E' - |\vec{k}|}{\sqrt{(E' - |\vec{k}|)^2 + m_1^2 + m_2^2}} \, \dfrac{a_-(\vec{k})}{\sqrt{2}} \end{pmatrix}. \tag{11b}$$

We again change the normalization according to

$$\mathcal{U}'_\sigma(\vec{k}) = \left(\frac{E'}{m_2} \right)^{1/2} U'_\sigma(\vec{k}) \,, \qquad \mathcal{V}'_\sigma(\vec{k}) = \left(\frac{E'}{m_2} \right)^{1/2} V'_\sigma(\vec{k}) \,, \qquad E' = \sqrt{\vec{k}^2 - m_1^2 - m_2^2} \,. \tag{12}$$

The sum rule, which projects onto positive-energy and negative-energy states, reads as follows,

$$\sum_\sigma (-\sigma) \, \mathcal{U}'_\sigma(\vec{k}) \otimes \overline{\mathcal{U}}'_\sigma(\vec{k}) \, \gamma^5 = \frac{\not{k} + \mathrm{i}\,m_1 - \gamma^5 \, m_2}{2m_2} \,, \quad \sum_\sigma (-\sigma) \, \mathcal{V}'_\sigma(\vec{k}) \otimes \overline{\mathcal{V}}'_\sigma(\vec{k}) \, \gamma^5 = \frac{\not{k} - \mathrm{i}\,m_1 + \gamma^5 \, m_2}{2m_2} \,. \tag{13}$$

Here, σ is a 'good' quantum number and characterizes the helicity, not chirality. Indeed, σ is equal to the helicity for positive-energy states, and equal to minus the helicity for negative-energy states.

V. FOR MASSLESS SPIN-1/2 PARTICLES, HELICITY EQUALS ±CHIRALITY

Massless particles always are in a helicity eigenstate, if they are in an energy eigenstate. The massless Dirac equation and its Hamiltonian H_0 read, quite simply,

$$\mathrm{i}\gamma^\mu \, \partial_\mu \, \psi(x) = 0 \,, \qquad\qquad H_0 = \vec{\alpha} \cdot \vec{p} \,. \tag{14}$$

The Hamiltonian is Hermitian as well as pseudo-Hermitian,

$$H_0 = H_0^+ \,, \qquad H_0 = \gamma^5 \, H_0^+ \, \gamma^5 \,. \tag{15}$$

The fundamental eigenspinors are well known,

$$u_+(\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} a_+(\vec{k}) \\ a_+(\vec{k}) \end{pmatrix} , \quad u_-(\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} a_-(\vec{k}) \\ -a_-(\vec{k}) \end{pmatrix} , \quad v_+(\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} -a_+(\vec{k}) \\ -a_+(\vec{k}) \end{pmatrix} , \quad v_-(\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} -a_-(\vec{k}) \\ a_-(\vec{k}) \end{pmatrix} . \tag{16}$$

They simultaneously fulfill sum rules which are akin to both Eqs. (7) and (13),

$$\sum_\sigma \, 2 \, |\vec{k}| \, u_\sigma(\vec{k}) \otimes \overline{u}_\sigma(\vec{k}) = \sum_\sigma \, 2 \, |\vec{k}| \, v_\sigma(\vec{k}) \otimes \overline{v}_\sigma(\vec{k}) = \not{k} \,, \tag{17}$$

$$\sum_\sigma \, 2 \, |\vec{k}| \, (-\sigma) \, u_\sigma(\vec{k}) \otimes \overline{u}_\sigma(\vec{k}) \, \gamma^5 = \sum_\sigma \, 2 \, |\vec{k}| \, (-\sigma) \, v_\sigma(\vec{k}) \otimes \overline{v}_\sigma(\vec{k}) \, \gamma^5 = \not{k} \,. \tag{18}$$

This observation ensures that both Eqs. (7) and (13) have a smooth massless limit [12].

VI. REAL SUPERLUMINAL SPIN-1/2 PARTICLES ARE ALWAYS LEFT-HANDED

One might think that the sum rules are somewhat arbitrary, and have no physical significance, and that the appearance of the 'mysterious' factor $(-\sigma)$ is arbitrary. However, that is not the case. Namely, if we postulate that the time-ordered product of field operators should give a propagator whose Fourier transformation is the inverse of the Hamiltonian, then we have to sum over the eigenspinors, as is done in any derivation of a Green function, and obtain the positive-energy and negative-energy projectors over the eigenspinors as a result of the spin sums [12]. So, we have to postulate that a relation of the type

$$\sum_{\rm spin} (\text{prefactor}) \times (\text{tensor product of states}) \times (\text{scalar or pseudo-scalar matrix}) = \text{projector} \tag{19}$$

holds. Here, the prefactor can only come from the fundamental anticommutator of the field operator, which in turn can only involve the 'good' quantum numbers. If we postulate a relation of the form

$$\left\{ b_\sigma(k), b_\rho^+(k') \right\} = f(\sigma, \vec{k}) \; (2\pi)^3 \, \frac{E}{m} \delta^3(\vec{k} - \vec{k}') \, \delta_{\sigma\rho} \,, \tag{20}$$

where the b and b^+ annihilate and create particles of the respective quantum numbers, and corresponding relations for the creators and annihilators of antiparticles, then the 'prefactor' in Eq. (19) has to be equal to $f(\sigma, \vec{k})$, as a quick calculation [10–12] shows,

$$(\text{prefactor}) = f(\sigma, \vec{k}) \,. \tag{21}$$

One simply has to generalize the derivation that leads from Eq. (3.169) to Eq. (3.170) in the standard textbook [42] in a straightforward way [12]. In turn, an inspection of Eqs. (7) and (13) shows that for the choices

$$f(\sigma, \vec{k}) = 1 \qquad (\text{tardyonic choice}) \,, \qquad f(\sigma, \vec{k}) = -\sigma \qquad (\text{tachyonic choice}) \,, \tag{22}$$

the tardyonic (subluminal) and tachyonic (superluminal) sum rules are fulfilled. We have been unable to fulfill the sum rules with different choices. A more detailed discussion can be found in Ref. [12].

We have previously mentioned that virtual superluminal spin-1/2 particles come in two helicities. However, if we accept the anticommutator relation (20), then the right-handed helicity state acquires negative norm. This is seen as follows,

$$\langle 1_{k,\sigma} | 1_{k,\sigma} \rangle = \left\langle 0 \left| b_\sigma(k)\, b_\sigma^+(k) \right| 0 \right\rangle = \left\langle 0 \left| \left\{ b_\sigma(k), b_\sigma^+(k) \right\} \right| 0 \right\rangle = (-\sigma)\, V \,\frac{E}{m}\,, \tag{23}$$

where $V = (2\pi)^3\, \delta^3(\vec{0})$ is the normalization volume in coordinate space. This is negative for $\sigma = 1$. Therefore, right-handed particle states are excluded from the spectrum by a Gupta–Bleuler condition. This is analogous to virtual photons which can be scalar or longitudinal, but physical photons must be transverse.

VII. CONCLUSIONS

Recently, we have been working on extensions of the Dirac equation [10–12]. Here, we attempt to describe the intuitive picture which underlies our work, and to identify its philosophical implications. In particular, we attempt to formulate the intuitive picture on which recent attempts to construct a spin-1/2 field theory of tachyonic particles and antiparticles are based. The superluminal Hamiltonians are found to have a mathematical property known as pseudo-Hermiticity, or γ^5 Hermiticity, which means that energy levels are real or come in complex-conjugated pairs. Following Pauli [32], we identify these Hamiltonians as physically meaningful. Furthermore, we find (unique?) sum rules for the fundamental spinor solutions which suggest that the right-handed helicity component is suppressed for superluminal spin-1/2 particles. These sum rules are crucial in the derivation of the time-ordered propagator for the (infinitesimally) superluminal fields, as outlined in Eqs. (46)–(56) of Ref. [12].

A superluminal neutrino has the potential to address at least three paradoxons connected with neutrinos: (i) the observation of only one helicity component (left-handed for neutrinos, opposite for antineutrinos) due to a Gupta–Bleuler condition, (ii) the 'autobahn helicity paradox' because even infinitesimally superluminal particles remain superluminal upon Lorentz transformation. One cannot overtake an ever-so-slightly superluminal neutrino. This follows from the Lorentz transformation, by which superluminal particles always remain superluminal under a transformation between subluminal reference frames. (iii) The tachyonic Dirac equation allows for plane-wave solutions, so that essentially nothing has to be altered in electroweak theory if we accept the fact that subluminal and superluminal particles may couple through Lorentz-invariant interactions. Note, in particular, that a massive, subluminal Dirac neutrino is excluded because of the autobahn paradox, but an infinitesimally superluminal neutrino is not. The neutrino can only be an infinitesimally superluminal spin-1/2 particle, or it must be a Majorana particle.

We have presented [10–12] the following alternative to the commonly assumption that the neutrino is a Majorana particle. (i) Neutrinos are superluminal but not as superluminal as recent false and retracted experimental claims. One has to measure the neutrino speed much more precisely in order to determine whether the neutrino is tachyonic, exceeding the accuracy of the 1979 Fermilab [43] and the 2007 MINOS experiments [44]. (ii) Neutrinos are described by the tachyonic Dirac equation, which follows from Eq. (8) by setting $m_1 = 0$ and keeping only the m_2 term (see Refs. [10–12, 45–47]). Mass eigenstates of the tachyonic Dirac field also constitute momentum eigenstates. (iii) The 'wrong' helicities are suppressed in the physical spectrum because of negative norm; it is not necessary to invoke a seesaw mechanism. (iv) The eigenspinors of the subluminal and superluminal Dirac equations fulfill simple and beautiful tensor sum rules, given here in Eqs. (7) and (13), and there is a meta-paradigm in physics that to every surprising, structurally simple mathematical relations, there might be a corresponding physical interpretation.

The possible existence of superluminal particles which travel faster than light raises a couple of physical, conceptual, as well as philosophical issues. No attempt has been made here to resolve those questions, but an account has been given of some arguments brought forward in the discussion. In particular, it is pointed out that if we assume that if the neutrino is superluminal, then the neutrino is not equal to its own antiparticle, and neutrinoless double beta decay is forbidden. While this conclusion somewhat depends on the precise equation proposed for the description of the neutrino, none of the currently proposed equations [10–12, 45–47] is charge conjugation invariant, so that there are no charge-conjugation-invariant Majorana solutions of the tachyonic spin-1/2 equations. We recall that experimental evidence for the observation of neutrinoless double beta decay [13–19] is disputed. We also recall a few more findings:

- If we accept a Lorentz-noninvariant vacuum and the Feinberg–Sudarshan [8, 9, 27–30] reinterpretation principle, then we can incorporate superluminal spin-1/2 particles into field theory without breaking Lorentz invariance and without questioning the special theory of relativity, while breaking one of the Osterwalder–Schrader [31] axioms.

- The Hamiltonians describing faster-than-light particles are pseudo-Hermitian and the precise analysis of their spectrum in mathematical terms may yield to further insight into this domain of physics, both on the level of quantum mechanics as well as on the level of field theory. The concept of pseudo–Hermiticity is as old as 1943, when it was introduced by Pauli in Ref. [32].

- Coincidentally, the only particle which is a candidate for superluminality, the neutrino, comes in only one helicity (left-handed), while antineutrinos come in right-handed helicity. This is precisely what our anticommutator relation (20) predicts, and postulating this anticommutator is the only possibility we found in order to complete the spinor sum in Eq. (13), where the prefactor $(-\sigma)$ is necessary in order to obtain the positive-energy and negative-energy projector. It is then not required to invoke the see-saw mechanism in order to assign large mass to heavy neutrinos of the 'wrong' helicity; this follows naturally under the above assumptions.

- No further alterations are necessary for the theory of weak interactions because Lorentz invariance is fully conserved. The tachyonic mass term could be determined using more precise experiments on neutrino flight times or by looking at the end point of the tritium beta decay experiments more accurately.

Experimental determinations of the neutrino mass square have typically resulted in negative expectation values, yet, compatible with zero within experimental error bounds [48–54]. Many experiments on the neutrino propagation velocity have resulted in best estimates with $v_\nu > c$, yet, compatible with $v = c$ within error bounds [43, 44]. Recently, conservatism has been held high by physicists as a guiding principle in directed and guided research, and has been advocated as a further guiding principle in clear hierarchies within the physics community. One might ask what is more conservative: (i) To abandon the cherished concept of 'lepton number', to assume that the neutrino is described by an equation that does not allow plane-wave solutions on the level of first quantization, and to assume that the neutrino masses are generated by a nonrenormalizable interaction with a concomitant hierarchy problem, suppressing just the right helicity component (it could have been the other [1]) or (ii) to assume that the neutrino is ever-so-slightly superluminal, to conserve the concept of 'lepton number', and to find a natural explanation for the observed helicity states of the neutrino, while allowing for plane-wave solutions and solving the 'autobahn paradox' in the most natural way possible, by fundamentally denying the CAGIVA driver the right to overtake the neutrino (in our gedanken experiment). Final clarification can only come from experiment, as already discussed. Let us mention another curious experimental possibility: If it were experimentally possible to trap neutrinos around a wire because of their magnetic moment (as with neutrons), this would conclusively imply that they must be subluminal and thus, Majorana particles. Independent of these questions, the above remarks and literature references may be of use for those attending the conference who study the philosophical and mathematical consequences of a few ideas brought forward into the realm of theoretical physics.

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Appendix A: Majorana Equation

Let $\omega(x)$ denote a two-component spinor amplitude. In the conventions of Ref. [1], where the chiral representation of the Dirac algebra is employed, the Majorana equation reads [see Eq. (6.25) of Ref. [1]]

$$\overline{\sigma}^\mu \, \partial_\mu \omega(x) + m \, \sigma^2 \, \omega^*(x) = 0 \,. \tag{A1}$$

Here, $\overline{\sigma}^\mu = (1, -\vec{\sigma})$. This is a two-component equation which clearly cannot be solve by a plane-wave ansatz because the first term would involve a factor $\exp(-\mathrm{i}\, k \cdot x)$, whereas the second one, which involves the complex conjugate $\omega^*(x)$, would go as $\exp(\mathrm{i}\, k \cdot x)$. Consequently, in the field operator of the Majorana particle, the plane-wave spinor wave functions multiplying the creation and annihilation operators in the Fourier decomposition are not plane-wave eigenstates of the Majorana equation. Let us consider the Fourier decomposition of $\omega(x)$ given in Eq. (6.43) of Ref. [1], which reads

$$\omega(x) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \, \frac{m}{E} \left(b_-(k) a_-(k) \, \mathrm{e}^{-\mathrm{i} k \cdot x} + \frac{m \, b_+(k)}{E + |\vec{k}|} \, a_+(k) \, \mathrm{e}^{-\mathrm{i} k \cdot x} + b_+^+(k) \, a_-(k) \, \mathrm{e}^{\mathrm{i} k \cdot x} + \frac{m \, b_-^+(k)}{E + |\vec{k}|} \, a_+(k) \, \mathrm{e}^{\mathrm{i} k \cdot x} \right) \tag{A2}$$

where $E = \sqrt{\vec{k}^2 + m^2}$ is the energy of the Majorana particle, and the helicity spinors have been defined in Eq. (5). [There is a typographical error in Ref. [1], the exponential in the last term should be $\exp(\mathrm{i} k \cdot x)$ as given above.] One cannot identify the Fourier amplitudes $b_-(k)$ and $b_-^+(k)$ as independent variables, and the spinors $a_-(k)\,\exp(-\mathrm{i}k\cdot x)$ and $[m/(E+|\vec{k}|)]\,a_+(k)\,\exp(\mathrm{i}k\cdot x)$ are not solutions of the Majorana equation. Again, this is because any wave function proportional to $\exp(-\mathrm{i}k\cdot x)$ cannot be an eigenfunction of the Majorana equation, which involves complex conjugation. Note that one can write down the Lagrangian of the Majorana field in a form reminiscent of the Dirac Lagrangian [see Eq. (6.23) of Ref. [1]], but there is a caveat: Because of the requirement of charge conjugation invariance, the four spinor components of the field effectively reduce to an equation for a two-component spinor, as written in Eq. (A1).

In Ref. [55], it is stated that the mass eigenstates of the Majorana neutrino are not plane waves but are of the form $\nu_L + \nu_L^c$, i.e., superpositions of left- and right-handed states [see Eq. (3) of Ref. [55]]. This is permissible if the neutrino is its own antiparticle. In this context, we also recall that the charge-conjugation invariant solutions of the massless Dirac equation,

$$\Psi_+(x) = u_+(\vec{k})\mathrm{e}^{-\mathrm{i}k\cdot x} + v_-(\vec{k})\mathrm{e}^{\mathrm{i}k\cdot x}\,, \qquad \mathcal{C}\overline{\Psi}_+(x) = \Psi_+(x)\,, \tag{A3}$$

and

$$\Psi_-(x) = u_-(\vec{k})\mathrm{e}^{-\mathrm{i}k\cdot x} + v_+(\vec{k})\mathrm{e}^{\mathrm{i}k\cdot x}\,, \qquad \mathcal{C}\overline{\Psi}_-(x) = \Psi_-(x)\,, \tag{A4}$$

are not plane waves with a definite four-momentum, but superpositions of plane waves which travel in opposite directions. Charge conjugation invariance holds because of the relation $C\,\overline{u}_\pm(\vec{k}) = v_\mp$ with $C = \mathrm{i}\,\gamma^2\,\gamma^0$ (in the standard representation). So an outgoing Majorana neutrino in a scattering process with a definite four-momentum cannot simultaneously be in an energy eigenstate of the Majorana equation. By contrast, all eigenspinors of the tachyonic Dirac equation denote energy and momentum eigenstates of the tachyonic Dirac Hamiltonian, in agreement with common wisdom for unperturbed (free) plane-wave electrons, positrons, heavy leptons, photons (plane-wave states $\vec{\epsilon}_{\vec{k}\lambda}\,\mathrm{e}^{-\mathrm{i}k\cdot x}$), heavy gauge bosons, and quarks.

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