The Epistemological Significance of Reducing the Relativity Theories to First-order Zermelo-Fraenkel Set Theory

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Introduction

There are three elements which I shall bring together in this paper. One is the project of Andréka, Madarász, Németi, Székely and others, represented in (Andréka *et. al*. 2002). The second is Molinini's philosophical work on the nature of mathematical explanations in science (Molinini 2011). The third is my pluralist approach to mathematics (Friend 2012a). I shall discuss each element separately in their corresponding section, and, in the last section, I shall put them all together by way of a conclusion.

In the first section I look at the project of Andréka, Madarász, Németi, Székely and others. The project is to reduce special and general relativity to Zermelo-Fraenkel set theory (henceforth, ZF). That is, they add some definitions and axioms to ZF and derive, in the language of set theory the "laws" of relativity theory.[1] But they do not only address the 'core' of the relativity theories (Andréka *et.al*. 2002, 8). They go beyond this, and derive many other results as well. We can think of these results as "theorems" or "predictions" of the relativity theories.

The second element and the second section concern the work of Molinini (2011). Molinini takes a pluralist approach to the notion of mathematical explanation for physical phenomena. What he means by 'pluralism' is that "What counts as a good explanation can vary from case to case, and we cannot design a single model able to capture all of these instances." (Molinini 2010, 16). What makes cases different are (a) the *intellectual tools* and (b) the *conceptual resources* provided by different mathematical theories. An intellectual tool is: "an ability to reason as used in the practice of explaining." (Molinini 2010, 352). A conceptual resource is "a concept which permits the use of our intellectual tools in a particular situation." (Molinini 2010, 352). Conceptual resources in mathematics give us mathematical concepts which allow us to analyse or see a physical situation in a certain way. We then use mathematics, as a tool to reason over that situation.

The third element concerns a pluralist philosophy of mathematics (Friend 2012). If we are pluralists in mathematics, then we do not accept that there is a foundation to mathematics in the sense of giving us the ontology, essence and absolute truths of all of mathematics. Rather, according to the pluralist, there are a number of mathematical theories which crosscheck each other. It is the crosschecking, along with the rigour of the proofs, which gives mathematics stability and objectivity – not in the metaphysical ontological sense, but in the sense of exercising cognitive command (Wright 1992, 92 - 3). What Wright means by this is

[1] For example, they derive the time dilation effect: that moving clocks slow down, for a 'stationary' observer. Similarly, they derive the length contraction effect: that moving objects contract in the direction of travel. From these they can then derive that clocks at the front and back of moving object, will get out of sync. (Andréka *et. al.* 2002, 90ff.

that we can reason well or poorly in mathematics. We can be corrected in our reasoning. This is a type of objectivity not grounded in ontology. Instead it is the objectivity which accompanies the notion of correction and error.[2] Central theories in mathematics, such as ZF, play a very prominent role in setting high standards of cognitive command, the reasoning is made plain, and we can easily correct error. More than that, ZF lends stability to the rest of mathematics through crosschecking, because mathematicians in their practice treat it as a basis for comparison. Wright refers to this as "width of cosmological role". This is another sort of objectivity which comes when elements of a discourse apply outside that discourse. ZF does this through crosschecking. What I mean by 'crosschecking' is that mathematical theories are often compared to ZF. Either they are reduced to ZF, or we have an equi-consistency proof between a theory and ZF, or we know what we would have to add to, or subtract from ZF in order to find ourselves in our new theory. In other words, the crosschecking might just be a question of making the relationship between a theory and ZF as explicit as possible. Crosschecking a theory with ZF is fairly standard practice in mathematics today.

I shall put the elements together in the fourth section. Roughly, the 'putting together' runs as follows. There is a philosophical literature on the topic of mathematical explanations for *physical phenomena*. The work of Andréka, Madarász, Németi, (2002) Székely (2012) and others does more than this. They give a mathematical explanation for whole *physical theories*, not just isolated phenomena. The interesting question concerns the *epistemological significance* of the explanation.[3] I use the account of Molinini to start the analysis and give solid sense of the epistemological significance as an explanation *of relativity theories*. However, I then deepen the analysis in section three to address the significance in terms of the particular *mathematical theory* they use. I assume a pluralist philosophy of mathematics. It turns out that the 'reduction' of relativity theories to ZF not only tells us something about the physical theory, it also tells us something about ZF, and its relationship to other areas of mathematics, and beyond this into philosophy.

1. The Relativity Theory Project

I draw on the work of Andréka, Madarász, Németi (2002) and Székely (2012) for the purposes of this paper. The first is a book of nearly 2,300 pages, based on a

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[2] Wright's definition of a discourse exhibiting cognitive command is as follows:

> A discourse exhibits Cognitive Command if and only if it is *a priori* that differences of opinion arising within it can be satisfactorily explained only in terms of the "divergent input" [different premises, axioms, definitions, assumptions], that is, the disputants' working on the basis of different information (and hence guilty of ignorance or error, depending on the status of that information), or "unsuitable conditions" (resulting in inattention or distraction, and so in inferential error, or oversight of data and so on), or "malfunction" (for example, prejudicial assessment of data, upwards or downwards, or dogma, or failings in other categories already listed). (Wright 1992, 92-3).

[3] A lot of the literature concerns the question whether or not there can be "genuinely" mathematical explanations for physical phenomena. In light of the logical relativity theory project, I take the answer to this question to be obvious.

course given at the University of Amsterdam in 1998. The two sources I use are part of a much larger project, involving other researchers, and many other papers. Call their work the 'logical relativity theory project'. The researchers involved in the logical relativity theory project give genuine mathematical explanations for physical phenomena and for two very important theories in physics: the theories of special and general relativity.

There are different ways of thinking about the project. One is chronological. Einstein, and later others, developed the two relativity theories. The idea was to explain space-time at the scale of the entire universe, and then explain phenomena on that scale. It turned out that Euclidean geometry was not a good model, that the speed of light was one of the few constants, that when particles, or objects, approach the speed of light, there are serious spatial and temporal "distortions" (with respect to our experience at a much smaller scale here on Earth), that the observer of phenomena plays a role in measuring, or noticing, the "distortions". Some "laws" were developed to reflect the nature of space-time and to predict the phenomena. The most famous law is $E = MC^2$. We were then able to predict, for example, the existence of black holes. We could then go about verifying the theory by looking for black holes, or, better, detecting them through their predicted effects on bodies which are relatively "close" to a black hole. One of the difficulties in obtaining empirical confirmation for the theory is that we are an observer, and we are not in a position to compare our observations with those of others (sufficiently "far" from us to make differences in measurement testable against the theory). The scale of the distances and objects in the theory are "too big", and our temporal and spatial limits are "too small". Nevertheless, we have obtained some empirical confirmation, and what we have discovered has helped us to correct the theory.

Regardless, there was always a *malaise* about the relativity theories, and this had to do with the predictions being different at this scale than at our limited human scale, where for example Euclidean space-time will do, and light can only travel in straight (Euclidean) lines. Thus, there was something we still did not understand about the relativity theories, and it was also not clear to everyone how the two fit together with each other.[4]

Confronting this *malaise*, some mathematicians and physicists proposed trying to give 'deeper' explanations. On the one hand, it was not clear how to do this. On the other hand, it is evident that the direction to go in, for a deeper explanation, is to look at the fundamental principles and try to explain them. How could we do this? Not through observation and prediction, but rather, in the other direction: we had to look at the mathematics. There is no other more fundamental (and acceptable) explanation.[5] Acting on a suggestion by Suppes, Ax and Goldblatt, worked on a first-order formulation of Minkowski space-time (Andréka *et. al.* 2002, 6), since Minkowski space-time sits at the heart of special relativity theory. The Ax

[4] 'Clarity' has two aspects, a phenomenological (private) aspect and a public communicative aspect. This is an important distinction because the claim is that some physicists might well report that for them the relationship between special and general relativity is quite clear (phenomenologically), but it was still not clear to everyone in the wider community. This indicates a lack of clarity in communication.

[5] An unacceptable explanation would be an unscientific one, such as a mystical explanation.

and Goldblatt project is limited in two directions. First they rest content with only deriving Minkowski space-time, and not developing the first-order theory in the direction of the *predictions* of special relativity theory, so they give no 'logical' explanation of the *phenomena* of special relativity, only of the pre-conditions for the phenomena (Andréka *et. al.*, 2002, 8). The second limitation is in the opposite direction. They do not *explain* Minkowski space-time. They take it as primitive, or as given. (Andréka *et. al.* 2002, 7). They *formulate* it; they do not *derive* it.

The logical relativity theory project develops the explanation of relativity theory in both directions. They give a logical explanation for the phenomena, and for Minkowski space-time. They use the first-order language of Zermelo-Fraenkel set theory (henceforth: ZF), and develop axioms in only that language, without adding constants and primitives from outside logic. Thus, their explanation is deeper (more fundamental) and reaches further. They even defend their choice of logic! (Andréka *et. al.* 2002, 1245 ff.) And they do this in the only way possible: through philosophical considerations and justifications. This is a great historic achievement in the development of our scientific concepts. This was the chronological way of thinking about the project.

There is another way of thinking about the project; and this is more philosophical. Andréka *et. al. reduce* the relativity theories *to* first-order ZF. The conceptual reverse of the reduction is that they give an *explanation* of the relativity theories *in terms of* ZF. This way of thinking about the project invites certain philosophical questions.

One question is: what is explained by what? Or, what is the *nature* of the reduction/ explanation? To examine the 'nature' we are after the philosophical characteristics of the *explicans* (the explanation) and the *explicandum* (the thing explained). The other question is: What is the *significance* of the explanation? To answer this question we want to discuss the philosophical context of the *explicans*. In section 2 I answer the first question. In section 3, I answer the second.

2. The Nature of the Logical Explanation of the Relativity Theories

Molinini joins Batterman and Pincock against Van Fraassen,[6] Friedman, Kitcher and Steiner in taking a pluralist view of mathematical explanations of physical phenomena (henceforth: MEPP). Molinini, Batterman and Pincock's pluralism contrasts with the more common monistic view held by, for example, (Friedman 1974), (Kitcher 1981), (Steiner 1978). The monistic view is that there has to be a single theory, or single model, of MEPPs.

The monistic view does not hold for two types of reason: one is that for every particular monistic theory of MEPP available in the literature, there are convincing counter-examples. The other concerns mathematical explanations within

[6] The reason Van Fraassen is counted amongst the monists in giving his theory of explanation is that all explanations, for him fit into one model - his 'why-question' account of explanation. (Molinini 2011, 16 n. 14). It does not affect the argument here, if he is counted amongst the pluralists.

mathematics. Within the philosophy of mathematics there is dispute as to what counts as a mathematical explanation (Mancosu 2011). In mathematics we have proofs, and a lot of explanation at the meta-level. The proofs themselves are not presented in a single way (to the chagrin of the formalists), so what counts as a proof is in dispute, and whether a proof is an explanation is also in dispute. The disputes do not look as though they will be quickly resolved.[7] These are not definitive reasons against a unified theory of mathematical explanation, but they are sufficiently strong to suggest we consider a pluralist approach to mathematical explanation of both physical phenomena, and of mathematics *simplicitur*. Thus, rather than defend Molinini's pluralism in explanation further, I shall take his guidance in looking at the logical relativity theory project, and show, *how*, and in what sense the relativity theory project is a genuine MEPP. In the following section, I turn to the second issue.

Batterman, Pincock and Molinini are pluralist in mathematical explanation in the following sense: "What counts as a good explanation can vary from case to case, and we cannot design a single model able to capture all of these instances." (Molinini 2010, 16). What distinguishes Molinini from Batterman and Pincock is that, for him, what makes cases different are (a) the *intellectual tools* and (b) the *conceptual resources* provided by different mathematical theories. An intellectual tool is: "an ability to reason while used in the practice of explaining." (Molinini 2010, 352). A conceptual resource is "a concept which permits the use of our intellectual tools in a particular situation." (Molinini 2010, 352).[8] Conceptual resources in mathematics give us mathematical concepts which allow us to analyse or see a physical situation in a certain way. We then use mathematics, as a tool to reason over that situation.

Both: intellectual tools and conceptual resources vary from one community of investigators to the next, and the factors which influence these are: the subjective preferences and aptitudes of individual members and the historical context of the community.[9] What makes *better* or *worse* intellectual tools and conceptual resources is indicated by their fruitfulness (Molinini 2011, 352). The issue of fruitfulness is especially delicate in mathematics, since we cannot very well count the number of theorems of a theory, as a measure of fruit.[10] Rather, fruitfulness is a qualitative measure. It depends on the centrality, importance, naturalness and spread of the theory. To detect fruitfulness, we can look to new applications or predictions, especially with scientific applications, but we can also look to mathematical

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[7] The relationship between MEPPs and mathematical explanations of mathematical concepts is interesting, especially in light of the logical relativity theory project. They are extensionally equivalent. The difference lies in the intention and this is only indicated in the meta-language. The intention is meted out by the particular application. We confirm the theory mathematical theory against physical data.

[8] Molinini defends his view against the pluralism of Batterman and Pincock, so I shall not reproduce his defense here.

[9] 'Historical context' covers many things, ranging from the ideas one is exposed to as cultural currency, to political systemic influences on our thinking.

[10] For any reasonably complex theory there is an infinite number, and, under usual assumptions about the languages of the theories there is the same infinite number of theorems for any pair of theories.

fruitfulness: that a mathematical technique, or result, is borrowed in a 'foreign' context, or is used to develop a mathematical theory.

Following Molinini, to answer the question of *epistemological significance*, we have to investigate the project along each of (a), (b) and look at the fruitfulness of the work, and all this in the socio-historical context of the work. It turns out that the significance in the case of the work of the logical relativity theory project is very high.

For example, take a fragment of the project. Székely proves the consistency of superluminal particles with the theory of special relativity. The conceptual resources in this case are those available to us through first-order ZF, and the reasoning, the intellectual tool, is impeccable logical reasoning, as we find it in this part of mathematics. One striking thing about this project is that the very question (about consistency) is a logician's question. This testifies to the use of the conceptual resource provided by set theory, namely, the very special types of questions asked by logicians. Moreover, Székely's question is not the only example. Throughout the project we see many instances of logician's questions being asked, "What are the weakest axioms needed to prove some prediction or result of the theory?" "What part of relativity theory is independent of the light constant?" "What is the logical relationship between special and general relativity?" "If we change one of the parameters on an axioms what else changes?" These are all logician's questions, so members of the project approach the relativity theories as both a branch of mathematics and as a physical theory. ZF provides tremendous conceptual resources to investigate such questions. It is also a perfect tool for explanation since the explanation is a logical explanation.

For example, to answer his question about consistency, Székely's gives a mathematical proof. The *significance* of the logical reasoning in the proof concerns ease of communication of ideas. Explanations in science are acts of communication. The more basic the concepts, the more people are able to understand them, and set theory and logic are conceptually more basic than physics. Thus, we can conclude that, under Molinini's conceptions of explanation in science, the logical relativity theory project is a perfect example of a mathematical explanation for physical phenomena. This is the start of the analysis, we now move on to the delicate matter of the philosophical significance of the mathematical theory when we are mathematical pluralists.

3. The Significance of Explanation in ZF for the Mathematical Pluralist

We should be careful in assessing the significance of the logical relativity theory project. Philosophers *usually* think of Zermelo-Fraenkel set theory as foundational to mathematics. Here, I understand 'foundation' to mean that the foundational theory gives the essence, the ontology and all the truths of mathematics.[11] I disagree with this. It is *central* to mathematics, but not in a *foundational* way. A pluralist, like me, believes that the evidence is not strong

[11] Not all *mathematicians* use the term 'foundation' in this way, but philosophers do.

enough to claim that *any* particular mathematical theory is a foundation. Instead, there are simply a number of encompassing theories, to which a lot of other mathematical theories can be reduced. (Friend 2012b) The encompassing theories can be used to measure and recognise each other. So, under this pluralist view, we might think that the logical relativity project is just one of reducing the theory to one, amongst many, encompassing mathematical theories.

This is not quite right, since the pluralist recognises the *centrality* of Zermelo-Fraenkel set theory to mathematics. This is why the pluralist's analysis of the significance of the relativity theory project is more interesting than the usual one. It might also be more accurate with respect to how mathematicians think of ZF.[12] For the pluralist, ZF acts, not as a foundation, but as a *lingua franca*, and as a measure of reliability of a theory. As a *lingua franca*, it is one of the best tools for communication and dissemination of ideas in mathematics. This is not because of its truth, it is a result of two things, the generality and power of the theory together with its popularity. The latter is a socio-historical fact. We can easily observe that ZF plays a central role in mathematics. The popularity and centrality re-enforce each other. ZF receives a lot of confirmation in the form of crosschecking against other theories. The confirmation gives the theory a sense of objectivity and robustness. This is because the theory exercises cognitive command, in Wright's (1992) sense of the term. That is, it is not a subjective, or individual, matter whether 3 is a subset of 8. We prove this, and proofs are public displays of reasoning. This makes ZF internally robust and objective, that is, each theorem follows from the axioms. The pluralist can recognise more than this.

The reliability, objectivity or robustness, *of ZF* itself is tested by reductions to, and crosschecking against other independent theories, through equi-consistency proofs, for example. These are an 'external' check on reliability, objectivity and robustness. These checks are also types of explanation. They are re-conceptualisations. When we can perceive the same object or theory from different angles, through the lens of different theories, we have another type of objectivity, what Wright calls 'wide cosmological role'. For the pluralist, ZF reaches out to all of mathematics, even though not all mathematics is reducible to ZF! One of the advantages of the pluralist position lies in exactly being able to draw this distinction between reducing theory and a central theory. The logical relativity theory takes advantage of this pluralist attitude. We shall see this in the concluding section.

Before turning to the conclusion, let me introduce another aspect of mathematical pluralism. Communication and explanation goes in two directions. The reduction of relativity theories to ZF further confirms the reliability of ZF set theory.[13] Someone learning relativity theory should be interested in learning ZF as well. As far as I know there are not yet any 'purely' mathematical results which have come out of the logical relativity theory project: ones that tell us something new about other areas of *mathematics*, but it is really a matter of time and focus before

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[12] Of course, this depends on the mathematician one talks to (Friend 2012, 35ff.)

[13] For the purposes here, applications are the reverse of reductions. The intention is different, but the technical result is extensionally the same.

such results are obtained. When this happens, we shall have genuine two-way confirmation.

Conclusion

The philosophical literature on the topic of mathematical explanations for physical phenomena has not (yet) engaged the logical relativity theory project. Instead the contributors rest content with scientific phenomena in which the explanation seems to be indispensable. But said literature *should* engage this project because Andréka, Madarász, Németi, (2002) Székely (2012) give a mathematical explanation for whole *physical theories*, not just isolated phenomena. Once we bring the logical relativity theory project to the attention of those who write on MEPPs, the interesting question concerns the *epistemological significance* of the explanation.

Using Molinini's analysis of MEPPs, we discover that the conceptual resource provided by ZF concerns the sorts of questions being asked. The questions are logician's questions. Moreover, the explanations are mathematical proofs, which, in principle, most mathematicians and logicians can understand. Thus, the explanations reach beyond the scientific community out to the mathematical community directly.[14] Since the explanations take the form of proofs together with a more intuitive discussion in the meta-language, we have both the intuitive element, and the precision of mathematical proof together in the explanations. The intuitive discussion is to remind us of what it is we are explaining through the mathematical proofs. The intuitions test the mathematics; the mathematics *explains* the intuitions.

Focusing on the mathematical proofs and the language used in the derivations of the predictions, we find that ZF is a very interesting choice of mathematical theory, and it is more interesting if we are mathematical pluralists. For a pluralist, ZF is central to mathematics, but not foundational. This allows us to have a more nuanced and sensitive analysis of the significance of ZF.

Why does this pluralist approach to foundations in mathematics make the case for the significance of the logical relativity theory project more *interesting* than if we were foundationalists about mathematics? One reason is that because ZF is a *lingua franca*, and because it is tested against other theories, the spread of the results of the logical relativity theory project includes all of mathematics, as it is practiced today (beyond the bounds of ZF). So the conceptual resource includes all of mathematics, but through the lens of the language of ZF. This is why, in carrying out the logical relativity theory project, it was conceivable to also use a non-well-founded set theory to produce some results. The excursus outside ZF is only reasonable under a pluralist conception of mathematics, not under a foundationalist conception.

The second reason is that ZF is a perfect tool for reasoning. This makes the logical relativity theory project objective, not in the metaphysically absolute sense, but in the sense that the project is subject to *logical* correction. This is related to

[14] In fact, Andréka and Neméti were asked to give a plenary lecture at the Association of Symbolic Logic, Logic Colloquium in 2009 in Sofia. This is the largest logician's meeting in Europe.

three ideas in the literature, one is Wright's ideas of 'cognitive command' (we have to follow the reasoning) and 'width of cosmological role' (that the ideas reach beyond the particular theory). These senses of objectivity are re-enforced by the reduction of the relativity theories to ZF, and further re-enforcement will be completed when the project begins to give us purely mathematical results, and I think this is a matter of time. The third is the idea of error and logical justification. Let me expand on this a little.

The rigour of a justification should match our degree of scepticism. That is, we do not normally need to prove anything. We only need to give a proof, or a part of a proof, when there is the suspicion of error (Sundholm 2012). While some physicists will claim that they are quite comfortable with the relativity theories, and they convince their students to feel the same way (or leave the course of study); outside these circles, there is a more general *malaise* about the theories. This *malaise* is a call for further and deeper justification. It is not enough to end a justification for the relativity theories with stipulations of physical constants and laws. This deeper, and more basic, explanation is provided by the logical relativity theory project. But this is not all.

For a pluralist, like me, even logical justification in the form of a deductive proof is not an end. It is an invitation to investigate further and more deeply. This further investigation is already being carried out in the logical relativity theory project when they argue for their choice of mathematical theory (Andréka *et. al.* 2002, 1245ff). Logical justification is an invitation from the highest and purest ranks of thinking, where only our reasoning can guide us. This is especially important when we cannot *rely* on 'feel' or 'intuitions' about the original physical theory. History, and experience tells us that relativity theories, or theories about large-scale space, time and matter have proved elusive to our intuitions (*pace* Kant).[15] Nevertheless, we have several resources for carrying out this investigation, but these are logical, and sometimes metaphysical or aesthetic, resources. For this reason, the tool for reasoning, ZF, reaches further than the mathematical theory. It invites an interplay between logic and philosophy, and we only develop our understanding of the relativity theories by extending our explanations and justification, not only in the direction of mathematics and logic, but beyond these to philosophy.

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[15] Kant makes a clever philosophical move. He makes it a conceptual matter that we reason about space in a Euclidean way! So that we rely on intuition is just in the nature of spatial reasoning. The justification is logical and metaphysical. As we know, it turned out that Euclidean geometry is not well adapted for our theories about space and time on either a much smaller, or a much larger scale.

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