# Approximating the jump number of interval orders using 3-set cover algorithms

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When certain job scheduling problems are modeled with posets, a need arises to compute the jump number of a given partial order $P$. That means, a linear extension for $P$ is to be found, which minimizes the number of incomparable adjacent elements. This problem is also interesting from theoretical point of view. The computational complexity status has been settled by Pulleyblank [7], who established its NP-hardness in general case. We are considering a restricted, although still rich subclass formed by interval orders [4], in which the problem remains NP-complete, but admits a constant-ratio polynomial-time approximation algorithms.

The purpose of this work is to improve the approximation ratio, which is equal to 3/2 in each of three different algorithms, published by Felsner [3], Sysło [8], and Mitas [6]. We exploit the approach proposed by Mitas, where the problem has been reduced to packing vertex-disjoint edges and odd cycles in a graph based on the canonical representation of an interval order. A cycle of higher weight implies more saved bumps (i.e., comparable adjacent elements). The algorithm of Cornuéjols et al. [1] has been applied to approximate saved bumps, which maximizes the number of vertices in packed edges and cycles (so it does not take into account the weigths associated with cycles).

We propose to consider a dual approach, that is: to cover all vertices of obtained graph with edges or odd cycles (not necessarily disjoint). We show that such covering may be transformed into a proper linear extension of a given poset, and that the number of paid bumps is equal to the total weight of a covering. We further cast this graph covering problem as a set covering problem. The minimum number of paid bumps may be then approximated by the semi-local optimization algorithm of Duh and Fürer [2] for 3-set covering. Our approach is justified by the observation that the covering problem instances arising from interval orders are of particular nature. Pairs and triples (or triangles in a graph) always cost 1, the price for a pentagon is 2, and so on. We are left with an open question of how well the optimum covering by cycles of all sizes can be approximated using only pairs and triples. We have conducted a series of computer experiments which have shown that the proposed algorithm performs considerably better than the previously known ones (see [5]).

## References

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