BOOK OF ABSTRACTS

3rd Workshop in Fourier Analysis and Related Fields

Budapest, MTA Rényi Institute, 2009, September 17-23.

List of Participants

Gautami Bhowmik, Université Lille, Gautami.Bhowmik@math.univ-lille1.fr

András Bíró, Rényi Institute, Budapest, biroand@renyi.hu

István Blahota, College of Nyíregyháza, blahota@nyf.hu

Aline Bonami, Université d'Orléans, aline.bonami@gmail.com

Tamás Erdélyi, Texas A&M University, terdelyi@math.tamu.edu

György Gát, College of Nyíregyháza, gatgy@nyf.hu

Ágota P. Horváth, Technical University of Budapest (BME), ahorvath@math.bme.hu

Philippe Jaming, Université d'Orléans, philippe.jaming@gmail.com

Tamás Keleti, ELTE University, Budapest, tamas.keleti@gmail.com

Sergei Konyagin, Moscow State University, konyagin23@gmail.com

Miklós Laczkovich, ELTE University, Budapest, laczk@cs.elte.hu

Péter Maga, Central European University, Budapest, magapeter@gmail.com

Máté Matolcsi, Rényi Institute, Budapest, matemato@gmail.com

Lozko Milev, University of Sofia, milev@fmi.uni-sofia.bg

Ferenc Móricz, SZTE University, Szeged, moricz@math.u-szeged.hu

Károly Nagy, College of Nyíregyháza, nkaroly@nyf.hu

Nikola Naidenov, University of Sofia, nikola@fmi.uni-sofia.bg

Stefan Neuwirth, Uni. de Franche-Comté, stefan.neuwirth@univ-fcomte.fr

Alexander Olevskii, Tel Aviv University, olevskii@yahoo.com

Szilard Révész, Rényi Institute, Budapest, revesz@renyi.hu

Imre Z. Ruzsa, Rényi Institute, Budapest, ruzsa@renyi.hu

Sándor Szabó, Technical University of Budapest (BME), sszabo@math.bme.hu

Laszlo Székelyhidi, University of Debrecen, lszekelyhidi@gmail.com

Rodolfo Toledo, College of Nyíregyháza, toledo@nyf.hu

Jim Wright, University of Edinburgh, J.R.Wright@ed.ac.uk

Gautami Bhowmik (Lille)
Analytical properties of some Dirichlet Series

We will discuss the analytic continuation of certain Dirichlet series, particularly those that admit an Euler product. As consequences, we will obtain asymptotic expansions of some number-theoretic functions.

István Blahota (Nyíregyháza)
Maximal operators on Vilenkin systems

The topic of the talk is the boundedness of maximal operator $\sigma^* = \sup |\sigma_n|$ on Vilenkin- and double Vilenkin-systems with Fejér and with the Marcinkiewicz-Fejér means. We also proved similar statements on the d-dimensional Walsh-Paley system.

In this presentation we take a survey of our published and submitted papers listed below.

REFERENCES

[1.] Blahota, I., Gát, G., Goginava, U. : Maximal operators of Fejér means of Vilenkin-Fourier series, Journal of Inequalities in Pure and Applied Mathematics (JIPAM) 7 (4) 2008.

[2.]Blahota, I., Gát, G., Goginava, U. : Maximal operators of Fejér means of double Vilenkin Fourier series, Colloq. Math. 107 (2007), no. 2, 287-296.

[3.]Blahota, I., Goginava, U. : The Maximal Operator of the Marcinkiewicz-Fejr Means of the 2-Dimensional Vilenkin-Fourier Series, Studia Scientiarum Mathematicarum Hungarica, 45 (3) 2008.

[4.]Blahota, I., Goginava, U. : The martingale Hardy type inequality for the maximal operator of the (C, α) means of cubic partial sums of the d-dimensional Walsh-Fourier series, submitted.

Aline Bonami (Orléans)
Uncertainty Principle for finite Abelian groups (joint work with S. A. Ghobber)

We will be interested in determining how small the support and the spectrum of a non zero function f may be simultaneously on a finite Abelian group. A first well-known estimate (sometimes known as Stark-Donoho Uncertainty Principle) has been obtained by Matolcsi and J. Szücs: the product of the two cardinals is bounded below by the cardinal of the group. More recently, Tao has observed that one can say much more when the cardinality of the group is a prime number p. More precisely, for A and B of cardinality k and l, the subspace of functions with support in A and spectrum in B has dimension $\min\{0, k+l-p\}$. Using this, Meshulam has given in the general case a bound below for the cardinal of the spectrum of a non zero function, knowing the cardinal of its support. We will discuss on the sharpness of the bound of Meshulam and characterize functions for which there is equality.

Tamás Erdélyi (Texas)

Some Recent Results on Littlewood Polynomials

A Littlewood polynomial is a polynomial with each of its coefficients in -1,1. We focus on various problems about Littlewood polynomials such as bounding the number of their (distinct) real zeros, the largest possible multiplicity a zero may have at 1, and their size on the unit circle of the complex plane in various norms. A special attention will be paid to cyclotomic Littlewood polynomials. A few fairly new results will be stated and many questions will be raised.

György Gát (Nyíregyháza)

One and two dimensional Fejér means on the Walsh and Vilenkin groups

Let $m := (m_k, k \in \mathbf{N})$ ($\mathbf{N} := \{0, 1, \dots\}$ be a sequence of integers each of them not less than 2. Let Z_{m_k} denote the discrete cyclic group of order m_k. Let G_m be the complete direct product of the groups $Z_{m_k} (k \in \mathbf{N})$. If the sequence m is a bounded one, then we call G_m a bounded Vilenkin group. Otherwise, G_m is an unbounded one.
The characters of G_m, $\psi := (\psi_n : n \in \mathbf{N})$ is called a Vilenkin system. Define the nth Fejér means as follows

$$\sigma_n f := \frac{1}{n} \sum_{k=0}^{n-1} S_k f,$$

where $S_k f$ is the kth partial sum of the Fourier series of f. One of the most celebrated problems in dyadic harmonic analysis is the pointwise convergence of the Fejér means of functions on one and two-dimensional unbounded Vilenkin groups. The methods known in the trigonometric or in the Walsh, bounded Vilenkin case are not powerful enough. One of the main problems is that the proofs on the bounded Vilenkin groups heavily use the fact that the L^1 norm of the Fejér kernels are uniformly bounded. This is not the case if the group G_m is an unbounded one.

The aim of this talk is to give a résumé of the recent developments concerning this matter both in the point of view of one and two dimensional cases, convergence and divergence. Also will have a look into some convergence and divergence results of other summation methods.

Ágota P. Horváth (Budapest)

Abel-summation, Dirichlet-problem, Biorthonormal Systems with Singularities

Investigating the connection of the weighted norm of the Hardy-Littlewood maximal function with the weighted norm of the original function the following question arised by Benjamin Muckenhoupt in 1972 : There is an orthonormal system ($\{\varphi_n\}$) in a space/ with respect to a weight w on $[0, 2\pi)$, and there is another weight u on the same interval. The Poisson integral of a function f is defined by $P_r(f, x) = \sum_n r^n a_n(f)\varphi_n$, where $a_n(f)$-s are the Fourier coefficients of f with respect to w. The question is the following: Under what conditions will this Poisson integral converge to the function (with $r \to 1-$) according to the weighted norm with u? B. Muckenhoupt gave the answere in two cases: in the trigonometric case (that is $w \equiv 1$), and in ultraspheric, or Gegenbauer case ($w(\theta) = \sin^{2\lambda}(\theta)$). In these cases the necessary and sufficient condition was that u had to fulfil the A_p- or the weighted A_p-condition.

If u is an A_p-weight, then u may has only "week" zeros. The whole situation changes, when u has "strong" zeros, like $\sin^k \frac{x-x_0}{2}$. On the trigonometric system the question was generalized (in this direction) by Kazaros S. Kazarian in 1987 . Developing the multiplicative completion method of R. P. Boas and H. Pollard, he gave a method for giving the fundamental system in the weighted space with respect to u with "strong" zeros, and for giving the modified Poisson kernel here. Roughly speaking the new system (with respect to u) can be get from the old one by deleting some consecutive φ_n−s, and the number of the members has to be deleted depends on the zeros of u. The characterization of the existence of the solution of Dirichlet's problem in a weighted L^p-space on the unite disk was given also by K. S. Kazarian.

A sufficient condition for the similar problem in the continuous case was given by K. S. Kazarian and Á. P. H. in 2006.

In 2007 a sufficient condition for Abel-summability was given on the real line (Á. P. H.), when the number of zeros of u is finite.

This talk will deal with the case of infinitely many zeros. In this case the above-mentioned "removing procedure" implies an interpolation problem on infinitely many nodes. Besides of the existence, the convergence of the solution is also a quetion.

Philippe Jaming (Orléans)

The phase retrieval problem for the Fractional Fourier Transform

he Fractional Fourier Transform (FrFT) is a one-parameter family of transforms on $L^2(\mathbb{R})$ defined by

$$\mathcal{F}_\theta f(\xi) = c_\theta e^{-i\pi|\xi|^2 \cot\alpha}\mathcal{F}[e^{-i\pi|\cdot|^2 \cot\alpha}u](\xi/\sin\alpha).$$

where c_α is a normalisation constant and $\mathcal{F}$ is the usual Fourier transform. This transform appears in optics (Fresnel diffraction and so-called ABCD systems) and is also, up to renormalization, a solution of the Free Shrödinger equation. Moreover, as $\mathcal{F}_0 f = f$, $\mathcal{F}_{\pi/2} f = \mathcal{F} f$ and $\mathcal{F}_{\theta+\theta'} = \mathcal{F}_\theta\mathcal{F}_{\theta'}$, the parameter θ can be seen as an "angle" in the time-frequency plane.

The problem that we study here comes from optics where one is only able to measure $|\mathcal{F}_\theta f|$. More precisely, we will explore the following problem

Phase Retrieval Problem (PRP). *Let us fix a set of angles $\tau \subset [0,\pi/2]$ and a class of functions $\mathcal{C} \subset L^2(\mathbb{R})$. Assume that $f, g \in \mathcal{C}$ are such that $|\mathcal{F}_\theta f| = |\mathcal{F}_\theta g|$ for every $\theta \in \tau$. Is there a constant c such that $g = cf$? (And ideally design an algorithm to construct f from the data $|\mathcal{F}_\theta f|$, $\theta \in \tau$.)*

When $\tau = \{0, \pi/2\}$ this problem dates back to Pauli and negative answers are obtained in various classes of functions.

We will give a positive answer to the PRP when $\tau = [0, \pi/2]$ and $\mathcal{C} = L^2(\mathbb{R})$. When $\mathcal{C} = PW_K(\mathbb{R})$ is the set of functions with Fourier transform supported in a (fixed) compact set K, we will show that a conveniently chosen discrete set leads to a positive answer. Further, for $\tau = \{0, \theta\}$ with θ well chosen, the classes in which negative answers for the Pauli problem were found now lead to positive solutions!

Tamás Keleti (Budapest)

Continuous Besicovitch sets in $\mathbb{R}^n$ (joint work with Esa Järvenpää, Maarit Järvenpää, András Máthé)

In 1920 Besicovitch constructed a compact set of Lebesgue measure zero in the plane that contains a unit line segment in every direction. Later he showed that some modification answers the question Kakeya asked in 1917:

there exists a planar set of arbitrarily small area within which a unit line segment can be continuously rotated by a full rotation. It was already mentioned by Besicovitch that one cannot rotate the unit segment continuously within a planar set of Lebesgue measure zero but a proof of it has been published only recently by T. Tao on his web page.

We study the question whether this continuous rotation of a segment inside a set of measure zero can be done in higher dimension. We also try to parametrize continuously unit segments (or even full lines) in $\mathbb{R}^n$ by their direction so that we get segments/lines in every direction but their union is small.

Sergei Konyagin (Moscow)
On convergence of greedy approximations for the trigonometric system

We study in this the following nonlinear method of summation of trigonometric Fourier series. Consider a periodic function $f \in L_p(\mathbb{T})$, $1 \le p \le \infty$, ($L_\infty(\mathbb{T}) = C(\mathbb{T})$), defined on the torus $\mathbb{T}$. Let a number $m \in \mathbb{N}$ be given and Λ_m be a set of $k \in \mathbb{Z}$ with the properties:

$$\min_{k\in\Lambda_m} |\hat{f}(k)| \ge \max_{k\notin\Lambda_m} |\hat{f}(k)|, \quad |\Lambda_m| = m,$$

where

$$\hat{f}(k) := (2\pi)^{-1} \int_{\mathbb{T}} f(x) e^{-ikx} dx$$

is a Fourier coefficient of f.

We define

$$G_m(f) := S_{\Lambda_m}(f) := \sum_{k\in\Lambda_m} \hat{f}(k) e^{i(k,x)}$$

and call it an m-th greedy approximant of f with regard to the trigonometric system $\mathcal{T} := \{e^{i(k,x)}\}_{k\in\mathbb{Z}}$. We discuss convergence of greedy approximants $S_{\Lambda_m}(f)$ to f as $m \to \infty$.

Péter Maga (Budapest)
Two aspects of the largeness of sets

In measure theory, we can measure a set using for example Lebesgue measure or Hausdorff dimension. From a geometrical point of view transformations can be used to measure sets: Does the set contain every finite set as a pattern? How many translated copies of the set can cover the space? On the one hand, there are connections: a set of positive Lebesgue measure contains the similar copy of any finite set; the space cannot be covered with countably many copies of a Lebesgue nullset. On the other hand, there are differences: it can happen to a set that its complement is a Lebesgue nullset and the space cannot be covered with countably many copies of it. This talk is about to answer these questions and similar ones. What happens, if we measure the size in dimension? Are there 2 dimensional compact sets on the plane that do not contain the vertices of any regular triangle? If a set is of zero dimension, does it follow that the space can be covered only with continuum many copies of it?

Máté Matolcsi (Budapest)

Improved bounds on the supremum of autoconvolutions

Investigations of the maximal possible cardinality of g-Sidon sets in $[1,N]$ lead to the following purely analytic problem: given a nonnegative function f with integral 1, supported on the interval $[-\frac{1}{4},\frac{1}{4}]$, what lower bounds can we give on the supremum of $f*f$? We will describe the best existing lower bound to date, as well as a surprising failure of a conjecture of Schinzel and Schmidt on the extremal function.

Lozko Milev (Sofia)

Strictly definite extreme points of the unit ball in a polynomial space

Let Δ be the standard simplex in $\mathbb{R}^2$, i.e. the triangle bounded by the lines $x=0,\ y=0,\ x+y=1$. Denote by π_2 the set of all real bivariate algebraic polynomials of degree at most two. Let $\pi_2(\Delta)$ be the space of the polynomials $f\in\pi_2$, endowed with the supremum norm on Δ.

We give a full description of the strictly definite extreme points of the unit ball of $\pi_2(\Delta)$.

Ferenc Móricz (Szeged)

On the uniform convergence of double sine integrals over $\mathbb{R}^2_+$

We investigate the convergence behavior of the family of double sine integrals of the form

$$\int_0^\infty \int_0^\infty f(x,y) \sin ux \sin vy dx dy, \quad \text{where} \quad (u,v) \in \mathbb{R}_+^2 := \mathbb{R}_+ \times \mathbb{R}_+,\ \mathbb{R}_+ := (0,\infty),$$

and f is a monotonically nonincreasing function. We give necessary and sufficient conditions for the uniform convergence of the 'remainder' integrals $\int_{a_1}^{b_1} \int_{a_2}^{b_2}$ to zero in $(u,v) \in \mathbb{R}_+^2$ as $\max\{a_1, a_2\} \to \infty$, where $b_j > a_j \geq 0$, $j = 1, 2$ (called uniform convergence in the regular sense). This implies the uniform existence of the finite limits of the partial integrals $\int_0^{b_1} \int_0^{b_2}$ in $(u,v) \in \mathbb{R}_+^2$ as $\min\{b_1, b_2\} \to \infty$ (called uniform convergence in Pringsheim's sense).

Our basic tool is the second mean value theorem for certain double integrals over a rectangle.

Key words: regular convergence of double integrals over $\mathbb{R}_+^2$, convergence in Pringsheim's sense, monotonically nonincreasing functions on $\mathbb{R}_+^2$, integration by parts for double Riemann-Stieltjes integrals, second mean value theorem for double integrals, double sine integrals over $\mathbb{R}_+^2$, uniform convergence, uniform boundedness.

Károly Nagy (Nyíregyháza)

Approximation by Nörlund means of Walsh-Fourier series

In this talk we would like to investigate the rate of the approximation by Nörlund means of Walsh-(Kaczmarz-)Fourier series of a function in L^p ($1 \leq p \leq \infty$). We will investigate the rate of the approximation by this means, in particular, in $\mathrm{Lip}(\alpha, p)$, where $\alpha > 0$ and $1 \leq p \leq \infty$. In case $p = \infty$ by L^p we mean C_W, the collection of the uniformly W-continuous functions.

In special cases, we obtain the earlier result by Skvortsov [2]. Earlier results on the Walsh-Fourier series was given by Móricz and Siddiqi [1].

After this we define the Nörlund means of cubical partial sums of Walsh-(Kaczmarz)-Fourier series of a function in L^p.

Our main theorems state that the approximation behavior of the two-dimensional Walsh-(Kaczmarz)-Nörlund means defined by us is soo good as the approximation behavior of the one-dimensional Walsh-(Kaczmarz)-Nörlund means.

REFERENCES

[1] F. Móricz, A. Siddiqi, *Approximation by Nörlund means of Walsh-Fourier series*, Journal of Approx. Theory 70 (3) (1992), 375-389.

[2] V.A. Skvortsov, *On Fourier series with respect to the Walsh-Kaczmarz system*, Analysis Math. 7 (1981), 141-150.

Nikola Naidenov (Sofia)

Indefinite extreme points of the unit ball in a polynomial space

Let Δ be the triangle in $\mathbb{R}^2$ bounded by the lines $x = 0,\ y = 0,\ x + y = 1$ (the standard simplex in $\mathbb{R}^2$). We denote by π_2 the set of all real bivariate algebraic polynomials of total degree at most two. Let $\pi_2(\Delta)$ be the space of the polynomials $f \in \pi_2$, endowed with the supremum norm on Δ.

We describe the set of all indefinite extreme points of the unit ball of $\pi_2(\Delta)$.

Stefan Neuwirth (Besancon)

Transfer of Fourier multipliers into Schur multipliers

We will review well-known techniques and a new theorem that permit to associate a Toeplitz-Schur multiplier to a Fourier multiplier, and vice versa, and provide some applications.

Alexander Olevszkii (Tel Aviv)

Wiener's "Closer of Translates" problem and Piatetski's uniqueness phenomenon (joint work with Nir Lev)

Wiener characterized the cyclic vectors (with respect to translations) in lp(Z) and Lp(R) ,p= 1,2, in terms of the zero set of Fourier transform. He conjectured that a similar characterization should be true for 1¡p¡2. I'll discuss this conjecture.

Szilárd Gy. Révész (Budapest)

On the notion of uniform asymptotic upper density on LCA groups

If $S \subset \mathbb{R}$ is a sequence, then its uniform asymptotic upper density (u.a.u.d. for short) is

$$\overline{D}(S) := \limsup_{r\to\infty} \sup_{x\in\mathbb{R}} \frac{\#\{s \in S\ :\ |s - x| \leq r\}}{2r}.$$

Actually, the lim sup is a limit, for the quantity "essentially decreases".

The origin of the notion goes back to Pólya and Beurling. It was used first in function theory, but later on many other applications emerged. Some people call $\overline{D}(S)$ "Banach density". A typical area of application is related to investigation of differences: for if $\overline{D}(S) > 0$, then $S - S$ has positive uniform lower density, and its asymptotic lower density is $\delta(S - S) \geq \overline{D}(S)$.

Note a slight "ambiguity": sometimes in the continuous situation of $\mathbb{R}^d$, a set A of positive measure is considered in place of a sequence, and then the definition of the asymptotic upper density is, with K the unit ball or box,

$$\overline{D}(A) := \limsup_{r\to\infty} \frac{\sup_{x\in\mathbb{R}^d} |A \cap (rK + x)|}{|rK|} \ .$$

That motivates the general formulation: u.a.u.d. of *measures*, say measure ν with respect to measure μ. For keeping the notion translation invariant, μ is always the normalized Haar measure, however. E.g. in the discrete case $\nu := \#|_S$ is the counting measure of S, while $\mu := |\cdot|$ is just the volume.

The general formulation in $\mathbb{R}^d$ (or $\mathbb{Z}^d$) is thus

$$\overline{D}(\nu) := \limsup_{r\to\infty} \frac{\sup_{x\in\mathbb{R}^d} \nu(rK + x)}{|rK|} \ . \tag{1}$$

We want to extend this notion to general locally compact Abelian groups. Although the extension looks a bit surprising, it works quite well and several classical results referring to the notion of u.a.u.d. extends to L.C.A. groups.

Imre Z. Ruzsa (Budapest)

Sqares, cubes, and positive cosine polynomials

We discuss the possibility of forming positive cosine polynomials with possibly small constant term and using only (a) squares, (b) cubes as frequencies. These cases seem to behave differently.

László Székelyhidi (Debrecen)

Fourier series of mean periodic functions

Mean periodic functions are generalizations of periodic functions. In early works of Carleman and Kahane different transforms have been introduced for this class of functions in order to generalize classical Fourier series and Fourier transforms. In this talk we introduce a Fourier transform for mean periodic functions and exhibit its connections with other classical transforms.

Rodolfo Toledo (Nyíregyháza)

The maximal value of Dirichlet kernels with respect to representative product systems

The Walsh-Paley system is formed by the characters of the dyadic group, i.e., the complete product of the discrete cyclic group of order 2 with the product of topologies and measures. Vilenkin in 1947 generalized this structure studying the complete product of arbitrary cyclic groups. In Vilenkin groups the order of the cyclic groups appeared in the product can be unbounded. A natural generalization of the Vilenkin groups is the complete product of arbitrary groups, non necessarily commutative groups. In this case we use representation theory in order to obtain orthonormal systems, taking the finite product of the normalized coordinate functions of the continuous irreducible representations appeared in the dual object of the finite groups. These systems are named representative product systems.

In this talk I deal with the differences between a Vilenkin system and a general representative product system based in the comportment of Dirichlet kernels.

Jim Wright (Edinburgh)

The Fourier restriction problem in rings of integers

We explore the Fourier restriction phenomenon in the setting of the ring of integers modulo N for general N and observe a striking similarity with the corresponding euclidean problem. One should contrast this with known results in the finite field setting.