# Relativity theory via a network of logic theories

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We investigate relativity theory (special, general, cosmological) in form of a category of first-order logic (FOL) theories as objects and interpretations between them as morphisms. The common aspect in these theories is that they all concern relativity theory with different emphasis, different details, different aspects, different formalisms, different resolutions. This is a pluralistic approach in the sense of [1].

Example 1. Take two axiom systems for special relativity, the first one MinkTh describing Minkowskian metric in the spirit of [2] while the second one SpecRel based on the constancy of the speed of light [3]. These FOL theories are definitionally equivalent in the sense of mathematical logic, i.e., they express the same physical content via "different words" and a theory-morphism (interpretation) specifies a precise translation between the two formalisms. MinkTh is well-suited for making computations, but it is not so well-suited for answering the why-type questions, for teaching the theory to non-physicists, for giving insights towards possible modifications of the theory. On the other hand, SpecRel is well-suited for all these latter, and the computational aspects are available in it via the theory-morphism that connects it with MinkTh. The explanation and understanding versus calculation dilemma can be resolved by using this theory morphism.

Example 2. SpecRel of [3] is coordinate system-, or reference frame-oriented, while SigTh of [4] uses meager resources, it talks only about experimenters sending and receiving signals, even quantities for measuring are not available in it. SigTh focuses on what the experimenters can experience, i.e., on "sensory data/impressions", on outcomes of experiments, on "detectation" while SpecRel talks about how things are there in "reality", using highly abstract notions such as quantities (real numbers). The two theories use disjoint languages and talk about different kinds of entities. Yet, a precise comparison is possible by using theory morphisms, see [5] . A byproduct of establishing these theory-morphisms is a concrete operational semantics for special relativity theory, e.g., concrete algorithms for how an observer of SpecRel can set up his/her reference frame. Looking at these morphisms from the other direction, we get tools for investigating the emergence and ontological statuses of abstract mathematical concepts such as, e.g., the real numbers in rather experiment-oriented theories such as SigTh.

Similar investigations can be done for the general and the cosmological theories of relativity.

Richard Feynman said in his Nobel laureate inaugural lecture: "Theories of the known, which are described by different physical ideas may be equivalent in all their predictions and are hence scientifically indistinguishable. However, they are not psychologically identical when trying to move from that base into the unknown. For different views suggest different kinds of modifications which might be made and hence are not equivalent in the hypotheses one generates from them in one's attempt to understand what is not yet understood. I, therefore, think that a good theoretical physicist today might find it useful to have a wide range of physical viewpoints and mathematical expressions of the same theory (for example, of quantum electrodynamics) available to him."

We believe that interpreting one theory in another, i.e., investigating the network of FOL theories as opposed to investigating particular/fixed FOL theories, is a flexible methodology for connecting physical theories with each other and with "physical reality". See [6,7].

**References**

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