Understanding Gauge

James Owen Weatherall

Note: This abstract is being submitted along with two related papers: one by Thomas Barrett (Princeton -- presenter) and Hans Halvorson (Princeton); and another by Sarita Rosenstock (Irvine -- presenter), Thomas Barrett, and myself. Our hope is to present the three papers together, perhaps with other related work if appropriate, as part of a symposium session.

A central theme in philosophy of physics has concerned how to understand the structure associated with or posited by a physical theory. For instance, Stein (1965) and Earman (1989) have shown how debates concerning substantivalism and relationism in Newtonian gravitation can be recast as a question of whether a classical spacetime is endowed with certain geometrical structure. Similarly, North (2009), Curiel (2013), and Barrett (2013) have debated whether one should prefer a Hamiltonian or a Lagrangian formulation of classical mechanics, in part on the basis of which formulation postulates more or less structure. One can understand recent debates concerning interpretations of Yang-Mills theory (cf. Belot [1998, 2003], Healey [2007], Arntzenius [2012]) along similar lines.

Discussions of structure are especially salient in the context of so-called "gauge theories," which are often described as positing *excess* structure, beyond what is needed to represent a physical situation; this excess structure is associated with a class of "gauge transformations" between apparently distinct models of the theory that are believed to represent identical physical configurations. The archetypal example of a gauge theory is classical electromagnetism, but many of our best physical theories—including general relativity, Newtonian gravitation, and Yang-Mills theory—have also been described as gauge theories. Yet the very notion of gauge presents significant conceptual puzzles: What does it mean to say that some structure is unnecessary or "excess"? Can we formulate gauge theories in a way that eliminates excess structure, and if so, what is the significance of such re-formulations?

Recent work by Halvorson, Barrett, and Weatherall (much of which will be presented in this symposium) has attempted to develop new formal tools for studying scientific theories. One application of these tools is in comparing the structure posited by scientific theories. Here I will apply these tools to the questions of what makes mathematical structure "excess" structure, and to what the relationship is between a formulation of a theory with excess structure and a formulation without that structure.

The starting point for the analysis will be two representations of electromagnetism as categories of models, one that explicitly mentions the "gauge structure" and one that is "gauge invariant" in the sense that the purportedly excess structure never appears. It will turn out that these theories are *not* equivalent as categories—indeed, the first posits "more structure"—unless one supplements the first with additional arrows, corresponding to the gauge transformations described above. I take this to be a way of recovering the standard way of speaking about gauge. I will then show that Newtonian gravitation is a gauge theory in the same sense, but that general relativity is not. I will conclude by turning to Yang-Mills theory, a generalization of classical electromagnetism that has recently vexed philosophers. Though Yang-Mills theory is often cited as *the* standard example of a gauge theory, I will argue that neither of the formulations standardly discussed by philosophers are gauge theories in the sense that electromagnetism and Newtonian gravitation are.

References

Arntzenius, Frank (2012). *Space, Time, and Stuff*. New York: Oxford University Press.

Barrett, Thomas (2013). "On the structure of classical mechanics." Forthcoming in *The British Journal for the Philosophy of Science*.

Belot, Gordon (1998). "Understanding Electromagnetism." *The British Journal for the Philosophy of Science* **48**(4), 531-555.

Belot, Gordon (2003). "Symmetry and Gauge Freedom." *Studies in History and Philosophy of Modern Physics* **34**(2), 189-225.

Curiel, Erik (2013). "Classical Mechanics is Lagrangian; It is Not Hamiltonian." Forthcoming in *The British Journal for the Philosophy of Science*.

Earman, John (1989). *World Enough and Spacetime*. Cambridge, MA: MIT Press

Healey, Richard (2007). *Gauging What's Real*. New York: Oxford University Press.

North, Jill (2009). "The 'Structure' of Physics: A Case Study." *Journal of Philosophy* **106**(2), 57-88.

Stein, Howard (1965). "Newtonian Space-Time." *The Texas Quarterly* **10**, 174-200.