Farewell to causality?

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Paper submitted to the
First International Conference on Logic and Relativity:
honoring István Németi's 70th birthday
September 8 - 12, 2012, Budapest

Abstract

Foundations of QED were elaborated (Dirac, 1928, 1929, 1951, 1962) with the precondition that the theory should be causal. Causality meant that „... the wave function at any time determines the wave function at any later time”. The latter, simultaneously with the requirement of Lorentz invariance of the theory, involved the requirement of invariance under time-reflection. A few years following the first publication of the theory, the first paradoxes were made themselves apparent, followed later by others (EPR, Aharonov-Bohm, Bell). Since causality was a prerequisite of the theory, causal paradoxes could not be explained in the framework of QED (Darvas, 2009).

Causality in the above sense works in flat space-time. This is not the case in real non-classical physical situations. Theories, like GTR, QED, assume non-Euclidean geometry. Invariance of the infinitesimal arc-length under reflection is ambiguous in curved spaces. It is the case already in constant curvature spaces, it holds more strongly in Riemannian geometry, and gets much more apparent in Finsler geometries, where the curvature changes not only point by point, but also according to direction in each point. One cannot disregard even the last cases in gauge theories. When reflection of a segment is ambiguous, unambiguity of reflection gets damaged. So does causality.

Keywords: causality, non-Euclidean geometries, invariance under reflection, ambiguity of reflection

Introduction

(Darvas, 2009) explicated that causal paradoxes of QM cannot be explained in the framework of QED. The arguments included that causality was a precondition of QED (logical paradox), ambiguity of parallels in curved spaces (mathematical background), and violation of causality by reflection of the infinitesimal arc-length, what is to be reflected into a bundle of infinitesimal arc-lengths in the future cone (invariance paradox).

1 Reflection in curved spaces

Let us consider a point *P* and a line segment *AB* along a straight line *m* not containing *P*. Given a point *Q* on *AB*, draw a line *x* parallel to *m* through *P*, and determine the mirror image of *Q* reflected in *x*. In Euclidean geometry the mirror image of *Q* will be a well defined point *Q’*. This holds for all inner points of the segment *AB*. So the mirror image of a line segment *AB* (the set of all points *Q*) will be an unambigously defined single line segment *A’B’* (the set of *Q’*).

This is not the case in spaces with a non-Euclidean geometry. Non-Euclidean geometries can describe spaces with a constant curvature – this is the simplest case, but with few physical relevance –; spaces with curvatures changing point by point (Riemann geometry) – this is the case described by semiclassical physical theories –; and spaces with curvature changing in every point and in each point according to directions (Finsler geometries) – this is the most complicated case applied in modern field-theories (e.g., Darvas, 2012).

Nevertheless, all the latter geometries possess the common feature that the mirror image of a line segment reflected in a point outside its line is a boundle of line segments. The geometry of this bundle may vary according to the geometry of the given space, but the ambiguity of the reflection operation is common for all.

Existence of mirror symmetry was a so strong assumption of Euclidean geometry, what went without saying, that it was even not included among the postulates. The symmetry properties of curved spaces have not been studied in due measure in geometry, although modern physical theories deserved to pay more attention to them.

2 The logical paradox

Deformation of our apprehension on causality in the logic of physical systems (first of all in gauge theories) deserve also deeper studies. We can say that causality is distorted, but one needs to learn more than simply state that damage.

In order to answer the question why can't causal paradoxes of QM be explained in the framework of QED first let me state that the framework in which a paradox can be explained, and the framework in which it has its origin may be different. The roots of the reasons lay in the ontology, while the possibility (or that's absence) to explain them belongs to the domain of epistemology. My intention is not "to explain" the specified causal paradoxes. But my intention is neither "to identify their reasons". I intend to investigate "*the possibility* of the explanation", or in other words whether there "can" be given an explanation in the given framework. This goal is wider than to find the reasons, but narrower than to explain.

Paradoxes raise questions that cannot be answered in a given contextual framework, but they may be answered in other frameworks. Previous attempts to explain causal paradoxes of QM have tried to solve the problems within the framework of QED. (The literature tackling the theme is very wide. Similar to my earlier paper in this theme [2009] I refer only to a few papers that give, at least partially, a state of the art overview of the concerned literature, cf., for example, papers by Fine and Szabó, 1991-2008.) As regards concrete paradoxes, I mean first of all the Einstein-Podolsky-Rosen- (EPR-), Bell-, and Aharonov-Bohm-paradoxes (Einstein, Podolsky, Rosen, 1935; Bell, 1967; Bohm, Aharonov, 1957). There are treated apparent problems of causality and coincidences to be explained by the assumption of hidden variables (or in other ways).

In the preamble of the original theory of QED, Dirac (1928) formulated four preliminary requirements that the new theory should meet.

> The first of these requiremets was that (1) the theory must be *causal*. In proposing a quantum theory of the electron (overcoming the problems left by the Klein-Gordon interpretation), he formulated in § 1 (p. 612): „We should expect the interpretation of the relativity theory to be just as general as that of the non-relativity theory. The

general interpretation of non-relativity quantum mechanics is based on the transformation theory, …, so that the wave function at any time determines the wave function at any later time." I will refer to this statement as the *causality precondition*. I must notice that satisfying the causality condition is interpreted also that there cannot occur closed (or semi-closed) time-like curves in the described system.

I mention, for the sake of completeness, that the three other preliminary requirements to formulate Dirac's QED were the following: (2) *charge conjugation*: „The true relativity wave equation should … be such, that its solutions split up into two non-combining sets, referring respectively to the charge –*e* and the charge *e*." (p. 612). (3) *Lorentz invariance*: „Our problem is to obtain a wave equation … which shall be invariant under a Lorentz transformation …" (p. 613). (4) *homogeneity of the empty space*: „… all points in space are equivalent," (p. 613).

Let me add another preliminary notice, which gains special importance in respect of the conclusion of this paper. The simultaneous requirements of causality and of Lorentz invariance of the theory involve the requirement of *invariance under time-reflection*. (In a more rigorous formulation of the conditions, the four requirements together demand *CPT* invariance. In this paper I will restrict myself to the requirement of time-reflection invariance.)

Since the listed conditions are formulated as *preconditions* for constructing the theory, they *cannot be* treated or derived as *consequences* of the resulting theory. This holds first of all for causality. Since causality was a precondition of the formulation of the theory, that is, *the theory has been constructed so that it be causal*, therefore, causal paradoxes logically cannot be explained within the framework of QED. If a phenomenon violates causality, the reasons (and explanations) for it should be sought outside QED.

To transcend this problem we should consider the following four points:

(a) Dirac himself stated in his original paper (1928) that his theory is only an approximation and that it does not give an answer to all questions. After listing the problems left open by the Klein-Gordon theory and to be solved, he closed the introduction to his paper with these words: „The resulting theory is therefore still only an approximation, …" (p. 612). A year later he repeated: „Further progress lies in the direction of making our equations invariant under wider and still wider transformations." (Dirac, 1930)

(b) When he returned to improve the theory later (Dirac, 1951), he noted that the new theory „involves only the ratio *e/m*, not *e* and *m* separately" (p. 296). This is a sign that, although the electromagnetic effects (whose source is *e*) are magnitudes stronger than the gravitational effects (whose source is *m*), they are coupled.

(c) A decade before Dirac's (1928, 1929) first QED theory, Einstein (1919) had already noted that „the elementary formations which go to make up the atom" are influenced by gravitational fields (introductory paragraph). Although in that form the statement proved not to be exactly correct (Einstein's approximation of the extent of the effect of the gravitation on the electromagnetic processes [§ 3] could be questioned later), the effects of the gravitation on QM phenomena have been established. He applied first the field equations elaborated for the GTR and processes in which gravitation plays a role to the „Maxwell-Lorentz theory" of the electron, as he called it. According to him, „in regions where only electrical and gravitational fields are present" (§ 2), the electromagnetic and gravitational processes are coupled in the presence of a curvature tensor.

(d) In his latest extension to the theory of the electron, Dirac (1962) applied a space-time dependent metric to the electromagnetic field (p. 58 and on), and a Finsler-like geometry (p. 62).

So, we take into consideration the curvature of an elctromagnetic field as introduced by Einstein and then Dirac. This approach is motivated by our goal, namely to investigate the conditions of invariance under time-reflection, for – as we saw – the simultaneous requirements of causality and of Lorentz invariance of the theory involved to demand that invariance.

3. Possible clues for the violation of causality

Choose an arbitrary space-time point *P*, and an infinitesimal arc-length d*s* near to it in its past cone. Reflecting a past d*s* through *P* – as through the origin of the time-like cones in the future *in STR* – the reflection of d*s* will be a definite infinitesimal arc-length d*s*' in the future cone, which conserves its square length. The reflection is one-to-one unambiguous from the future to the past, too.

The reflection (Figure 1) will be ambiguous *in GTR* and *QED*, etc., which assume curvature in the space-time.

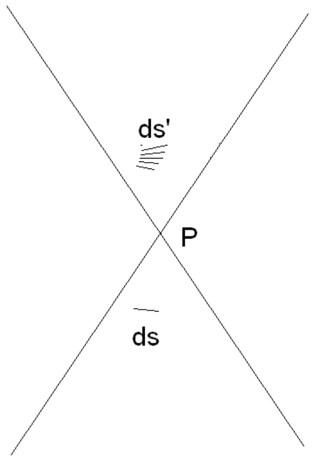

Figure 1

In these cases, *P* will represent a point, in which all $t = t_P$ lines intersect. In curved, that is non-Euclidean, geometries – as it has been shown in Section 1 – the reflection of a past time-like infinitesimal arc-length d*s* to the future (and vice versa) will form a bundle of arc-lengths, that conserve the square length of the original, but provide no unambiguous reflection image. Reflection symmetry has been violated; that is, causality is lost.

The consequence is that *causal paradoxes* (as specified above), which cannot be explained in STR (due to the starting logical considerations), *cannot be resolved in the framework of GTR due to the violation of causality*, either.

One can treat this problem as a consequence either of hidden variables, or of the probabilistic character of the realisation of one of the possible reflections in curved spaces (or even further: to apply space-times with different geometries, compared to those that had been introduced for GTR), nevertheless none of them can elude the question to leave behind the frameworks of QED.

There are two options. One can either construct a QED without the listed preliminary requirements and remain in the domain of STR. This seems less probable. Or, one can accept that causal paradoxes in QM are no longer „paradoxes" but normal phenomena in nature; that is, in the logical framework where they can be interpreted in the domain of GTR, causality really does not work, at least in its traditional sense.[1] We have to give up causality in both cases.

More precisely, causality works in one direction. Either it works from the past to the future so that one cause involves many effects, and it does not work in the opposite direction. Or it works from the future to the past so that one effect could be brought about by many causes, but this relation also cannot be reflected.

The described clue allows several interpretations, both in philosophy and in physics. These possible interpretations require us to reconsider our approach to causality.

Such possible reconsiderations of causality occur, for example, in respect to the interperetation of local theories. There appear in the literature several, sometimes contradicting approaches, sometimes not yet completely elaborated theories, both by physicists and by philosophers. The consistency of these approaches to causality is debated, or at least can be questioned. Discussion of those would go beyond, and is not the task of this paper.

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[1] Here I refer to late works by Dirac. Time reflection is a symmetry transformation. Dirac formulated this in his first requirement for „the relativity theory" to be applied in his original QED. Relation of symmetries, equivalence classes and transformation groups to each other, in the light of Dirac's theory, has been carefully studied by E. Castellani (2004). She studies the „irrelevant elements" in the theory in Section 2, where she analyzes the clue expressed by Dirac: „physical symmetries are related to the presence of irrelevant elements in the physical description." … „ Symmetries are … connected with the presence of non observable quantities in the physical description." (p. 1506). Multiple parallels belong to this category, since there is only one (arbitrary parallel) among them which can be observed in a concrete physical situation. Her analyzis resulted in the statement, that Dirac's later (1950s) theory of constrained systems started either from the above mentioned (physical) indeterminism, or from the mathematical representation, namely the choice of unphysical degrees of freedom giving rise to arbitrary functions of time (p. 1508).

As she quotes from the mature theory of Dirac (1964, p. 17): „…arbitrary functions of the time must mean that we are using a mathematical framework containing arbitrary features," (…) „As a result of this arbitrariness in the mathematical framework, the dynamical variables at future times are not completely determined by the initial dynamical variables, and this shows itself up through arbitrary functions appearing in the general solution." (Cf. this with his preliminary requirements formulated in 1929.) Castellani (2004) concludes, that all first-class constraints are generators of gauge transformations. In this light one can put the question, whether all gauge transformations are related with 'scale choice arbitrariness', or some of them originate in 'surplus structure arbitrariness' (p. 1507)? Seemingly, some do. Anyway, the elements of indeterminism appearing in the theory cannot be questioned in any of the two cases. Causal paradoxes, at least in this sense, cease to exist as paradoxes.

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