Mass and Energy in Special Relativistic Dynamics

Abstract

This paper considers the special relativistic relationship between mass and energy as embodied in Einstein's famous equation $E = mc^2$. I argue that the most natural candidates for understanding this relationship are inadequate, and thus that important conceptual and philosophical work on the dynamics of special relativity remains to be done. This paper forms part of a larger project concerning the interpretation and ontology of special relativity.

1 Introduction

How should we understand the special-relativistic relationship between mass and energy, canonically representation in Einstein's famous equation $E = mc^2$? Traditionally, Einstein's equation has been interpreted as expressing the *equivalence* of mass and energy—that they are fundamentally *the same thing* and that one can, at least in certain circumstances, be *converted* into the other. Thus, for example, Taylor and Wheeler (1966)[1] write that

> [J]oules and kilograms are two units–different only because of historical accident–for one and the same kind of quantity, mass-energy.... The conversion factor c^2, like the factor of conversion from ...miles to feet, can today be counted, if one wishes, as a detail of convention, rather than as a deep new principle. (p.137)

But this understanding of Einstein's equation is not universally accepted, and remains a topic of dispute even amongst physicists (see, e.g., Bondi and Spurgin (1987)[2]). Recently, the philosopher Marc Lange has proposed a new and radical interpretation of Einstein's equation—one that jettisons any equivalence between mass and energy, and moreover denies that the so-called 'conversion' between mass and energy is anything more than an

[1] E. Taylor and J. A. Wheeler. 1992. *Spacetime Physics*. New York: Freeman.

[2] H. Bondi and C. B. Spurgin. 1987. "Energy has mass". *Physics Bulletin*. **38**: 62–63.

artifact of the perspective that we use to describe a system.[3] While I'm sympathetic to many of Lange's criticisms against existing interpretations, Lange's own proposal for understanding $E = mc^2$ is entirely inadequate. My aim in this paper is to substantiate this charge. This short paper forms part of a broader project aimed at rethinking the interpretation and ontology of special relativity.

2 A Clarification

We must start by clarifying exactly which equation is up for interpretation. We know of course that the dynamics of special relativity defines the *relativistic energy* as $E = \gamma m_0 c^2$, where m_0 is the so-called rest mass of a particle and γ is the Lorentz factor. Many textbooks then go on to define the *relativistic mass* $m_r = \gamma m_0$, in which case the relativistic energy can be expressed as $E = m_r c^2$. But this is not the equation whose content we aim to interpret! After all, that the energy of a particle is a function of its mass is as true in classical physics as it is in special relativity, yet we do not infer from the classical kinetic energy expression $T = \frac{1}{2}mv^2$ that mass and kinetic energy are somehow identical and interconvertible. What makes the special relativistic situation different? The answer lies in the fact that the relativistic energy expression can be rewritten as

$$E = \gamma m_0 c^2 = m_0 c^2 + \frac{1}{2}mv^2 + ...$$

with the relativistic kinetic energy given by

$$T_r = (\gamma - 1)mc^2.$$

The relativistic energy can thus be simply expressed as

$$E = m_0 c^2 + T_r = E_0 + T_r.$$

What the first term suggests, in complete variance with classical dynamics, is that a body possesses energy simply in virtue of having mass, independently of its motion. (In a nod to the most prevalent understanding of the mass-energy relationship, the relativistic energy E is often called the *mass-energy* of a system—see, e.g., Faraoni (2013), p.164.[4]) The form of

[3] M. Lange. 2001. "The Most Famous Equation". *Journal of Philosophy.* **98**: 219–238.
[4] V. Faraoni. 2013. *Special Relativity.* Dordrecht: Springer.

Einstein's equation in need of interpretation is thus the form concerning an object's "rest mass":

$$E_0 = m_0 c^2.$$

This is the form of Einstein's equation that has suggested to many physicists that energy and mass are at root identical and interconvertible.

3 Seven Interpretations

To situate the discussion, it will be helpful to outline briefly several possible interpretations of Einstein's equation $E_0 = m_0 c^2$.

Option One: Mass and energy are fundamentally a single underlying thing–'mass-energy'–that can be manifestly differently in different contexts. Einstein's equation expresses how those manifestations are related.

Option Two: Mass and energy are distinct fundamental features of the world, but are of a common ontological category and are empirically related according to Einstein's equation.

Option Three: Mass and energy are distinct fundamental features of the world and occupy distinct ontological categories, but energy *possesses* mass in the sense of having mass as a property. The magnitude of that property is given by Einstein's equation.

Option Four: Mass and energy are distinct fundamental features of the world and occupy distinct ontological categories, but mass *possesses* energy in the sense of having energy as a property. The magnitude of that property is given by Einstein's equation.

Option Five: Mass (but not energy) is a fundamental feature of the world, and Einstein's equation defines the derived quantity E_0 in the same way that the classical equation $\mathbf{p} = m\mathbf{v}$ defines the (derived quantity) momentum.

Option Six: Energy (but not mass) is a fundamental feature of the world, and Einstein's equation implicitly defines the derived quantity m_0 just as the classical equation $\mathbf{p} = m\mathbf{v}$ defines the (derived quantity) momentum.

Option Seven: Neither energy nor mass are objective and fundamental features of the world. Einstein's equation expresses the relationship between two derivative quantities.

Most of these interpretations can be found, implicitly or explicitly, in the physics literature. So far as I know, this list is exhaustive.

One immediate problem concerns the fact that neither mass nor energy are generally considered, from a contemporary point of view, to be *substances*. They are, if anything, *properties of* objects, not things to which properties accrue. This immediately rules out Options Three and Four as resting on misunderstandings about the nature of mass and energy, despite the language that physicists often use when talking about mass and energy. Additionally, option Seven faces a battery of problems—e.g., what are the fundamental features grounding mass and energy? How are we to understand the prominent role that Einstein's equation plays in special relativity? etc.—and will not be considered further in this short paper.[5] That leaves us with four interpretive options from which to choose.

Note that all remaining options require some revision or other to the way we customarily think about the *conversion* of mass to energy (or vice versa). One is used to reading—e.g., in the context of various particle decay processes—that the so-called "mass defect" or "missing mass" after decay shows that mass has been *converted into* energy. The thought is that what was once in the form of mass is now, after decay, in the form of energy. However, this sort of identification across time only makes sense if there is some sort of thing or substance that persists and which can be identified at different times. As noted at the end of the previous section, such a metaphysics simply misunderstands the concepts of mass and energy. If we're to make sense of Einstein's equation, then, we have no choice but to re-think what it means to say that energy and mass are interconvertible.

4 Lange's Puzzle

Lange (2001) proposes to narrow does the remaining options by appealing to the notion of Lorentz-invariance:

> [U]nder a standard interpretation, relativity theory denies the objective reality to various properties that we ordinarily assign to material bodies (such as their length and velocity) and to events (such as their

[5]In fact, though, Option Seven is the view I'm developing as part of a larger research project. More details soon!

> separation in space and their separation in time). Each of these quantities is *frame dependent*; none is "Lorentz invariant"—that is, the same in every inertial frame of reference. Only what is the same in every inertial frame is a genuine feature of reality. The value that any frame-dependent quantity assumes in a given inertial frame reflects not just reality, but also that reference frame's own particular perspective. The Lorentz invariant quantities are exactly those which depend only on how the universe really is, uncontaminated by any contribution from us in describing the universe.
>
> Consider a body's mass m. (Here I mean what is sometimes called the body's "rest mass"...) This quantity is Lorentz invariant, and hence objectively real. But the body's energy is not, since its energy depends on its speed, and its speed v is plainly frame dependent. (p.225)

Invoking the notion of frame-dependence (or Lorentz-invariance) as a litmus test for whether a quantity is objectively real leaves proponents of the most widespread interpretation of Einstein's equation with something of a problem: if mass is an objectively real property of an object but energy is not, how could mass and energy be 'fundamentally the same thing'? This puzzle does more than rule out Option One, though. It also rules out Options Two and Six, both of which posit energy as an objectively real property of material objects. The only remaining interpretation of $E = mc^2$ is Option Five—naturally, the one Lange defends.

5 Mass-Energy Conversion as Unphysical?

Of course, the challenge is to make sense of the idea that mass and energy are interconvertible, given that one property (mass) but not the other (energy) is an objectively real feature of the world. How are we to understand claims about, say, missing mass?

It will help to start by considering a standard textbook example designed to illustrate the interconversion of mass and energy. Suppose we have an inelastic collision between identical bodies of (rest) mass m_0 such that, in the $p = 0$ frame, each object is moving with speed v. Prior to collision the total mass of the system is $2m_0$, whereas after collision it is M_0. How is M_0 related to m_0? It is a trivial algebraic exercise covered in most textbooks to show that the following result is implied by the conservation of relativistic energy:

$$M_0 = \gamma(2m_0) > 2m_0$$

and thus that the (rest) mass of the resulting stationary object is greater than the (rest) mass of the system before collision. According to the standard story, the kinetic energy of the incoming objects has been *converted* into mass. But this picture of interconversion is only compatible with a certain interpretive option—Option One—and, as we saw in the last section, that interpretive option can't be correct. How is Lange to understand this textbook example through the lens of Option Five?

To answer this question, we must indulge in a brief digression. According to Lange, (rest) mass is not an additive notion:

> [L]et us work in an inertial frame in which $p = 0$...Then (as I have already mentioned)
>
> $$E_0 = m_0 c^2$$
>
> Consider a system of finitely many constitents, where each exerts on the others only negligible forces when they are not in contact—such as a gas consisting of many molecules. Since the system's total energy $[E_0]$ is the sum of the energies $E_1, E_2, \ldots$ of its constituents, we have
>
> $$M_0 = (1/c^2)E_0 = (1/c^2)[E_1 + E_2 + \ldots]$$
>
> Recall that for any constituent (say, the ith one),
>
> $$E_i = m_{0i}\gamma_i c^2 = m_{0i}c^2/\sqrt{[1 - (v_i^2/c^2)]}$$
>
> ...and so we can use the approximation
>
> $$E_i \approx m_{0i}c^2 + (1/2)m_{0i}v_i^2$$
>
> Notice that this is just $m_{0i}c^2$ plus the ith constituent's classical kinetic energy. Substituting for $E_1, E_2, \ldots$
>
> $$M_0 \approx (1/c^2)[m_{01}c^2 + (1/2)m_{01}v_1^2 + m_{02}c^2 + (1/2)m_{02}v_2^2 + \ldots]$$
>
> and so
>
> $$M_0 \approx [m_{01} + m_{02} + \ldots] + (1/c^2)[(1/2)m_{01}v_1^2 + (1/2)m_{02}v_2^2 + \ldots]$$
>
> Hence, the system's [rest] mass M_0 exceeds the sum of its constituents' [rest] masses. (pp.229-230; notation adapted)

So although the rest mass of an object is an objective and fundamental property of that object, the rest mass of a composite system *isn't* simply the sum of the rest masses of its components. Notice that in the above calculation there is no physical process or interaction that might suggest an energy–mass conversion. The difference between the rest mass of the composite system (M_0) and the sum of the individual rest masses of its components ($m_{01} + m_{02} + \ldots$) is one that is thus brought about by how the system is *described*, not by any *physical* process that might convert energy into mass. We are misled by the notion of inter convertibility because we overlook the non-additive nature of rest mass.

Lange wants to tell the same story in the context of the inelastic collision case. Kinetic energy hasn't been converted into mass. Rather, we have inadvertently switched our perspective and failed to make the necessary non-additive adjustments to mass:

> This "conversion" of energy into mass is not any kind of real physical process taking place in nature. *We* "converted" energy into mass simply by *changing our perspective...*
>
> Suppose we begin by treating [a] gas as a single body. The body is heated. Heat energy flows into it and its mass increases by an equivalent amount. It looks like energy is being converted into mass; fluid, gossamer energy has "solidified" or "congealed" into matter, "the extended hard 'stuff' with which we are all familiar" (in Zahar's words). I argue, however, this is not a real process; rather, it is just an artifact of the perspective we have adopted. No such 'conversion' occurs on a different perspective. Let us being the gas's story again and this time, let us treat the gas as many bodies. We find no energy being transformed into matter as the heat is being added to the gas—so long as we continue to regard the gas as many bodies; none of those bodies increases its mass while the gas is heating up. The heat energy goes into their kinetic energies relative to the gas's center of mass.
>
> So on the first perspective, energy was converted into mass, whereas on the second, no such conversion occurred...This "conversion" of energy into mass is not a physical process. Thus, whether and when a conversion of energy into matter occurs in the story of the gas depends on the perspective from which we elect to tell that story and any shifts of perspective we make in the course of telling it (pp.235–236)

Applied to the case of the inelastic collision, the story Lange presumably wants to tell is that we shift our perspective in describing the system—from seeing it as made of components to seeing it as a single body—and as a result mistakenly attribute the non-additive increase in mass that accompanies this shift in perspective to a specious conversion of energy into mass.

Suppose we decide to view the single body of rest mass M_0 that results from the inelastic collision as a composite object consisting of two halves, each of which is formed from one of the colliding objects. Each half has rest mass m_0, and because each half is at rest in the $p = 0$ frame, the object *when viewed as a composite* has total rest mass

$$M_0' \approx [m_{01} + m_{02} + \ldots] + (1/c^2)[(1/2)m_{01}v_1^2 + (1/2)m_{02}v_2^2$$

$$= [m_0 + m_0] + [0 + 0] = 2m_0.$$

Since whether we view the resulting object as a single body or a composite makes no difference to the physics, but leads to different assessments of the rest mass, the lost mass that is 'converted' into energy would just seem to be an artifact of how the object is described. That, at least, is how Lange proposes that we understand Einstein's equation in the face of the puzzle outlined in the previous section.

6 A Sketch of Some Objections

In this final section I briefly sketch a series of objections to Lange's interpretation of $E = mc^2$.

OBJECTION ONE: Whatever the plausibility of Lange's analysis in the case of inelastic collisions, his 'descriptive' approach can't account for the conversions between mass and energy that arise in cases of photon pair production and annihilation:

$$\gamma + \gamma \to e^- + e^+$$

and

$$e^- + e^+ \to \gamma + \gamma.$$

In the first of these cases, two photons collide to generate an electron-positron pair and there's a conversion of energy to mass that can't be attributed to a shift in the level at which the system is described, for both the incoming and outgoing systems are taken to be composite and thus are described in the same way. However, the incoming system has zero rest mass, whereas the outgoing system has a rest mass of $2m_e$. (A similar problem arises for the increase in mass that results from trapping light in a box.)

OBJECTION TWO: Lange's interpretation is unable to accommodate the mass–energy conversions that arise from internal changes in a system's *potential* energy. As Faraoni (2013) notes, the mass of a spring increases in accordance with Einstein's equation upon compression on account of the resulting increase in potential energy. Yet this interaction involves no shift in the way in which the spring is described. Indeed, it's not at all clear how Lange's approach could handle cases of mass conversions with forms of energy that can't be readily reduced to kinetic energy.

Objection Three: In asserting that rest mass is an objectively real property of objects but not relativistic energy, Lange implicitly privileges the rest frame of an object over all other inertial frames, and thereby violates the principle that objectively real properties must be Lorentz-invariant. Of course, it's true that the rest mass remains the same in all other frames, but it's equally true that the relativistic mass-in-frame-S remains the same in all other frames. Lange has provided no reason to take the rest mass seriously, given that all inertial frames are on a par. Should we not also take an object's *proper length* to be an objectively real property?

Objection Four: If we accept rest mass as an objectively real property, how is that property to be understood? It can't be understood as a body's resistance to change of motion, for experimentally that's given by the *relativistic* mass. When considering the gravitational interaction between two bodies, which mass is relevant—the 'single body mass' or the 'composite body mass'? Lange's interpretation is in the awkward position of positing a fundamental property whose physical content is entirely unclear.

Objection Five: In what way does it follow from the failure of Lorentz-invariance that kinetic energy isn't an 'objective' property of objects? It's true that there is no fixed *value* of kinetic energy that a body possesses objectively and independently of a chosen frame, but there are objectively real (Lorentz invariance) facts about *differences* in kinetic energy between objects (and in how the kinetic energy of an object changes). These objective facts could surely form the basis of a competing interpretation of Einstein's equation. Consider an analogy: it's a well-known fact that the value of an object's acceleration is a frame-dependent quantity in special relativity, yet *whether* an object is accelerating is itself an objective and frame-independent fact.

Objection Six: It doesn't follow from the failure of Lorentz-invariance for kinetic energy that there can be no objective facts about *all* forms of energy. Even if kinetic energy is a frame-dependent quantity, it's consistent with that conclusion that there might be other forms of energy–e.g., rest energy–that are objectively real properties of objects.

The extremely qualified conclusion I draw from these objections is that we have as yet no clear understanding of exactly how mass and energy should be understood within the context of special relativistic dynamics, despite the proclamations often made about Einstein's famous equation.