BOOK OF ABSTRACTS

4th Workshop on Fourier Analysis and Related Fields

Budapest, MTA Rényi Institute, 2013, August 25-31.

Useful Information for Participants

Wifi is available at our Institute. The password is renyiwireless. If you require specifically, we can also set up a temporary account for you, so that you can use the computer terminals of the Institute.

The exchange rate between EUR and HUF is currently around 300. Therefore, if you are ever offered a rate below 290 you can be sure that you are being cheated. There are good exchange offices nearby the Institute at Kossuth Lajos street.

There are no talks on Wednesday afternoon. We offer two social programs for this period for interested participants. One is a sightseeing cruise on the Danube which can be further extended by a walk to the Castle area. The other is a visit to a famous spa in Budapest, the Szechenyi spa – please bring your swimming suits. Alternatively, we also draw your attention to the National Museum which is just 3 minutes from the Institute (at Muzeum körút), and the Museum of Fine arts which is at Hero's square.

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ABSTRACTS OF TALKS

Christine Bachoc (Bordeaux)
Fourier analysis and some extremal problems in geometry

Given a distance in Euclidean space $\mathbb{R}^d$, we consider the problem of estimating the maximal density of a measurable subset A of $\mathbb{R}^d$ with the property that no two points of A are at distance 1 apart. We will explain how Fourier analysis applies and how it compares to the more classical graphic approach. In the case of the Euclidean distance we will see applications to the so-called measurable chromatic number of Euclidean space.

Aline Bonami (Orléans)
Cumulants on the Wiener Space and quantitative Central Limit Theorems

Let F_n a sequence of centered random variables whose variance is 1. Assume that F_n converge in distribution to $N \sim \mathcal{N}(0,1)$. A natural question concerns the speed of convergence. A striking theorem of Nualart and Peccati says that, whenever F_n belongs to some fixed Wiener chaos, then F_n converges in law to $\mathcal{N}(0,1)$ if and only if $\mathbb{E}(F_n^4)$ tends to 3. This is an easy consequence of the following estimate (Nourdin, Peccati 2009): for F in some Wiener chaos,

$$d_{\mathrm{Kol}}(F,N) \leq \sqrt{\mathbb{E}[F^4]-3}.$$

Here, the Kolmogorov distance is defined as

$$d_{\mathrm{Kol}}(X,Y) := \sup_z |P(X<z) - P(Y<z)|.$$

One does not know whether this estimate is sharp. But it is not the case when one uses a smoother distance, defined by:

- Wasserstein distance

$$d_{\mathrm{Wass}}(X,Y) := \sup_{\|h'\|_\infty \leq 1} |\mathbb{E}[h(X)] - \mathbb{E}[h(Y)]|,$$

- "smooth" distance

$$d(X,Y) := \sup_{\|h''\|_\infty \leq 1} |\mathbb{E}[h(X)] - \mathbb{E}[h(Y)]|.$$

We have the following:
Theorem (BBNP). *Whenever F is in some Wiener chaos and satisfies $\mathbb{E}(F) = 0$, $\mathbb{E}(F^2) = 1$, then*

$$d_{\mathrm{Wass}}(F,N) \leq C \max\left\{|\mathbb{E}[F^3]|, (\mathbb{E}[F^4]-3)^{3/4}\right\}.$$

$$d(F,N) \leq C \max\left\{|\mathbb{E}[F^3]|, \mathbb{E}[F^4]-3\right\}.$$

The last statement is sharp.

For the proof, one uses Malliavin calculus to reduce the problem to estimates on cumulants, which are defined from the characteristic function $\phi_F(t) = \mathbb{E}[e^{itF}]$. The jth *cumulant* of F, denoted by $\kappa_j(F)$, is

$$\kappa_j(F) = (-i)^j \frac{d^j}{dt^j} \log \phi_F(t)|_{t=0}.$$

The main argument comes from the estimate, valid for $s > 4$,

$$\kappa_s(F) \leq c_s(q) \left[\kappa_4(F)\right]^{\frac{s}{4}},$$

which can be compared to the classical hypercontractivity estimates for moments.

These estimates can be used in particular in the context of the Breuer Major Theorem, with applications in signal processing and statistics.

This is a joint work with H. Biermé, Y. Nourdin and G. Peccati.

Javier Cilleruelo (Madrid)

Combinatorial problems in finite fields and Sidon sets

We use Sidon sets to present an elementary method to study some combinatorial problems in finite fields, such as sum product estimates, solvability of some equations and the distribution of their solutions.

Christian Elsholtz (Graz)

Hilbert cubes in multiplicatively defined sets

Let $a_0, a_1, \ldots, a_d$ denote positive integers and let $a_0 + \{0, a_1\} + \cdots + \{0, a_d\} \subset S$ be a so called Hilbert cube, which is inside a given set of positive integers S. (If $a_0 = 0$ these cubes are also known under the name subsetsums.) Such cubes been studied for example by Hilbert, Szemerédi, Hegyvári, Sárközy, Gyarmati, Stewart, Wood, and are connected to the Gowers norm.

For various sets S we give an improvement of the maximal dimension of a cube:
1) If S is the set of squares $x^2 \leq N$, then $d = O(\log \log N)$, improving on Hegyvári and Sárközy's $O((\log N)^{1/3})$.
For the set of k-th powers a bound of $d = O_k(\log \log N)$, holds, even though the proof is not quite the same.
2) If S is a set without an arithmetic progression of length k, then $d = O_k(\log N)$, which is best possible.
3) (Maybe we discuss some other multiplicatively defined sets as well).

We discuss various methods, and observe that a lemma guaranteeing in case 1) above an exponential growth of the iterated sum sets is the key to the upper bound.

This is joint work with R. Dietmann.

Dmitry Gorbachev (Tula)

Asymptotic lower bound for cardinality of weighted spherical designs

An asymptotic lower bound for the minimal number of points in a weighted spherical design is proved:

$$N(d,\tau) \geq \frac{1}{2^d \Gamma(d) A(d)} \cdot \tau^d (1 + o(1))$$

where $A(d)$ is LP-bound of sphere packing density Δ_d:

$$\Delta_d \leq A(d) = \frac{|B^d|}{2^d} \min_f f(0).$$

Here f is radial positive defined function with mean value $\widehat{f}(0) = 1$, $f|_{\mathbb{R}^d \setminus B^d} \leq 0$, B^d is the unit ball in $\mathbb{R}^d$.

For high order designs the estimate is better than known estimates of Delsarte–Goethals–Seidel and Yudin.

This research was partially supported by the RFFI 13-01-00045 and Dmitry Zimin's Foundation "Dynasty".

György Gát (Nyíregyháza)

Summation of one and two-dimensional Walsh-Fourier series

Let x be an element of the unit interval $I := [0, 1)$. The $\mathbb{N} \ni n$th Walsh function is

$$\omega_n(x) := (-1)^{\sum_{k=0}^{\infty} n_k x_k} \quad (n = \textstyle\sum_{k=0}^{\infty} k_i 2^i, \ x = \sum_{k=0}^{\infty} \frac{x_i}{2^{i+1}}).$$

The Walsh-Fourier coefficients, the n-th partial sum of the Fourier series, the n-th $(C, 1)$ mean of $f \in L^1(I)$:

$$\hat{f}(n) := \int_I f(x)\omega_n(x)dx, \quad S_n f := \sum_{k=0}^{n-1} \hat{f}(k)\omega_k, \quad \sigma_n f := \frac{1}{n}\sum_{k=0}^{n-1} S_k f.$$

It is of main interest that how to reconstruct a function from the partial sums of its Walsh-Fourier series. Fine proved [1] that for each integrable function we have the almost everywhere convergence of Fejér means $\sigma_n f \to f$.

In the talk we give a brief résumé of the recent results (see e.g. [2]) with respect to summability of Walsh-Fourier series of one and two dimensional functions. Among others, we talk about the convergence properties of Marczinkiewicz means and its generalizations. The Marcinkiewicz means [3] are defined as

$$t_n f(x) := \frac{1}{n}\sum_{k=0}^{n-1} S_{k,k} f(x).$$

[1] N.J. Fine, *Cesàro summability of Walsh-Fourier series*, Proc. Nat. Acad. Sci. U.S.A. **41** (1955), 558–591.

[2] G. GÁT,*Almost everywhere convergence of Fejr and logarithmic means of subsequences of partial sums of the Walsh-Fourier series of integrable functions*, Journal of Approximation Theory **162 (4)** (2010), 687–708.

[3] L.V. Zhizhiasvili, *Generalization of a theorem of Marcinkiewicz*, Izv. Akad. nauk USSR Ser Mat. **32** (1968), 1112–1122 (Russian).

Valery I. Ivanov (Tula)

Fourier–Dunkl harmonic analysis and sharp Jackson inequality in L_2-space with power weight

Let $d \in \mathbb{N}$, $\mathbb{R}^d$ be d-dimensional real Euclidean space with inner product (x, y) and norm $|x| = \sqrt{(x, x)}$,

$$v_k(x) = \prod_{\alpha \in R_+} |(\alpha, x)|^{2k(\alpha)}$$

generalized power weight, defined by positive subsystem R_+ of root system $R \subset \mathbb{R}^d$ and function $k(\alpha) : R \to \mathbb{R}_+$ invariant with respect to reflection group $G(R)$, generated by R,

$$c_k = \int_{\mathbb{R}^d} e^{-|x|^2/2} v_k(x)\, dx$$

Macdonald–Mehta–Selberg normalizing constant, $d\mu_k(x) = c_k^{-1} v_k(x)\, dx$,
$L_{2,k}(\mathbb{R}^d)$ Hilbert space of complex functions f on $\mathbb{R}^d$ with norm

$$\|f\|_{2,k} = \left(\int_{\mathbb{R}^d} |f(x)|^2 \, d\mu_k(x) \right)^{1/2} < \infty.$$

Harmonic analysis of the space $L_{2,k}(\mathbb{R}^d)$ is carried out by means of direct and inverse Dunkl integral transforms

$$\widehat{f}(y) = \int_{\mathbb{R}^d} f(x)\, \overline{e_k(x, y)} d\mu_k(x), \quad f(x) = \int_{\mathbb{R}^d} \widehat{f}(y)\, e_k(x, y) d\mu_k(y),$$

where $e_k(x, y)$ is generalized exponential, defined by means of differential-difference Dunkl operators

$$D_j f(x) = \frac{\partial f(x)}{\partial x_j} + \sum_{\alpha \in R_+} k(\alpha)(\alpha, e_j) \frac{f(x) - f(\sigma_\alpha(x))}{(\alpha, x)}, \quad j = 1, \ldots, d.$$

Generalized exponential $e_k(x, y)$ has properties similarly to properties of usual exponential $e^{i(x,y)}$ and harmonic analysis of the space $L_{2,k}(\mathbb{R}^d)$ is similar Fourier harmonic analysis of the space $L_2(\mathbb{R}^d)$ [1]. A great contribution to the development of harmonic analysis in spaces with power weight have C.F. Dunkl, M. Rösler, M.F.E. de Jeu, K. Trimeche, Y. Xu and others.

Let V be convex centrally symmetric compact body, invariant with respect to reflection group $G(R)$, $\sigma > 0$,

$$E(\sigma V, f)_{2,k} = \inf \left\{ \|f - g\|_{2,k} : g \in L_{2,k}(\mathbb{R}^d), \quad \operatorname{supp} \widehat{g} \subset \sigma V \right\}$$

best approximation of function $f \in L_{2,k}(\mathbb{R}^d)$ by entire functions of exponential type σV^* (with spectrum in σV). Here V^* is polar of V.

By means of any sequence of complex numbers

$$M = \{\mu_s\}_{s \in \mathbb{Z}}, \quad \sum_{s \in \mathbb{Z}} \mu_s = 0, \quad \sum_{s \in \mathbb{Z}} |\mu_s| < \infty$$

and convex centrally symmetric compact body U, invariant with respect to reflection group $G(R)$, define generalized modulus of continuity $\omega_M(\tau U, f)_{2,k}$. If

$$\nu_s = \sum_{l \in \mathbb{Z}} \mu_{l+s} \overline{\mu_l}, \quad \nu_0 = \sum_{l \in \mathbb{Z}} |\mu_l|^2,$$

then function

$$\varphi\left(t,y\right)=\sum_{s\in\mathbb{Z}}\nu_{s}e_{k}\left(st,y\right)\geq 0,\quad t,y\in\mathbb{R}^{d}.$$

Let

$$\omega_{M}(\tau U,f)_{2,k}=\sup_{t\in\tau U}\left(\int_{\mathbb{R}^{d}}\varphi\left(t,y\right)|\widehat{f}(y)|^{2}d\mu_{k}(x)\right)^{1/2},\quad \tau>0.$$

In the case of unit weight ($k\left(\alpha\right)\equiv 0$) we get generalized modulus of continuity, generated by infinitely-difference operator

$$\Delta_{t}^{M}f\left(x\right)=\sum_{s\in\mathbb{Z}}\mu_{s}f\left(x+st\right).$$

We proved the following theorem. THEOREM. *There is a constant* $\gamma=\gamma\left(k,M,V,U\right)>0$ *such that for any function* $f\in L_{2,k}\left(\mathbb{R}^{d}\right)$ *sharp Jackson inequality*

$$E\left(\sigma V,f\right)_{2,k}\leq\frac{1}{\sqrt{\nu_{0}}}\omega_{M}\left(\frac{\gamma}{\sigma}U,f\right)_{2,k} \tag{1}$$

is true.

In the case of unit weight this theorem was proved by S.N. Vasilyev [2].

Least value of γ in (1) is called as optimal argument. Optimal argument depend on power weight v_k, sequence M and on geometry of bodies V, U.

Our lecture will be devoted to the discussion of the optimal argument problem for sequence $M_r=\left\{(-1)^s\binom{r}{s}\right\}_{s\in\mathbb{Z}}$, as well as of the dual Logan type extremal problem for entire functions with spectrum in a body U [3, 4]. Sequence M_r define the modulus of continuity of order $r\in\mathbb{N}$.

This paper was supported by RFBR (grant No. 13-01-00045).

[1] *Rösler M.* Dunkl Operators: Theory and Applications // Lecture Notes in Math. Berlin, Heidelberg, New York: Springer–Verlag, 2002. V.1817. P.93–135.

[2] *Vasil'ev S.N.* Jackson inequality in $L_2(\mathbb{R}^n)$ with generalized modulus of continuity // Proceedings of the Steklov Institute of Mathematics. 2011. V. 273. Issue 1 Supplement. P. 163–170.

[3] *Ivanov A.V.* Some extremal problems for entire functions in weighted spaces // Proceedings of Tula State University. Natural Sciences. 2010. Issue 3. P. 26–44. (in Russian)

[4] *Ivanov A.V., Ivanov V.I.* Optimal arguments in Jackson inequality in $L_2(\mathbb{R}^d)$-space with power weight // Math. Notes. 2013. V.94. No. 3. P.338–348. (in Russian)

Philippe Jaming (Bordeaux)

Heisenberg uniqueness pairs

Let C be a smooth curve in the plane and A be a set of lines in the plane. (S,A) is a Heisenberg uniqueness pair (HUP) if the only finite measure m that is absolutely continuous with respect to arc length on C and such that its Fourier transform vanishes on A is $m=0$. In this talk we will show how this notion can be reformulated in geometric terms. This allows us to extend results of Hendelmalm, Sjolin and Lev to rather general curves.

This is joint work with Karim Kellay.

Mihalis Kolountzakis (Heraklion)

In which domains can one do Fourier Analysis?

We all know how to do Fourier Analysis on an interval

with the orthonormal basis

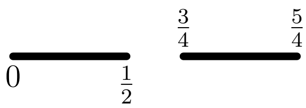
$$e^{2\pi i n x}, \ n \in \mathbb{Z}.$$

With a little effort we can do the same with the domain

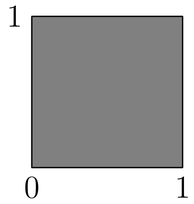
and the orthonormal basis

$$e^{2\pi i 2nx}, \ e^{2\pi i (2n - \frac{1}{2})x}, \ n \in \mathbb{Z}.$$

but there is no such orthogonal basis for the domain

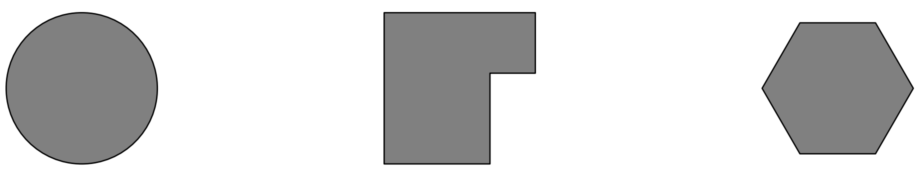


In higher dimensions we know that the unit square



admits the orthogonal basis $e^{2\pi i (m,n)\cdot(x,y)}$, $(m,n) \in \mathbb{Z}^2$.

But what about the following domains?



The answer is negative for the disk and affirmative for the other two domains.

Domains that have such an orthogonal basis of exponentials $e^{2\pi i \lambda \cdot x}$, for λ in some set of frequencies Λ, are called *spectral* and the question of which domains are spectral has been investigated since the 1970s.

We will cover some of the history of this problem and show some recent results (with Alex Iosevich) regarding the periodicity of spectra in dimension 1.

Vilmos Komornik (Strasbourg)

Observability of rectangular membranes and plates on small sets

We present several applications of Fourier series to control theory, obtained in collaboration with P. Loreti. First we consider vibrating membranes. We establish various observability theorems for rectangular membranes by applying Mehrenberger's recent generalization of Ingham's theorem.

Next we investigate vibrating plates. Since the works of Haraux and Jaffard we know that rectangular plates may be observed by subregions not satisfying the so-called geometrical control condition. (The latter condition is essentially necessary and sufficient for hyperbolic equations by a theorem of Bardos, Lebeau and Rauch.) We improve these results by observing only on an arbitrarily short segment inside the domain. The estimates may be strengthened by observing on several well-chosen segments.

Péter Kórus (Szeged)

Convergence of trigonometric series in $\mathbf{L}^\mathbf{p}(\mathbf{0} < \mathbf{p} < \mathbf{1})$

This is a joint work with Xhevat Z. Krasniqi and Ferenc Móricz.

We give necessary conditions in terms of the coefficients for the convergence of a double trigonometric series in the L^p-metric, where $0 < p < 1$. The results and their proofs were motivated by the recent papers [1] and [2]. Our basic tools in the proofs are the Hardy-Littlewood inequality for functions in H^p and the Bernstein-Zygmund inequalities for the derivatives of trigonometric polynomials and their conjugates in the L^p-metric $(0 < p < 1)$.

[1] A. S. Belov: On conditions of the average convergence (upper boundedness) of trigonometric series. J. Math. Sci. (N. Y.) 155 (2008), 5–17.

[2] F. Móricz: Necessary conditions for L^1-convergence of double Fourier series. J. Math. Anal. Appl. 363 (2010), 559–568.

Miklós Laczkovich (Budapest)

Local spectral synthesis on Abelian groups

We say that spectral synthesis holds on the Abelian group G if every closed and translation invariant linear subspace of the set of complex valued functions defined on G is spanned by polynomial-exponential functions. It is known that spectral synthesis holds on some countable Abelian groups (if the torsion free rank of G is finite) but fails, e.g., on the free Abelian group of countably infinitely many generators. We prove that local spectral synthesis (when we use local polynomials instead of polynomials) holds on every countable Abelian group. More precisely, there is a cardinal number κ such that $\omega_1 \leq \kappa \leq 2^\omega$, and local spectral synthesis holds on G if and only if the torsion free rank of G is less that κ.

Frederick Manners (Cambridge)

On the "Pyjama Problem"

The “pyjama stripe” is the subset of $\mathbb{R}^2$ consisting of a vertical strip of width 2ε around every integer x-coordinate. The “pyjama problem” asks whether finitely many rotations of the pyjama stripe around the origin can cover the plane. It turns out the answer is yes. I will attempt to give an overview of the problem and present the main features of a proof.

Máté Matolcsi (Budapest)

A Fourier approach to the circulant Hadamard conjecture

The circulant Hadamard conjecture states that there exist no circulant $n \times n$ Hadamard matrices for $n > 4$. I will present an approach to this conjecture based on discrete Fourier analysis.

Lozko Milev (Sofia)

Semidefinite Extreme Points In A Set Of Quadratic Bivariate Polynomials

Let Δ be the triangle in $\mathbb{R}^2$ bounded by the lines $x = 0,\ y = 0,\ x + y = 1$ (the standard simplex in $\mathbb{R}^2$). Denote by π_2 the set of all real bivariate algebraic polynomials of total degree at most two. Let B_Δ be the unit ball of the space π_2 endowed with the supremum norm on Δ.

We describe the semidefinite extreme points of B_Δ. This completes the description of the set E_Δ of all extreme points of B_Δ started by L. Milev and N. Naidenov in two papers, concerning strictly definite and indefinite elements of E_Δ.

Ferenc Móricz (Szeged)

Statistical convergence of sequences and series of complex numbers with applications in Fourier Analysis and Summability

This is a survey paper on recent results indicated in the title. In contrast to the famous examples of Kolmogorov and Fejér on the pointwise divergence of Fourier series, the statistical convergence of the Fourier series of any integrable function takes place at almost every point; and the statistical convergence of the Fourier series of any continuous function is uniform. Furthermore, Tauberian conditions are also presented, under which ordinary convergence of any sequence of real or complex numbers follows from its statistical summability.

Nikola Naidenov (Sofia)

On The Path-Connectedness Of The Set Of Extreme Points In Certain Polynomial Spaces

Let Δ be the standard simplex in $\mathbb{R}^2$. Denote by π_2 the set of all real bivariate algebraic polynomials of total degree at most two. Let B_Δ be the unit ball of the space π_2 endowed with the supremum norm on Δ.

Recently L. Milev and N. Naidenov completed the description of the set E_Δ of all extreme points of B_Δ. We present here a result concerning the path-connectedness of E_Δ and compare E_Δ with other known sets of extreme points in spaces of algebraic and trigonometric polynomials.

Stefan Neuwirth (Besancon)

On the (Fourier analytic) Sidon constant of $\{0, 1, 2, 3\}$

This constant is the maximum of the sum $|c_0| + |c_1| + |c_2| + |c_3|$ of the moduli of the coefficients of a trigonometric polynomial $c_0 + c_1 e^{it} + c_2 e^{2it} + c_3 e^{3it}$ bounded by 1. Its value is still unknown, but I will present some ideas on how to compute it.

István Prause (Helsinki)

Bilipschitz maps and spirals

Bilipschitz maps, by definition, do not change distances significantly. Nevertheless, they may change the local geometry by creating spirals out of straight segments. We study this behaviour in a quantitative fashion in terms of the bilipschitz distortion. We give sharp answers to the questions how fast and how often spiralling can occur. This is joint work with K. Astala, T. Iwaniec and E. Saksman.

Sinai Robins (Singapore)

Tiling the integer lattice by translated lattice sublattices

If we represent $\mathbb{Z}^d$ as a disjoint, finite union of translated integer sublattices, then the translated sublattices must possess some special properties. We call such a representation a lattice tiling. Here we develop a theoretical framework, based on multiple residues and dual groups, which allows up to prove, for example, that if we have any lattice tiling, and if p^k divides the determinant of any of any one translated sublattice, then p^k also divides the determinant of (at least) one of the other translated sublattices, for any prime p. We will cover other related results as well, from first principles. We also investigate the question, for a lattice tiling in general dimension, of when there must be at least two translated sublattices which are translates of one another, following a question of Erdős in the 1-dim'l case, and we give in general dimension one such sufficient condition in terms of cyclic lattices.

Although the case $d = 2$ for the latter question remains open, we give a bound from below for the determinant of any potential counterexample.

This is joint work with Maciej Borodzik, and Danny Nguyen.

Anne de Roton (Nancy)

Short survey on sets with no solution to a three linear equation

When can we say that a subset of integers or a subset of the interval $[0,1]$ does not contain any solution to a linear equation of the form $ax + by = cz$ with a, b and c given positive integers ? What kind of big sets with no solution can we build ?

To answer these questions, various tools in Fourier analysis, number theory and combinatorics are used. We shall see how these tools are used in this context and we shall emphasize both the differences and the similarity between the continuous and the discrete cases.

Imre Z. Ruzsa (Budapest)

Squares and difference sets in finite fields

For infinitely many primes $p = 4k + 1$ we give a slightly improved upper bound for the maximal cardinality of a set $B \subset \mathbb{Z}_p$ such that the difference set $B - B$ contains only quadratic residues. Namely, instead of the "trivial" bound $|B| \leq \sqrt{p}$ we prove $|B| \leq \sqrt{p} - 1$, under suitable conditions on p. The new bound is valid for approximately three quarters of the primes $p = 4k + 1$. Joint work with Máte Matolcsi and Christine Bachoc.

Angel San Antolín (Alicante)

A family of smooth compactly supported tight wavelet frames

This is a joint work with R. A. Zalik.

For any dilation matrix with integer entries, we construct a family of smooth compactly supported tight wavelet frames in $L^2(\mathbb{R}^d)$, $d \geq 1$.

Estimates for the degrees of smoothness of these framelets are given. Our construction involves some compactly supported refinable functions, the Unitary Extension Principle and a slight generalization of a theorem of Lai and Stöckler.

If the required degree of smoothness of our framelets increases, then we need a bigger number of generators. However, for any, A, 2×2 dilation matrix with integer entries and $|\det A| = 2$, we show a method to construct a family of smooth compactly supported tight wavelet frames with only three generators.

Ilya D. Shkredov (Moscow)

The eigenvalues method in Combinatorial Number Theory

In the talk a family of operators (finite matrices) with interesting properties will be discussed.

These operators appeared during attempts to give a simple proof of Chang's theorem from Combinatorial Number Theory.

At the moment our operators have found several applications in the area connected with Chang's result as well as another problems of Number Theory such as : bounds for the additive energy of some families of sets, new structural results for sets with small higher energy, estimates of Heilbronn's exponential sums and others.

Ilona Simon (Pécs)

On the almost everywhere convergence of two-dimensional Cesaro means on the 2-adic additive group

This is a joint work with György Gát.

In 1935 Jessen, Marcinkiewicz and Zygmund[5] proved that the Fejér means $\sigma^1_{n,m}f$ of the trigonometric Fourier series of two variable functions $f \in L\log^+ L$ converge almost everywhere to the function when $\min\{n,m\} \to \infty$.

Several results were published concerning Fejér and Cesàro means of Fourier series with respect to the characters of the 2-adic additive group. In 1997 Gát[2] proved the a.e. convergence $\sigma^1_n f \ \to \ f$ for integrable functions f. Gát[1] also proved the a.e. convergence of Cesáro means $\sigma^\alpha_n f \ \to \ f$ for functions $f \in L\log^+ L(\mathbb{I}^2)$ and $\alpha > 0$.

Now we investigate this problem for two-dimensional functions: Let $f \in L\log^+ L(\mathbb{I}^2)$. Then for $\alpha, \beta > 0$ we have a.e. convergence $\sigma^{\alpha,\beta}_{n,m}f \to f$ as $\min\{n,m\} \to \infty$.

[1] Gát, Gy., *Almost everywhere convergence of Cesàro means of Fourier series on the group of 2-adic integers*, Acta Math. Hungar., 116 (3) (2007), pp. 209-221.

[2] Gát, Gy., *On the almost everywhere convergence of Fejér means of functions on the group of 2-adic integers*, J. Approx. Theory, 90 (1997), pp. 8896.

[3] Móricz, F., Schipp, F., Wade, W., R., *Cesàro summability of double Walsh-Fourier series* Trans. Amer. Math. Soc., 329 (1992), pp. 131-140.

[4] Schipp, F., Wade, W.R., Simon, P., Pál, J., *Walsh Series, An Introduction to Dyadic Harmonic Analysis*, Adam Hilger, Ltd., Bristol and New York, (1990).

[5] Jessen, B., Marcinkiewicz, J., Zygmund, A., *Note on the differentiability of multiple integrals.* Fund. Math., 25 (1935), pp. 217-234.

László Székelyhidi (Gaborone & Debrecen)

Spectral Analysis and Synthesis on Abelian Groups

The classical Uniqueness Theorem on the Fourier transform of 2π-periodic continuous complex valued functions on the real line says that if the function is nonzero, then it has at least one nonzero Fourier coefficient. This can be reformulated by saying that the smallest translation invariant linear space including the function, and closed under uniform convergence, that is, the *variety* of the function, contains at least one complex exponential function. From Fejér's famous result on the uniform convergence of Fejér means to the function one derives the stronger property: the complex exponentials in the variety of the function actually span a dense subspace. The first reformulation above is a typical *spectral analysis* result, while the second one is a *spectral synthesis* theorem. In general, in spectral analysis and spectral synthesis one studies similar problems, where the functions are defined on locally compact Abelian groups and the exponentials are replaced by appropriate functions, called exponential monomials. In this talk we present some recent results and methods on spectral analysis and spectral synthesis on discrete Abelian groups.

Endre Szemerédi (Budapest & New Brunswick)

On the density of sequences of integers, the sum of no two of which is a square

We are going to prove that if $S \subset [1, N]$ and $x + y \neq z^2$ $x, y \in S$ $z \in \mathbb{N}$, then $|S| \leq \dfrac{11}{32}N$.

The result is tight and we are going to show that there is only one set S for which $|S| = \dfrac{11}{32}N$.

Unfortunately we will use very little Fourier analysis.

Joint work with Ayman Khalfalah and Simaoh Herdade.

Ferenc Weisz (Budapest)

Summability of multi-dimensional Fourier series

The triangular, circular and cubic partial sums and summability of higher dimensional Fourier series are investigated. Both norm and almost everywhere convergence of the partial sums, Fejér and Riesz means are considered. Some inequalities are proved for the boundedness of the summability means and the maximal operators on L_p spaces and Hardy spaces.

László Zsidó (Roma)

Splitting non-commutative dynamical systems in almost periodic and weakly mixing parts

By a classical theorem of B. O. Koopman and J. von Neumann, for any linear contraction T on a Hilbert space H, denoting by H_{AP} the closed linear span of all eigenvectors of T corresponding to eigenvalues of absolute value 1, T is almost periodic on H_{AP} (the orbits are relatively compact) and it is weakly mixing to 0 on $H \ominus H_{AP}$ (the orbits converge weakly to 0 in density).

This result was extended to more general semigroups of linear contractions on more general Banach spaces by K. Jacobs, K. de Leeuw and I. Glicksberg.

A similar splitting was recently proved by C. Niculescu, A. Ströh and L. Zsidó for state preserving endomorphisms of C*-algebras. Their proof actually works in the case of general commutative, locally compact semigroups of state preserving endomorphisms.

In this talk we shall discuss the case of not necessarily commutative, locally compact semigroups of state preserving endomorphisms.