# How an axiomatic Bohmian field theory could look

A first-order axiomatic theory SB is proposed that leads to the construction of a relativistic dynamics in the spirit of Bohmian mechanics..

Relativistic spacetime is constructible ab initio by a first-order theory ST that is a conservative extension of Zermelo's set theory with urelemente (Benda 2008, 2013). ST has worldlines as primitive objects, which are set-theoretical urelemente. Spacetime points are sets of worldlines. Frome here, a semi-Riemannian manifold is constructed. A physical theory has thereby not yet been created. What is missing are dynamical magnitudes which give rise to experiences. The right-hand side of the Einstein equation provides them, and it is a task for the physicist to specify the magnitudes therein according to some accepted dynamical theory. That has, for example, been conducted using the electromagnetic stress-energy tensor. But also a stress-energy tensor derived from  Bohmian mechanics offers itself as a candidate. I propose to employ such a tensor to obtain a dynamical relativistic theory SB, which is an extension of ST.

Briefly sketched, the sought extension is achieved by furnishing every worldline with a smooth wave function from $\mathbb{R}^4$ to $\mathbb{C}$, which proceeds in time along the worldline. The remaining three arguments are specific to that worldline. Thus a four-dimensional space is associated with each worldline. Wave functions vary smoothly among worldline curves. The Bohmian stress-energy tensor is constructed in analogy to that of an ideal, pressure-less fluid: Its *i*-*j*-th component is equal to the product of the *i*-th and the *j*-th component of the four-velocity of the fluid, which in turn is obtained by the Bohmian dynamic equation.

Association of worldlines and wave functions is done on a primitive level. The vocabulary of SB is unchanged from ST. So its elementary formulas are: $x = y$,; $x \in y$; and $v \nmid_x w$ , read "*w* intersects *v* at *x*". A modified axiom postulates that $v \nmid_x w$ implies that *x* is a relation on $\mathbb{C}^2$. In comparison, in ST, *x* is postulated to be a real number. Its counterpart in SB is the real part of the unchanging first component of each of its elements. The remaining physical axioms of ST are modified accordingly for SB. Given a worldline *w* and a set of worldlines *z* which do not mutually intersect in a local region (in a sense already defined in ST), the set $\{x : v \nmid_x w \wedge v \in z\}$ defines a relation. We call it an "*w*-intersection relation". For all worldlines *v* and all worldlines *w* that intersect *v*, the wave function associated with *v* is obtained from the set $\{x : v \nmid_x w\}$.

Three axioms are added to ST which state:

(1) for all worldlines *w*, every *w*-intersection relation is a smooth mapping;

(2) for each worldline, the wave function associated with it is unique;

(3) the Einstein equation holds, with its right-hand side being the Bohmian stress-energy tensor.

Thus the dynamical theory SB is derived from axioms which state no more than conditions of worldlines intersecting each other. Furthermore, SB is analogous to Bohmian mechanics. Every particle in the latter corresponds to the density along a worldline curve in SB, Both Bohmian particles and worldlines in SB are associated with a three-dimensional configuration space for each time point and each point on the worldline curve, respectively, The configuration space of each worldline in SB is distinct from spacetime in SB. The set of wave functions having configuration spaces as their domains determine the dynamics in SB. They are taken as a given in SB, without further physical motivation, which goes beyond the present scope. Nevertheless, a plausible model of SB has each wave function evolve according to the Schrödinger equation. SB is relativistically covariant since its bulding blocks, worldlines and their intersecting, are prior to coordinate systems. Yet in the limit case of tiny, mutually isolated particles, if each wave function evolves according to the Schrödinger equation, Bohmian mechanics is obtained.

## References


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Benda, Thomas (2013). “An axiomatic foundation of relativistic spacetime.” *Synthese*, online first, DOI **10.1007/s11229-013-0345-6**.