On the tail behaviour of the distribution function of the maximum for the partial sums of a class of i.i.d. random variables

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We take an L_1-dense class of functions $\mathcal{F}$ on a measurable space $(X, \mathcal{X})$ together with a sequence of independent, identically distributed X-space valued random variables $\xi_1, \dots, \xi_n$, and give a good estimate on the tail distribution of $\sup\limits_{f \in \mathcal{F}} \sum\limits_{j=1}^{n} f(\xi_j)$ if $\sup\limits_{x \in X} |f(x)| \le 1$, $Ef(\xi_1) = 0$, and $Ef(\xi_1)^2 \le \sigma^2$ with a number σ^2 for all $f \in \mathcal{F}$. Informally saying our estimate states that $\sup\limits_{f \in \mathcal{F}} \sum\limits_{j=1}^{n} f(\xi_j)$ is not much larger than the greatest term in the supremum. To get a more precise description of this result we have to understand better the content of some classical results like Bernstein's or Bennet's inequality about the tail distribution of partial sums of independent random variables.

The proof of this result may be interesting for its own sake. Generally, we expect that the partial sums of independent random variables are asymptotically Gaussian, hence a refinement of the methods worked out for the study of Gaussian random variables may be useful in their investigation. But there are cases when the 'Gaussian arguments' do not work, and we have to find new methods. In the study of the problem considered in this talk we get into such a situation.