On Einstein Algebras and Relativistic Spacetimes

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Note: This abstract is being submitted along with two related papers: one by Thomas Barrett (Princeton – presenter) and Hans Halvorson (Princeton); and another by James Owen Weatherall (Irvine). Our hope is to present the three papers together, perhaps with other related work if appropriate, as part of a symposium session.

John Earman has argued that certain algebraic structures—what Geroch (1972) calls *Einstein algebras*—might provide the mathematical setting for an appropriately "relationist" approach to general relativity (Earman, 1979). The motivation for this proposal was the hole argument, as redeployed by Stachel (1989) and Earman and Norton (1987). It seems Earman hoped (indeed, argued) that a single Einstein algebra would correspond to an equivalence class of relativistic spacetimes related by the isometries arising in the hole argument. One might then take the Einstein algebra formalism to do away with "excess structure" appearing in the formalism of relativistic spacetimes—that is, the formalism of manifolds and tensor fields—by equivocating between what Earman and Norton described as "Leibniz equivalent" spacetimes. This program ran aground, however, when Rynasiewicz (1992) observed that one can recover precisely the isometries used in the hole argument as isomorphisms between Einstein algebras. Thus—whatever one thinks of the merits of Earman's program in the first place—it seemed Einstein algebras could not do the work Earman hoped.

We think Rynasiewicz had this essentially right. But there is more to say about the relationship between Einstein algebras and relativistic spacetimes. There is a sense in which these are equivalent theories, according to a standard of equivalence recently discussed by Barrett and Halvorson (2015). More precisely we will show that, on natural definitions of the appropriate categories, the category of Einstein algebras is dual to the category of relativistic spacetimes, where the functor realizing the equivalence preserves the empirical structure of the theories.

One might worry that this result is little more than a repackaging of Rynasiewicz's point in the language of categories and functors—and in a sense, this is right.[1] Still, we take it that the point is worth making, for two reasons. For one, there has been a flurry of recent work on what it means to say that two theories are equivalent. Here we provide a novel example of two theories that are equivalent by one, but not all, of the standards of equivalence on the table. For instance, it is hard to see how these theories could be equivalent by Glymour's criterion of equivalence, since it is not clear how to make sense of "covariant definability" when one of the theories is not formulated in terms of fields

[1] That said, we do add a bit to Ryansiewicz' discussion insofar as he limited himself to topological spaces and algebras of continuous functions, and simply stipulated that similar remarks applied in the case of real interest, of Lorentzian manifolds; our results fulfill the promisory note.

on a manifold. The example is of particular interest in this regard because it differs from others that have been explored by, for instance, Weatherall (2014) and Rosenstock and Weatherall (2015), in two ways. For one, the categories in question are dual, rather than equivalent. And for another, the apparent differences between the theories in question are of a different character from other examples in the literature, which may be of probative value as we try to evaluate the merits of the criterion of equivalence.

The second reason the point is worth making is closely related to this last issue. As Earman has argued, the Einstein algebraic formalism does seem to capture a relationist intuition about how spacetime structure relates to possible configurations of matter. Implicit in such arguments—and explicit in Earman and Norton (1987) and Earman (1979)—is the idea that the standard formalism of relativistic spacetimes is somehow "substantivalist". Still others, such as Pooley (2010), have argued that nothing in the physics turns on the relationism/substantivalism debate. The results we present might then be taken as a way of making this idea precise, by showing that in a robust sense, the "relationist" theory, once one spells it out in sufficient detail, is equivalent to the "substantivalist" theory, properly construed. In particular, the sense of equivalence we consider here is often taken as a standard for determining when two mathematical theories have the same structure. And so, the equivalence we discuss suggests that a relationist theory of general relativity, spelled out in these terms, has precisely as much structure as the standard theory.

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