On a Hamiltonian Problem For Triple Systems

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(substituted by M. Schacht)

UAM Poznań and Emory University

joint work with
V. Rödl, M. Schacht and E. Szemerédi

Erdős Centennial Conference
Dirac-type questions

Theorem (Dirac 1952)

If an $n$-vertex graph $G$ with $n \geq 3$ satisfies $\delta(G) \geq n/2$, then $G$ is Hamiltonian.

Main Question

How to extend this result to $k$-uniform hypergraphs?

Problems:

What is a (Hamiltonian) cycle in a hypergraph?

What replaces minimum degree in hypergraphs?
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- $k$-uniform hypergraph $H = (V, E)$, i.e., $E \subseteq \binom{V}{k}$
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- minimum vertex degree $\delta_1(H)$
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Remarks:

$\ell = 0 \rightarrow$ perfect matchings

Bollobás, Daykin & Erdős 1976

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$h(3,0)$

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Given integers $k$, $\ell$, and $d$ determine the function $h_d^{(k,\ell)}(n)$ with the property

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for any $n$-vertex $k$-uniform hypergraph $H$. 

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Some known results

Theorems \((d = 2, n \text{ large})\)

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New bound (work in progress) \(h_2(n) \lesssim 4/5 n^2\)

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    - A. Ruciński (UAM Poznań & Emory)
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Hamiltonian cycles in 3-graphs

July 2013
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### New bound (work in progress)

\[
h^2_1(n) \lesssim \frac{4}{5} \binom{n}{2}
\]
Where do we stand?

New bound (work in progress)

\[
\frac{h}{1^n} \lesssim \frac{4}{5^{n/2}}
\]

Conjecture

\[
\frac{h}{1^n} \sim \frac{h}{0^n} \sim \frac{n}{4} \sim \text{"min. pair degree for matchings of size } \frac{n}{4}\text{"}
\]

\[
\frac{h}{1^n} \sim \frac{7}{16^{n/2}} \sim \text{"min. vertex degree for matchings of size } \frac{n}{4}\text{"}
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New bound (work in progress)

\[ h^2_1(n) \lesssim \frac{4}{5} \binom{n}{2} \]

Some evidence:

\[ h^2_2(n) \lesssim \frac{1}{2} \binom{n}{2} \]

\[ h^1_1(n) \lesssim \frac{7}{16} \binom{n^2}{2} \]

“min. pair degree for matchings of size \( n/4 \)”
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\[ h_1^2(n) \lesssim \frac{4}{5} \binom{n}{2} \]

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\[ h_1^2(n) \sim \frac{5}{9} \binom{n}{2} \]
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- $h_2^2(n) \sim h_2^0(n) \sim n/2$
- $h_2^1(n) \sim n/4$ “min. pair degree for matchings of size $n/4$”
- $h_1^1(n) \sim \frac{7}{16} \binom{n}{2}$ “min. vertex degree for matchings of size $n/4$”
Lower bound construction

Suppose $3|n$ and $|X| = n/3 - 1$
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- $e \in E(H) \iff e \cap X \neq \emptyset$
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$\Rightarrow \delta_1(H) \sim \binom{n}{2} - \binom{2n/3}{2} \sim \frac{5}{9} \binom{n}{2}$,
Lower bound construction

- Suppose $3|n$ and $|X| = n/3 - 1$
- $e \in E(H) \iff e \cap X \neq \emptyset$
- $\delta_1(H) \sim \binom{n}{2} - \binom{2n/3}{2} \sim \frac{5}{9}\binom{n}{2}$, but $H$ contains no perfect matching
Lower bound construction with perfect matching

Suppose $3|n$ and $|X| = n/3 + 1$
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$$\Rightarrow$$

- every edge of a $C^2_n$ intersects $Y$ in at least two vertices
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$\Rightarrow$ every edge of a $C_n^2$ intersects $Y$ in at least two vertices.
Absorbing method

1. Find an absorbing path $A$ in $H$ with $|V(A)| = c_1 n$.

2. Find almost Hamiltonian cycle $C$ containing $A$.

3. Apply absorbing property of $A$ to $U = V \setminus V(C)$ and obtain Hamiltonian cycle.
Absorbing method

1. Find an absorbing path $A$ in $H$ with $|V(A)| = c_1 n$:
   - $\forall U \subseteq V \setminus V(A)$ with $|U| \leq c_2 n \ll c_1 n$
   - $\exists$ path $A_U$ with the same endpairs and $V(A_U) = V(A) \cup U$.

2. Build a long cycle $C = T_m \supset A$, $m \geq n - c_2 n$

3. Apply the absorbing property of $A$ to $U = V \cap V(T_m)$ obtaining a Hamiltonian cycle $T_n$.
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Finding absorbers with $\delta_2(H) \geq (1/2 + \varepsilon)n$

Fact 1
For all $v \in V$ there are $\Omega \varepsilon(n^4)$ 4-tuples $(a, b, c, d)$ such that $abc, bcd, abv, bvc, vcd \in H$.

Proof: there are $\geq n \times n/2$ choices of $b, c$ and $\geq (2\varepsilon n)^2$ choices of $a, d$. 
Finding absorbers with $\delta_2(H) \geq (1/2 + \varepsilon)n$

Fact: For every $v \in V$ there are $\varepsilon^2 n^4$ absorbers $(a, b, c, d)$. 
1 Randomly select $\gamma n$ from all 4-tuples
Absorbing path

1. Randomly select $\gamma n$ from all 4-tuples
2. Select a pairwise disjoint subset of those 4-tuples forming paths
   $\Rightarrow$ w.h.p. for every $v \in V$ at least $\gamma^4 \varepsilon^2 n$ absorber were selected
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2. Select a pairwise disjoint subset of those 4-tuples forming paths $\Rightarrow$ w.h.p. for every $v \in V$ at least $\gamma^4 \epsilon^2 n$ absorber were selected
3. Connect the selected 4-tuples $P_i$ to obtain the path $A$
Some ideas

- remove hyperedges from $H$, that contain pairs $(x, y)$ with
  $\deg_H(x, y) \leq (1/2 + \varepsilon)n$
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\[ \leftarrow \text{requires } \delta_1(H) \geq \frac{5 - \sqrt{5}}{3} \binom{n}{2} \]
Some ideas

- remove hyperedges from $H$, that contain pairs $(x, y)$ with $\deg_H(x, y) \leq (1/2 + \varepsilon)n$

  $\leftarrow$ requires $\delta_1(H) \geq \frac{5-\sqrt{5}}{3} \binom{n}{2}$

- slightly more careful, remove only hyperedges which contain no pair of high degree
Some ideas

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- slightly more careful, remove only hyperedges which contain no pair of high degree

- balance between “finding absorbers” and “making connections between large pairs”
Questions