Intuitive Geometry,
László Fejes Tóth Centennial

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Alfréd Rényi Institute of Mathematics
Central European University

Book of Abstracts

Invited talks

Alexander Barvinok (University of Michigan)

Thrifty approximations of convex bodies by polytopes

We discuss how well a convex body can be approximated in the Banach - Mazur distance by a polytope with a given number of vertices. We consider both coarse (the number of vertices is bounded by a polynomial in the dimension) and fine (the distance is small) approximations.

András Bezdek (Alfréd Rényi Insitute of Mathematics and Auburn University)

On a question of L. Fejes Tóth concerning crossing pairs in a thinnest covering of the plane with convex disks

Two convex disks in the plane are said to *cross each other* if the removal of their intersection causes each disk to fall into disjoint components. Almost all major theorems concerning the covering density of a convex disk were proved only for crossing-free coverings. This includes the classical theorem of L. Fejes Tóth (1950) that uses the maximum area hexagon inscribed in the disk to give a lower bound for the covering density of the disk. From the early seventies, all attempts of generalizing this theorem were based on the common belief that crossings in a covering of the plane with congruent convex disks, being counterproductive for producing low density, are always avoidable. Unexpectedly, Heppes and Wegner (1980) constructed a series of examples, where a convex region was covered with congruent hexagons so that the hexagons could not be rearranged so as to cover the region without crossing. However, as was shown by Heppes (2003), the smallest covering density of the plane with congruent copies of a sufficiently fat ellipse can be achieved with a lattice covering, thus in a crossing free manner. G. Fejes Tóth (2005) generalized Heppes's result for r-fat convex discs, with r sufficiently close to 1 (r-fat convex discs are inscribed in a unit circle and contain a concentric circle of radius r). Exposing the true nature of the trouble with occurrence of crossing in the thinnest covering, in a joint paper with W. Kuperberg (2010) we presented an example of a convex pentagon with the property that in every thinnest covering of the plane with pentagons congruent to it crossings must occur. The example has no bearing on the validity of Fejes Tóth's bound in general, but it shows that any prospective proof must take into consideration the existence of unavoidable crossings. This talk is a report on the joint work with W. Kuperberg concerning coverings with crossings. The main idea used in the case of crossing pentagons will be revisited and it will be also used for describing a new family of convex spherical sets, so that i) their congruent copies do not tile S^2, ii) their covering densities can be determined and iii) their thinnest coverings contain crossing pairs.

Henry Cohn (Microsoft Research New England)

Genetics of the regular figures in projective space

László Fejes Tóth emphasized the role of optimization problems in characterizing geometric structure and symmetry. In this talk (based on joint work with Abhinav Kumar and Greg Minton), we'll examine the case of packings in projective spaces and Grassmannians. Some exceptional packings in these spaces are completely characterized by their symmetry, while others exist for seemingly more mysterious reasons.

Herbert Edelsbrunner (Institute of Science and Technology Austria)

Inclusion-Exclusion for Multiple Covers with Balls

Inclusion-exclusion is an effective method for computing the volume of a union of measurable sets. Its implementations are the software of choice for computing the volume, area, or other measures of biomolecules, which are usually modeled as a union of balls in 3-dimensional space.

Motivated by a related but larger-scale biological question, we extend this approach to multiple coverings, proving long and short inclusion-exclusion formulas for the subset of $\mathbb{R}^n$ covered by at least k balls in a finite set. Along the way, we generalize order-k Voronoi diagrams to diagrams defined by cotransitive partial orders, and we show that every such diagram is a weighted Voronoi diagram of weighted averages of the input balls.

This is joint work with Mabel Iglesias-Ham.

Zoltán Füredi (Alfréd Rényi Institute of Mathematics)

Triangles, angles, and more angles

This is a problem oriented talk illustrating that Intuitive Geometry is full of interesting problems. We especially consider questions proposed by L. Fejes Tóth.

Peter M. Gruber (Vienna University of Technology)

Voronoi type properties of lattice packings of convex bodies

The classical criterion of Voronoi: "A lattice packing of balls is (locally) of maximum density if and only if it is perfect and eutactic" is refined and extended. The refinements deal with refined maximum properties: No lattice packing of balls has stationary density. Each latice packing of balls of maximum density is of ultra maximum density. The extensions treat lattice packings of o-symmetric convex bodies: Again, no lattice packing has stationary density. A lattice packing is of ultra maximum density if and only if it is (what we call) c-perfect and c-eutactic (such packings exist). The maximum properties yield lower bounds for the kissing numbers, including the lower bound of Swinnerton-Dyer.

References

1. *Application of an idea of Voronoi to lattice packing*, Ann. Mat. Pura Appl. **193** (2014), 239–259.
2. *Application of an idea of Voronoi to lattice packing, supplement*, Ann. Mat. Pura Appl., in print
3. *Density and kissing numbers of lattice packings*, in preparation.

Thomas Hales (University of Pittsburgh)

A formal proof of the Kepler conjecture

The Kepler conjecture asserts that no packing of congruent balls in space can have density greater than the familiar cannonball arrangement, called the face-centered cubic packing. The first plausible proof-strategy of the conjecture was provided by L. Fejes Tóth in the 1950s. In 1998, Sam Ferguson and I announced a proof of the conjecture, which was published several years later.

In the meantime, I started a long-term project to give a formal proof of the theorem. A proof is formally verified if every step of the proof has been checked at the level of the primitive inference rules of logic and the foundational axioms of mathematics. In a major collaborative effort, the Kepler conjecture has now been formally verified. This is one of the largest formal proof projects ever completed.

Martin Henk (Technische Universität Berlin)

Cone-volume measure of convex bodies

The cone-volume measure of convex bodies is the $p = 0$ limit case of the general L_p–surface area measures whose characterization is a central problem in modern convex geometry. In the talk we will survey recent results on the cone-volume measure, and among others we show that the cone-volume measure of bodies with centroid at the origin satisfies the subspace-concentration-condition (scd). This has several consequences, e.g.,

– the "U-conjecture" is true,

– the scd is a necessary condition for the (in general still open) L_0-Minkowski problem.

If time permits we will also discuss an application to lattice points and Ehrhart polynomials.

The talk is based on joint works with Eva Linke and Károly Böröczky.

Gil Kalai (Hebrew University of Jerusalem)

Discrete geometry – Personal reflections on some works by Jiří Matoušek

László Fejes Tóth with H.S.M. Coxeter and Paul Erdős laid the foundations of discrete geometry, a wonderful area of mathematics with deep connections to other areas of mathematics and science. My talk will be about a young champion of this field, Jiří Matoušek, a great geometer, combinatorialist, and computer scientist, who untimely passed away a few months ago. I will start with fractional Helly theorems, continue with the combinatorics of linear programming, and end with connections between topology and discrete geometry.

Greg Kuperberg (University of California at Davis)

Geometric t-designs and their relations to packings and coverings

Combinatorialists, numerical analysts, and lately quantum information theorists have been interested in geometric t-designs, which are also known as t-cubature formulas. Ever since the work of Delsarte, Goethals, and Seidel, it has been understood that t-designs are conceptually dual to packings of convex bodies (or otherwise metric balls). In some cases this duality is very precise, more so than the rough duality between packings and coverings. In this talk I will discuss several results, both asymptotic upper bounds and lower bounds, that come from relations among t-cubature formulas, packings, and coverings. The cubature formulas are mostly geometric on spaces such as the cube and the simplex; the shapes to pack will be both combinatorial (on the Hamming cube and integer lattice) and geometric.

László Lovász (Eötvös University, Budapest)

The dimension of orthogonal representations

Monika Ludwig (Technische Universität Wien)

Valuations on Lattice Polytopes

Lattice polytopes are convex hulls of finitely many points with integer coordinates in $\mathbb{R}^n$. A function z from a family $\mathcal{F}$ of subsets of $\mathbb{R}^n$ with values in an abelian group (or more generally, an abelian monoid) is a valuation if

$$z(P) + z(Q) = z(P \cup Q) + z(P \cap Q)$$

whenever $P, Q, P \cup Q, P \cap Q \in \mathcal{F}$ and $z(\emptyset) = 0$. The classification of real-valued invariant valuations on lattice polytopes by Betke & Kneser is classical (and will be recalled). It establishes a characterization of the coefficients of the Ehrhart polynomial.

Building on this, a classification is established of Minkowski valuations on lattice polytopes, that is valuations with values in the abelian semigroup of compact convex sets with Minkowski or vector addition. For valuations that intertwine the special linear group over the integers and are translation invariant, we obtain in the contravariant case that the only such valuations are multiples of projection bodies. In the equivariant case, the only such valuations are generalized difference bodies combined with multiples of the newly defined discrete Steiner point.

Joint work with Károly J. Böröczky.

Oleg Musin (University of Texas at Brownsville)

Five Essays on the László Fejes Tóth Geometry

We consider the following topics:

(1) The Fejes Tóth book "Lagerungen in der Ebene, auf der Kugel und im Raum";

(2) The Fejes Tóth bound on the minimum distance between points on the sphere and Tammes' problem;

(3) The Fejes Tóth problem for maximizing the minimum distance between antipodal points on the sphere

(4) The Fejes Tóth – Sachs problem on the one–sided kissing numbers;

(5) The Fejes Tóth theorems on extreme properties of polytopes.

Rom Pinchasi (Technion, Haifa)

The odd area of planar sets

Let $\mathcal{F}$ be a family that consists of an odd number of parallel translates of a given (compact and with positive measure) set F in the plane. We are interested in the area of those points in the plane that belong to an odd number of sets in $\mathcal{F}$. The minimum (infimum) possible such area is called the odd area of F. We will resolve completely the cases where F is either a triangle, a parallelogram, or a trapezoid. We will study the odd area of F where F is any polygon with rational vertices and show that it is always positive. We will also prove the existence and construct a set F whose odd area is equal to 0. Many beautiful questions still remain open.

This is an ongoing project based on works with Assaf Oren and Igor Pak and with Uri Rabinovich.

Francisco Santos (Universidad de Cantabria)

Long paths in combinatorial abstractions of polytopes

One way to try to prove the "Polynomial Hirsch Conjecture" is by generalizing it to more general objects. For example, a conjecture of Hähnle stemming from his work with Eisenbrand et al. on "connected layer families" would imply the following strong version of the polynomial Hirsch conjecture: the diameter of every normal simplicial complex of dimension d with n vertices is bounded by $(n-1)(d+1)$.

Here, a pure simplicial complex is called *normal* if every link is strongly connected. That is, if for every two simplices s_1 and s_2 there is a dual path between them not leaving the star of $s_1 \cap s_2$. Normality also plays a role in Adiprasito and Benedetti's proof of the (original) Hirsch Conjecture for flag polytopes; indeed, what they prove is the Hirsch bound for the diameter of every flag, normal simplicial complex.

One of the proofs of it goes by showing that:

1) Between every pair of simplifies in a normal (not necessarily flag) complex there is always a special type of path that they call "combinatorial segment".

2) Combinatorial segments in flag normal complexes are non revisiting, hence they satisfy the Hirsch bound.

We here revisit this proof and show how to construct combinatorial segments of exponential length in normal simplicial complexes, and even in simplicial polytopes. Along the way we redefine Adiprasito and Benedetti's "combinatorial segments" as what we call conservative and monotone admissible paths.

We also report on a recent construction by Bogart and Kim of paths of almost quadratic length in the context of subset partition graphs.

This is joint work with Jean-Philippe Labbé and Thibault Manneville.

Rolf Schneider (Universität Freiburg)

Poisson hyperplanes and convex bodies

Stationary Poisson processes of hyperplanes in Euclidean space provide the most accessible examples of infinite discrete random systems of hyperplanes. They give rise to several models for random polytopes and pose a number of challenging geometric questions. The answers to such questions often depend on methods and results from the classical theory of convex bodies. We explain this and give some new examples.

József Solymosi (University of British Columbia)

Algebraic methods in discrete geometry

There are some exciting recent results in discrete geometry where the proof uses tools from algebra. Using algebra in discrete mathematics is certainly not new; however, what we see now is the development of algebraic methods which are applicable to various problems in discrete geometry. In this talk I will show some of the key results and list a few open problems of the field.

Endre Szemerédi (Alfréd Rényi Institute of Mathematics)

Maximum size of a set of integers with no two adding up to a square

Erdős and Sárközy asked the maximum size of a subset of the first N integers with no two elements adding up to a perfect square. In this talk we prove that the tight answer is $\frac{11}{32}N$ for sufficiently large N. We are going to prove some stability results also.

This is joint work with Simao Herdade and Ayman Khalfallah.

Asia Ivic Weiss (York University, Toronto)

Chirality in polyhedra, polytopes and thin geometries

This talk will overview the classification of regular and chiral polyhedra. These concepts will then be discussed in a more general setting of incidence geometries with the emphasis on characterization of groups of regular and chiral polytopes and thin geometries.

Günter M. Ziegler (Freie Universität Berlin)

Flag vectors of 4-polytopes and of 3-dimensional tilings

Very interesting 4-dimensional polytopes, polyhedral tilings of 3-space, as well as polyhedral surfaces can be constructed by projecting high-dimensional simple polytopes. In this lecture I plan to

1.) define "very interesting" (on the example for face count ratios for polyhedral tilings in $\mathbb{R}^3$),

2.) explain constructions that closely link the extremal problems for polyhedral surfaces, tilings of 3-space, and 4-polytopes,

3.) survey the parameter spaces and what we know about them (and what we would want to know),

4.) sketch some construction principles (linear algebra at work!), and comment on the difference between combinatorial, geometric, and topological models.

Contributed talks

Arseniy Akopyan (IST Austria)

Circle patterns and confocal conics

We construct some circle patterns which are naturally related with conics and quadrics in space. Our main new results are on checkerboard circumscribed nets in the plane and in spaces of higher dimension. We show how this larger class of these nets appears quite naturally in Laguerre geometry of oriented planes and spheres and leads to new remarkable incidence theorems.

This is joint work with Alexander Bobenko.

Alexey Balitskiy (Moscow Institute of Physics and Technology)

Billiards with almost all trajectories of equal lengths

A group of authors has reduced the classical conjecture of Mahler to a certain statement from symplectic geometry, having something to do with billiard dynamics. We will discuss a few results about billiards in convex bodies in connection with equality cases in Mahler's conjecture. In particular, we prove that all the billiard trajectories (in appropriate norm) in so-called Hanner bodies have equal lengths.

Florian Besau (Technische Universität Wien)

The spherical convex floating body

We introduce the spherical convex floating body for a spherical convex body on the Euclidean unit sphere. The asymptotic behavior of the volume difference of a spherical convex body and its floating body is investigated. This gives rise to a new spherical area measure, the floating measure. Remarkably, this floating measure turns out to be a spherical analogue of the classical affine surface area from affine differential geometry.

This is joint work with Elisabeth M. Werner.

Antonio Cañete (Universidad de Sevilla)

Divisions of rotationally symmetric planar convex bodies minimizing the maximum relative diameter

In this talk we shall study an optimization problem involving the diameter functional. More precisely, fix $k \in \mathbb{N}$, $k \geq 3$, and consider a k-rotationally symmetric planar convex body C. The question we shall focus on is: which is the division of C into k connected subsets minimizing the maximum relative diameter? We recall that the maximum relative diameter is the maximum of the diameters of the k subsets determined by the division. We shall see that the so-called *standard k-partition*, consisting of k inradius segments symmetrically placed, is a minimizing division for $k \leq 6$, but not when $k \geq 7$.

Moreover, for each $k \in \mathbb{N}$, $k \geq 3$, we shall characterize the optimal body for this problem (that is, the set with the division attaining the *lowest* value for the maximum relative diameter functional). Finally, we shall describe some open related problems.

This is part of a joint work with Uwe Schnell (University of Applied Sciences Zittau/Görlitz) and Salvador Segura (Universidad de Alicante).

References

1. A. Cañete, C. Miori, S. Segura, *Trisections of a 3-rotationally symmetric planar convex body minimizing the maximum relative diameter*, Journal of Mathematical Analysis and Applications, **418** (2014), 1030–1046.

2. A. Cañete, U. Schnell, S. Segura, *Subdivisions of k-rotationally symmetric planar convex bodies minimizing the maximum relative diameter*, preprint 2014, arXiv:1501.03907.

Boumediene Et-Taoui (Université de Haute Alsace)

On switching classes of graphs

A class of non-oriented simple graphs is called Seidel switching self-complementary if the complement of any representing graph is in the same equivalence class. In this talk we introduce the 3-signature (s, t) of a switching class of n-vertex graphs. The numbers s and t are the numbers of positive and negative triples within any representing graph of the class. It appears that, for any switching self-complementary class of n-vertex graphs, these numbers are equal, yielding $\binom{n}{3}$ even. Consequently if $n \equiv 3 \pmod 4$ then there is no switching self-complementary class of n-vertex graphs. It is known that the switching classes of Paley conference graphs with $4k+2$ vertices, $4k+1 = p^\alpha$, p an odd prime and α a positive integer are self-complementary. Here it is proven that all $4k$-vertex graphs contained in a $(4k+2)$-vertex Paley conference graph are switching equivalent and their class is still a switching self-complementary class. In addition, the 3-signature is generalized in view of obtaining a complete invariant of switching classes up to order 8.

This is joint work with Augustin Fruchard.

Ferenc Fodor (University of Szeged)

The packing density of the n-dimensional cross-polytope

We will give upper bounds for the packing density of the n-dimensional regular cross-polytope for $n \geq 7$. The upper bounds approach zero exponentially fast with the dimension n. Our main tool is a modified version of Blichfeldt's method.

This is joint work with G. Fejes Tóth and V. Vígh.

Augustin Fruchard (Université de Haute Alsace)

Short cages holding convex figures and convex bodies

A *cage* is the 1-skeleton of a convex polytope in $\mathbb{R}^3$. A cage G is said to *hold* a compact set K if no rigid motion can bring K in a position far away without meeting G on its way. The *length* of a cage G, denoted by $|G|$ is the sum of lengths of all its edges. Given a compact set K, let $L(K) = \inf\{|G|\,;\, G \text{ holds } K\}$. The purpose of this talk is to present some results about $L(K)$ for various compact sets K.

This is joint work with Prof. Tudor Zamfirescu.

Alexey Garber (The University of Texas at Brownsville)

Five-dimensional Dirichlet-Voronoi parallelohedra

In this talk we will report about full classification of combinatorially different five-dimensional Dirichlet-Voronoi parallelohedra for lattices.

The classification of affinely different Delone triangulations (L-type domains) can be done using Voronoi's second reduction theory. It was done completely for small dimensions up to 5. Dimensions 3 and 4 can be done without using the reduction theory, but already in dimension 5 it plays an important role for classification. The classification of five-dimensional L-type domains was made by E. Baranovskii and S. Ryshkov in 1973. They found 221 different triangulations, but later P. Engel in 1998 found that they missed one triangulation.

In this talk we will show how one can extend the Voronoi reduction theory to find all affinely non-equivalent lattice Delone decompositions and combinatorially different Dirichlet-Voronoi parallelohedra in arbitrary dimension and present our computational results in dimension 5.

Our main result is the following

Theorem. *There are* 110244 *affine types of lattice Delone triangulations and* 110244 *combinatorial types of Dirichlet-Voronoi parallelohedra in dimension* 5.

This is joint work with M. Dutour Sikirić, A. Schürmann, and C. Waldmann.

Jin-Ichi Itoh (Kumamoto University)

Quadratic surfaces as the surfaces generated by circles or rectangular hyperbolas

D. Hilbert and S. Cohn-Vossen wrote in their book "Anschauliche Geometrie" that ellipsoids and other quadratic surfaces are generated by parallel circles. Here we show how ellipsoids, one-sheeted hyperboloids, two-sheeted hyperboloids, and elliptic paraboloids are explicitly represented by two families of parallel circles (as circular surfaces) using coordinates. As circles are important objects in ellipsoids, rectangular hyperbolas (whose two asymptotes are orthogonal) are important objects in general hyperbolas. We found that many one-sheeted hyperbolas and two sheeted hyperbolas are represented by parallel circles (how they are constructed by rectangular hyperbola), and there are infinitely many families of parallel rectangular hyperbolas instead of only two as in circular cases. Also all hyperbolic paraboloids are generated by rectangular hyperbolas. Moreover we will discuss the general dimensional cases.

This is joint work with Yutaro Yamashita.

Matthieu Jacquemet (University of Fribourg)

Commensurability of hyperbolic Coxeter polyhedra with $n + 2$ facets

Finite-volume hyperbolic Coxeter polyhedra bounded by $n + 2$ hyperplanes in $\mathbb{H}^n$ are classified. In this talk, we shall give a survey of their commensurability. The methods used are of geometric, algebraic and arithmetic nature, and they can also be applied to polyhedra with more facets.

This is joint work with Rafael Guglielmetti and Ruth Kellerhals.

Balázs Keszegh (Alfréd Rényi Institute of Mathematics)

More on decomposing coverings by octants

We improve our upper bound given in 2012 by showing that every 9-fold covering of a point set in $\mathbb{R}^3$ by finitely many translates of an octant decomposes into two coverings, and our lower bound by a construction for a 4-fold covering that does not decompose into two coverings. The same bounds also hold for coverings of points in $\mathbb{R}^2$ by finitely many homothets or translates of a triangle. We also prove that certain dynamic interval coloring problems are equivalent to the above question.

This is joint work with Dömötör Pálvölgyi.

Robert Kozma (University of Illinois at Chicago)

New bounds for the optimal ball packing density of hyperbolic 4-space

In this talk we will consider ball packings of hyperbolic space. We begin by motivating the discussion with recent developments in three dimensions. We then show that it is possible to exceed the conjectured 4-dimensional packing density upper bound due to L. Fejes-Tóth (Regular Figures, 1964). We give several examples of horoball packing configurations that yield higher densities of ≈ 0.71644896 where horoballs are centered at the ideal vertices of certain Coxeter simplex tilings.

This is joint work with Jenő Szirmai.

David de Laat (TU Delft)

Moment methods in energy minimization

I will present a hierarchy of optimization problems which can be used to lower bound the ground state energy of a system of interacting particles. We construct this hierarchy by extending moment techniques as used in polynomial optimization to a functional analytic setting. We apply this to the Thomson problem, which asks for configurations of N points on the unit sphere which minimize the pairwise sum of reciprocal distances. I will show how harmonic analysis can be used to exploit the symmetry in the resulting optimization problems. This enables us to compute the second step of the hierarchy by semidefinite programming to obtain new bounds for the Thomson problem.

Zsolt Lángi (Budapest University of Technology and Economics)

An isoperimetric problem for d-polytopes

In this presentation we investigate the problem of finding the maximum volume polytopes, inscribed in the unit sphere of the d-dimensional Euclidean space, with a given number of vertices. We solve this problem for polytopes with $d+2$ vertices in every dimension, and for polytopes with $d+3$ vertices in odd dimensions. For polytopes with $d+3$ vertices in even dimensions we give a partial solution. Joint work with Á. G.Horváth.

Jin Li (Shanghai University and Technische Universität Wien)

Orlicz valuations

L_p Minkowski valuations were characterized as moment bodies, difference bodies and projection bodies by Ludwig [TAMS 2005] for $GL(n)$ compatible valuations and Haberl [JEMS 2012], Parapatits [TAMS 2014], [JLMS 2014] for $SL(n)$ compatible valuations. In this talk, I will present the classification of $SL(n)$ compatible Orlicz valuations. Unlike their L_p analogs, the identity operator and the reflection operator are the only $SL(n)$ compatible Orlicz valuations (up to dilations). The property that the Orlicz difference body operator is not an Orlicz valuation actually plays an important role in characterizing the identity operator and the reflection operator.

This is joint work with Gangsong Leng.

László Major (Eötvös University, Budapest)

The Unimodality Conjecture for cubical polytopes

Although the Unimodality Conjecture holds for some certain classes of cubical polytopes (e.g. cubes, capped cubical polytopes, neighborly cubical polytopes), it fails for cubical polytopes in general. A 12-dimensional cubical polytope with non-unimodal face vector is constructed by using capping operations over a neighborly cubical polytope with 2^{131} vertices. For cubical polytopes, the Unimodality Conjecture is proved for dimensions less than 11. The first one-third of the face vector of a cubical polytope is increasing and its last one-third is decreasing in any dimension.

Endre Makai, Jr. (Alfréd Rényi Institute of Mathematics)

A class of packings in $\mathbb{R}^n$ in which lattice packings have maximal density

Let $L \subset \mathbb{R}^3$ be the union of unit balls, whose centres lie on the z-axis, and are equidistant with distance $2d$. Then a packing of unit balls in $\mathbb{R}^3$ consisting of translates of L has a density at most $\pi/(3d\sqrt{3-d^2})$, with equality for a certain lattice packing of unit balls. Let $L \subset \mathbb{R}^4$ be the union of unit balls, whose centres lie on the x_3x_4-axis, and form either a square lattice, or a regular triangular lattice, of edge length 2. Then a packing of unit balls in $\mathbb{R}^4$ consisting of translates of L has a density at most $\pi^2/16$, with equality for the densest lattice packing of unit balls in $\mathbb{R}^4$. Our main tool for the proof is a theorem on (r, R)-systems in $\mathbb{R}^2$. If $R/r \leq 2\sqrt{2}$, then the Delone triangulation associated to this (r, R)-system has the following property. The average area of a Delone triangle is at least $\min\{A_0, r^2/2\}$, where A_0 is the infimum of the areas of the non-obtuse Delone triangles. This general theorem has applications also in other problems about packings: namely for $r^2/2 \geq A_0$ it is sufficient to deal only with the non-obtuse Delone triangles, which is in general a much easier task.

These results are joint with K. Böröczky and A. Heppes.

Máté Matolcsi (Alfréd Rényi Institute of Mathematics)

Improved bounds on the density of planar sets not containing unit distances

A 1-avoiding set is a subset of $\mathbb{R}^n$ that does not contain pairs of points at distance 1. Let $m_1(\mathbb{R}^n)$ denote the maximum fraction of $\mathbb{R}^n$ that can be covered by a measurable 1-avoiding set. We prove two results. First, we show that any 1-avoiding set in $\mathbb{R}^n$ ($n \geq 2$) that displays block structure (i.e., is made up of blocks such that the distance between any two points from the same block is less than 1 and points from distinct blocks lie farther than 1 unit of distance apart from each other) has density strictly less than $1/2^n$. Second, we use linear programming and harmonic analysis to show that $m_1(\mathbb{R}^2) \leq 0.258795$ – falling just short of the conjecture of Erdős asserting that $m_1(\mathbb{R}^2) < 1/4$.

Joint work with T. Keleti, F. Oliviera de Filho, and I. Z. Ruzsa.

Luis Montejano (National University of Mexico at Queretaro)

Homological Sperner-type theorems

Let K be a simplicial complex. Suppose the vertices of K are painted with $I = \{1, ..., m\}$ colours, that is; $V(K) = V_1 \cup ... \cup V_m$. A simplex $\sigma = \{v_1, ..., v_m\} \subset V(K)$ is rainbow if it contains exactly one vertex of every colour.

A homological Sperner-type theorem concludes the existence of a rainbow simplex of K under the hypothesis that certain homology groups of certain subcomplexes of K are zero.

We will discuss several Homological Sperner-type theorems and we will give some geometric applications.

Frank Morgan (Williams College)

Isoperimetric problems in $\mathbb{R}^n$ with density

Since its appearance in Perelman's proof of the Poincaré Conjecture, there has been a surge of interest in placing a positive density on a space. We focus on the isoperimetric problem in $\mathbb{R}^n$ with density (which weights both volume and perimeter) and include some recent results and open problems.

Márton Naszódi (EPFL (Lausanne) and Eötvös University, Budapest)

On a quantitative Helly-type theorem

Bárány, Katchalski and Pach proved the following quantitative form of Helly's theorem: If the intersection of a family of convex sets in $\mathbb{R}^d$ is of volume one, then the intersection of some subfamily of at most $2d$ members is of volume at most some constant $v(d)$. They gave the bound $v(d) \le d^{2d^2}$ and conjectured that $v(d) \le d^{cd}$. We confirm it.

Deborah Oliveros (Instituto de Matemáticas UNAM)

Helly numbers over subsets of $\mathbb{R}^d$

In this talk, we present some Helly-type theorems where the convex sets are required to intersect over subsets S of $\mathbb{R}^d$. This is a continuation of prior work for $S = \mathbb{R}^d$, $\mathbb{Z}^d$, and $\mathbb{Z}^{d-k} \times \mathbb{R}^k$ (motivated by continuous, integer, and mixed-integer optimization, respectively). We are particularly interested in the case when S has some algebraic structure, in particular when S is a subgroup or the difference between a lattice and some sublattices.

Joint work with J. A. De Loera, R. N. La Haye and E. Roldán-Pensado.

Alexandr Polyanskii (Moscow Institute of Physics and Technology)

On graphs diameters on spheres of small radii

Our talk is about the following question posed by M.Perles. What is the maximal number of edges in a diameter graph with n vertices on sphere of radius $r = 1/2 + \varepsilon$ for sufficiently small $\varepsilon > 0$ depending on n? We discuss this problem and some others that appeared during solving this one.

This is joint work with Andrey Kupavskii.

Daniel Reem (ICMC, University of Sao Paulo, Brazil)

The geometric stability of Voronoi diagrams with respect to small changes of the sites

The talk will address the question of the geometric stability of Voronoi diagrams: does a small perturbation or distortion of the sites (generators) yield a small change in the shapes of the corresponding Voronoi cells? This question is natural in various theoretical and real-world scenarios such as dynamical or random ones, scenarios in which imprecision or approximation are inherent, scenarios related to lattices, etc. It turns out that the answer is positive in a wide class of cases, but not in general, and explicit bounds can be given. More precisely, the setting is uniformly convex normed spaces of arbitrary dimension and (possibly infinitely many) positively separated sites of a general form. The uniform convexity assumption can be relaxed under some conditions, among them, the relationship between the location of the sites and the structure of the unit sphere.

Igor Shnurnikov (National Research University "Higher School of Economics", Moscow)

On the number of connected components in arrangements of submanifolds

Let us consider an arrangement of n submanifolds in a manifold M^d. We mean arrangements of lines in the plane, closed geodesics in manifolds with given metric, hyperplanes in projective or affine spaces, subtori in flat real torus, etc. Let M^d be d–dimensional and submanifolds be $(d-1)$–dimensional, then the complement to union of submanifolds consists of f connected components. Questions about values of f appeared in works of Schläfli, Grünbaum, Orlic, Terao, Solomon, Shannon, Martinov, Deshpande and many others. Let F be the set of possible values of f for given n and type of arrangements. We will study F, describing it completely or via bounds for f.

Theorem 1. *If M^d and submanifolds are connected and closed and submanifolds intersect each other transversally then*

$$f \geq n + 1 - \dim H_{d-1}(M^d, G),$$

where homology group is calculated for $G = \mathbb{Z}$ or $G = \mathbb{Z}_2$ if all submanifolds and M^d are orientable or not respectively.

Next theorem shows that this inequality is tight.

Theorem 2. *For flat subtori in a flat torus*

$$F = \{n - d + 1, \dots, n\} \cup \{l \in \mathbb{N} \mid l \geq 2(n - d)\}$$

for $n > d$. For $2 \leq n \leq d$ we have $F = \mathbb{N}$.

Let us note that for $d = 2$ we get geodesic arrangements in flat torus.

Theorem 3. *For arrangements of hyperplanes in the projective space, such that not all hyperplanes pass through one point, first four numbers of F in increasing order are*

$$(n-d+1)2^{d-1}, \quad 3(n-d)2^{d-2}, \quad (3n-3d+1)2^{d-2}, \quad 7(n-d)2^{d-3}$$

for $d \geq 3$ and $n \geq 2d+5$.

For arrangements of hyperplanes one could prove that for every d and sufficiently large n the set F contains almost all integers between minimal and maximal ones, i.e. the ratio of realizable integers to all integers between minimal and maximal numbers of F tends to 1.

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Pablo Soberón (University of Michigan)

Quantitative Helly-type theorems

During this talk we will discuss variants of Helly's theorem where we are interested in quantifying the size of the intersection of a family of convex sets in terms of its volume or number of lattice points. In particular, we seek to optimise the guarantee on the size of the intersection of a family of convex sets. We also discuss quantitative versions of the colourful Helly theorem and the (p,q) theorem of Alon and Kleitman.

Konrad Swanepoel (London School of Economics and Political Science)

Approximate Steiner trees

A *Steiner minimal tree* of a given a set N of n points in d-dimensional Euclidean space is a shortest tree that interconnects the set N, where we allow the tree to have additional vertices. These additional vertices, called *Steiner points*, have degree 3 and the angle between any two edges incident to a Steiner point is exactly 120°. In the plane, Steiner minimal trees can be constructed by ruler and compass, but in higher dimensions this is not true any more, and exact calculations are in practice replaced by numerical approximations. Rubinstein, Weng and Wormald (2006) studied the worst-case error in the length of *ϵ-approximate Steiner trees*, where all angles at Steiner points are within ϵ of 120°. We give an overview of what is known about this error, including some new results.

This is joint work with Charl Ras and Doreen Thomas (University of Melbourne).

László Szabó (University of West Hungary)

12-neighbour packings of unit balls in $\mathbb{E}^3$

A packing of unit balls in $\mathbb{E}^3$ is said to be a 12-neighbour packing if each ball is touched by 12 others. A 12-neighbour packing of unit balls can be constructed as follows. Consider a horizontal hexagonal layer of unit balls in which the centres of the balls are coplanar and each ball is touched by six others. Put on the top of this layer a second horizontal hexagonal layer of unit balls so that each ball of the first layer touches three balls of the second layer. The translation which carries the first layer into the second one, carries the second layer into a third one, and repeated translations of the same kind in both directions produce a packing of unit balls in which each ball has 12 neighbours. László Fejes Tóth conjectured that any 12-neighbour packing of unit balls in $\mathbb{E}^3$ is composed of such hexagonal layers. In September 2012 Thomas Hales posted a paper on the preprint server arXiv with a computer-assisted proof of this conjecture. The aim of this talk is to give a more geometric proof of the conjecture along a different line.

This is joint work with Károly Böröczky.

Jenő Szirmai (Budapest University of Technology and Economics)

Kepler-type problems in Thurston geometries

In mathematics sphere packing problems concern the arrangements of non-overlapping equal spheres which fill a space. Usually the space involved is the three-dimensional Euclidean space. However, ball (sphere) packing problems can be generalized to the other 3-dimensional Thurston geometries.

In an n-dimensional space of constant curvature $\mathbf{E}^n$, $\mathbf{H}^n$, $\mathbf{S}^n$ $(n \geq 2)$ let $d_n(r)$ be the density of $n+1$ spheres of radius r mutually touching one another with respect to the simplex spanned by the centres of the spheres. L. Fejes Tóth and H. S. M. Coxeter conjectured that in an n-dimensional space of constant curvature the density of packing spheres of radius r cannot exceed $d_n(r)$. This conjecture has been proved by C. Roger in the Euclidean space. The 2-dimensional case has been solved by L. Fejes Tóth. In an 3-dimensional space of constant curvature the problem has been investigated by Böröczky and Florian in [2] and it has been studied by K. Böröczky in [1] for n-dimensional space of constant curvature $(n \geq 4)$.

In [3], [8], [9], [4] and [6] we have studied some new aspects of the horoball and hyperball packings in $\mathbf{H}^n$ and we have realized that the ball, horoball and hyperball packing problems are not settled yet in the n-dimensional $(n \geq 3)$ hyperbolic space.

The goal of this talk to generalize the above problem of finding the densest geodesic and translation ball (or sphere) packing to the other 3-dimensional homogeneous geometries (Thurston geometries)

$$\widetilde{\mathbf{SL_2R}},\ \mathbf{Nil},\ \mathbf{S}^2\!\times\!\mathbf{R},\ \mathbf{H}^2\!\times\!\mathbf{R},\ \mathbf{Sol},$$

(see [10], [11], [12], [14], [15], [7]) and to describe a candidate of the densest geodesic and translation ball arrangement. The greatest density until now is ≈ 0.85327613 whose horoball arrangement is realized in the hyperbolic space $\mathbf{H}^3$. In this talk we show a geodesic ball arrangement in $\mathbf{S}^2 \times \mathbf{R}$ geometry whose density is ≈ 0.87499429 (see [13]).

We will use the unified interpretation of the Thurston geometries in the projective 3-sphere $\mathcal{PS}^3(\mathbf{V}^4, \boldsymbol{V}_4, \mathbf{R})$ introduced in [5].

This is joint work with Emil Molnár.

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Tibor Tarnai (Budapest University of Technology and Economics)

Packing of equal circles on spherical caps

We investigate the following problem: How must n equal circles be packed on a spherical cap of angular radius (half of the central visual angle) α without overlapping so that the angular radius of the circles will be as large as possible? If α is zero, the problem is reduced to finding the densest circle packing in a circle. If α is equal to 180°, than the problem is identical to the Tammes problem [1], that is, finding the densest circle packing on a sphere. It is apparent that if the angular diameter α varies from zero to 180° a transition from packing in a circle to packing on the sphere is obtained.

In this paper, on the basis of computer-based analysis, conjectured solutions to the problem for $n = 2, 3, 4, 5, 6, 7, 8$ will be presented for the complete range of α from zero to 180°. We will show how the packing density and the conjectured best circle configurations change with the angular radius α of the spherical cap. The results will be given in the form of packing graphs and density diagrams.

A special emphasis will be put on the case $\alpha = 90°$, that is, on the case of a hemisphere, since until now only point arrangements and not circle packings were studied on a hemisphere [2]. Practical importance of this problem at golf balls, geodynamic satellites, signal detecting devices, etc. will be shown.

This is joint work with András Lengyel. The research was supported by OTKA grant no. K81146.

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Viktor Vígh (University of Szeged)

On the diminishing process of Bálint Tóth

Let K and K_0 be convex bodies in $\mathbb{R}^d$, such that K contains the origin, and define the process (K_n, p_n), $n \geq 0$, as follows: let p_{n+1} be a uniform random point in K_n, and set $K_{n+1} = K_n \cap (p_{n+1} + K)$. Clearly, (K_n) is a nested sequence of convex bodies which converge to a non-empty limit object, again a convex body in $\mathbb{R}^d$. We study this process for K being a regular simplex, a cube, or a regular convex polygon with an odd number of vertices.

This talk is based on joint work with P. Kevei.

Aljosa Volcic (Università della Calabria)

Iterations of Steiner symmetrizations

In the attempt of solving the isoperimetric problem Steiner missed the important point of the existence of the solution. To fill the gap, W. Gross constructed, given a convex body K, a sequence of directions $\{u_n\}$ such that iterated Steiner symmetrals minimize the perimeter and converge, in the Hausdorff distance, to the ball K^* centered at the origin and having the same volume as K.

Peter Mani was the first to understand (in a paper dedicated to László Fejes Tóth in occasion of his 70^{th} birthday) that the convergence of the successive Steiner symmetrizations of a **convex body** K to K^* holds *almost surely.*

He conjectured that this happens also if we consider successive Steiner symmetrizations of a **compact set**. The conjecture was confirmed in 2006 by van Schaftingen. We provided in 2013 a different proof.

Recently several papers appeared concerning convergence of iterations of Steiner symmetrizations.

Bianchi, Klain, Lutwak, Yang and Zhang proved in 2011 the following result.

Theorem 1. *If K is a* **convex body** *and U is a countable set of directions, then it can be ordered in a sequence $\{u_n\}$ such that the successive Steiner iterations of K in that directions converge to K^*.*

We improved recently this result in two directions. On one hand the seed K of the iteration is allowed to be **compact** rather than convex, and on the other hand we proved that there exists a universal ordering of U which works for every seed.

The first step of the proof consists in the analogous statement for measurable sets (with L_1 convergence), then the result is extended to L_1 functions (and L_1 convergence), then to continuous functions with compact support (and uniform convergence) and finally to compact sets (with convergence in Hausdorff distance).

Jesús Yepes Nicolás (Instituto de Ciencias Matemáticas Madrid)

On a linear refinement of the Prékopa-Leindler inequality

The Prékopa-Leindler inequality states that, given $\lambda \in (0,1)$ and non-negative measurable functions $f, g, h : \mathbb{R}^n \longrightarrow \mathbb{R}_{\geq 0}$ such that, for any $x, y \in \mathbb{R}^n$,

$$h\big((1-\lambda)x + \lambda y\big) \geq f(x)^{1-\lambda} g(y)^{\lambda},$$

then

$$\int_{\mathbb{R}^n} h \,\mathrm{d}x \geq \left(\int_{\mathbb{R}^n} f \,\mathrm{d}x\right)^{1-\lambda} \left(\int_{\mathbb{R}^n} g \,\mathrm{d}x\right)^{\lambda}.$$

This result is closely related to a number of classical integral inequalities such as Hölder's inequality or the reverse Young's inequality and to some geometric ones like the well-known Brunn-Minkowski inequality.

In this talk we will show that under the sole assumption that f and g have a common *projection* onto a hyperplane (which is the analytic counterpart of the projection onto a hyperplane of a set), the Prékopa-Leindler inequality admits a linear refinement. That is, under such an assumption for the functions f and g, the right-hand side in the above integral inequality may be exchanged by the convex combination of the integrals, which yields a stronger inequality. Moreover, the same inequality can be obtained when assuming that both projections (not necessarily equal as functions) have the same integral. We will explore the main idea of the proof of the latter result, which has a strong geometric flavor.

This is joint work with A. Colesanti and E. Saorín Gómez.

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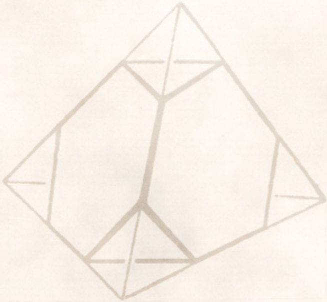

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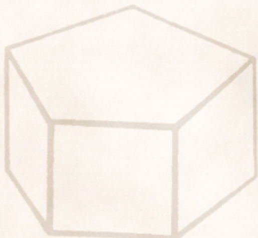

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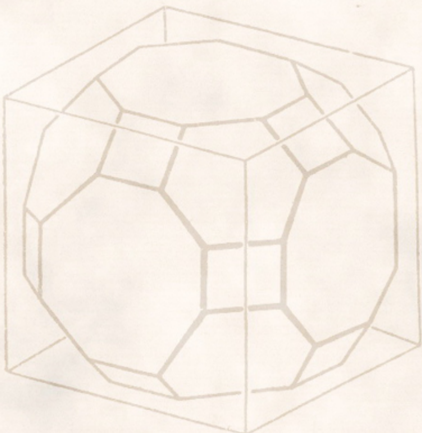

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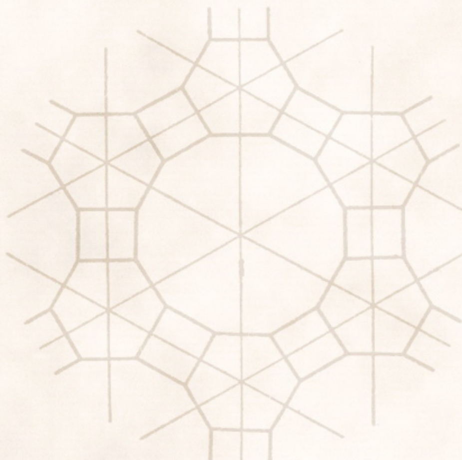

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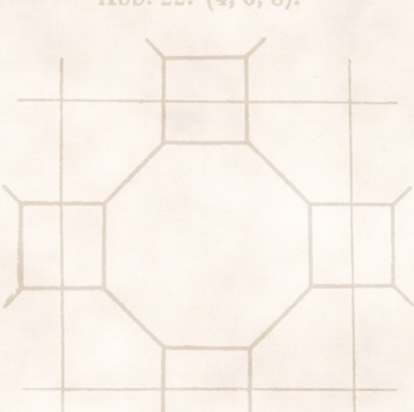

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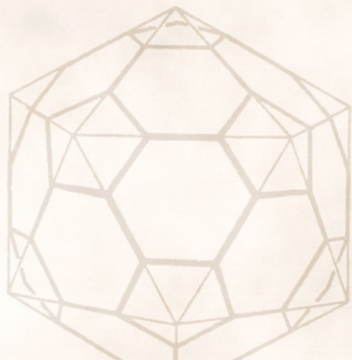

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